

Reduced Error Model for Learning-based Calibration of Serial Manipulators

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Keywords: Modeling, Parameter Identification, Calibration.

Abstract: In this work a reduced error model for a learning-based robot kinematic calibration of a serial manipulator is compared with a complete error model. To ensure high accuracy this approach combines the geometrical (structural inaccuracies) and non-geometrical influences like for e.g. elastic deformations that are configuration-dependent without explicitly defining all underlying physical processes that contribute to positioning inaccuracies by using a polynomial regression method. The proposed approach is evaluated on a dataset obtained using a 7-DOF manipulator KUKA LBR iiwa 7. The experimental results show the reduction of the mean Cartesian error up to 0.16 mm even for a reduced error model.

1 INTRODUCTION

For a robot manipulator that is mainly used in repetitive applications (e.g. pick-and-place operations) where the desired poses (position and orientation) of the manipulator's end-effector (EE) can be manually taught, high repeatability is important to successfully perform defined tasks. This ability to repeat a known pose has submillimeter values for modern manipulators. However if a task is unique, the robot is mostly given a target pose defined in some relative or absolute coordinate system. Such situations arise often when robot's poses are obtained through a simulation during which the layout of the working environment and a model of the robot are used. This requires special attention to the accuracy of the simulated robot model, and whether it corresponds to the actual kinematics of the robot.

The process of robot calibration that consists of developing a mathematical model and identification of parameters that are able to reflect the actual behavior of the investigated robot can be divided into three categories (Elatta et al., 2004). The first is joint calibration, which is also called first level calibration, where the difference between the actual joints displacements and the encoder signals is considered. Level two involves kinematic calibration, where the robots kinematic parameters are determined. Level three takes into account non-kinematic error sources like elasticity of the links or the backlash of the joints.

The main sources for positioning inaccuracies

can be divided into geometric and non-geometric errors. Geometric errors are present when nominal kinematic parameters of the robot do not correspond to actual parameters due to for ex. manufacturing errors.

Non-geometric errors include among others the link and joint compliance, elastic deformations, transmission nonlinearities, and thermal expansion. To include these effects into the robotic model the underlying processes can be expressed with gear or elastic models as in (Klimchik et al., 2015) or (Marie et al., 2013). Considering all sources that can potentially contribute to the end-effector positioning errors, it is difficult to model all relevant parameters explicitly.

Instead, learning-based approaches are proposed to model the behaviour of a robot. This allows us to solve the calibration problem as a function estimation task based on the measured data. In general errors of the robot's EE may originate from five factors (Liou et al., 1993): environmental (e.g. temperature or the warm-up process), parametric (e.g. Kinematic parameter variation due to manufacturing and assembly errors, influence of dynamic parameters, friction and other nonlinearities), measurement (resolution and nonlinearity of joint position sensors), computational (computer round-off and steady-state control errors) and application (e.g. installation errors). To be able to reflect different influences in the model, experiments that include variations of the relevant factors would have to be conducted. This work considers the influence of only a limited amount of

described factors, mainly concentrating on the internal parameters of the robot. There is a number of calibration approaches that use joint-dependent errors, but they often rely on complicated models with a big number of parameters (Ma et al., 2018).

In Section 2, a full and reduced error model is presented. Regression analysis is then used to find the values of the introduced error parameters. In Section 3, we applied this method to the performance evaluation of the KUKA LBR iiwa 7 manipulator to compare and analyze the two error models. We conclude our paper in Section 4.

2 JOINT-DEPENDENT ERROR MODELING

In this section, we describe a method used to include relevant processes influencing position and orientation of the end-effector in the modeling of a robot. First, D-H representation and generalized error modeling is used to establish the transformation from the base frame of the robot to the flange in presence of errors. Then, redundant error parameters are eliminated and regression analysis is used to include the joint-dependent influences.

2.1 Robot Kinematic Model

D-H parameters are used to describe the kinematics of a manipulator. The position and orientation of reference frame A_i with respect to the previous frame A_{i-1} is defined by a homogeneous transformation that depends on the geometric parameters of the manipulator: skew angles α_i , link lengths a_i , joint offsets d_i and joint angle offsets θ_i ($c_\alpha := \cos(\alpha)$, $s_\alpha := \sin(\alpha)$ for compact notation):

$$A_i = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

The transformation A_T from the base to the end-effector of a robot with n joints is then obtained by:

$$A_T = \prod_i^n A_i \quad (2)$$

2.2 Generalized Error Model

To include differences between the ideal robot and the actual one, the kinematic model represented by (2) can be extended by translational and rotational error parameters. The introduction of these errors leads to

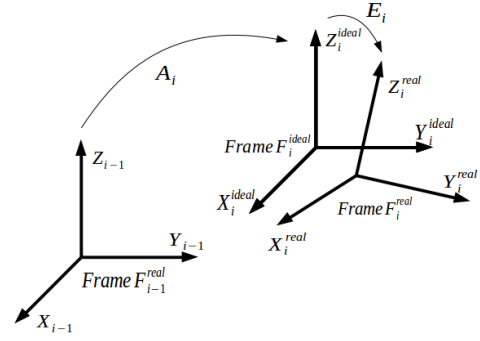


Figure 1: Frame transformations in presence of errors.

displacement of joint frames from their nominal locations as shown in Figure 1. For joint i this difference in frames can be represented by a homogeneous matrix E_i with 6 error parameters: $e = (e_{i1}, \dots, e_{i6})$. The rotational part of matrix E_i consists of e_{i4} , e_{i5} , e_{i6} , which denote rotation about X, Y and Z axes with respect to A_i . The e_{i1} , e_{i2} , e_{i3} represent translation in X, Y and Z direction respectively.

These errors for modern manipulators are expected to be small, so that the generalized error for a frame A_i can be expressed as:

$$E_i = \begin{bmatrix} 1 & -e_{i6} & e_{i5} & e_{i1} \\ e_{i6} & 1 & -e_{i4} & e_{i2} \\ -e_{i5} & e_{i4} & 1 & e_{i3} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

Considering the introduced error parameters, the general transformation model A_E in presence of translational and rotational errors for a robot with n joints, is given by:

$$A_E = \prod_i^n A_i E_i \quad (4)$$

When analyzing the effect rotational and translational components of the error parameters have on the resulting error of the EE , it is clear that e_{i4} , e_{i5} and e_{i6} have much greater influence. Taking this into account we can also further investigate the extend of these influences with the following error models:

1. Only rotational errors are present, resulting in $3 \times n$ error parameters (*RotXYZ-error model*):

$$E_i = \begin{bmatrix} 1 & -e_{i6} & e_{i5} & 0 \\ e_{i6} & 1 & -e_{i4} & 0 \\ -e_{i5} & e_{i4} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

2. Errors in translational as well as in rotational parts, corresponding to the most general case as in (3), leading to $6 \times n$ error parameters (*RotXYZ-TransXYZ-error model*).

2.3 Error Parameters Calculation

Now that we defined error models, the introduced error parameters should be estimated based on the experimental data. The nominal pose of the end-effector defined by (4) should be as close as possible to the measured reference pose $\mathbf{p}_{iT} \in \mathbf{P}_T$. It can be written in general form as:

$$\mathbf{p}_{iT} = \mathbf{f}_e(\mathbf{e}), \quad (6)$$

where \mathbf{f}_e is a non-linear function of error parameter vector \mathbf{e} . Since these errors are small, this function can be linearized at $\mathbf{0}$. If the difference between nominal position and measured position is Δp_i , for l measurements:

$$\Delta p = \begin{bmatrix} \Delta p_1 \\ \vdots \\ \Delta p_l \end{bmatrix} = \begin{bmatrix} \mathbf{J}_{e1} \\ \vdots \\ \mathbf{J}_{el} \end{bmatrix} \mathbf{e} = \mathbf{J}_e \mathbf{e} \quad (7)$$

where \mathbf{J}_e is a Jacobian function of \mathbf{f}_e with respect to the elements of error vector \mathbf{e} , evaluated at $\mathbf{0}$.

Under assumption that introduced error parameters are constant the above equation can be solved with for example a least squares technique, in which case the solution would be of the form:

$$\mathbf{e} = (\mathbf{J}_e^T \mathbf{J}_e)^{-1} \mathbf{J}_e^T \Delta p \quad (8)$$

where $(\mathbf{J}_e^T \mathbf{J}_e)^{-1} \mathbf{J}_e^T$ is a left pseudo-inverse matrix of \mathbf{J}_e . But considering the need to include the non-geometric influences (which are configuration dependent and as result can not be constant) into developed model, this approach can no longer be used. In this case we can consider each measurement individually:

$$\Delta p_i = \mathbf{J}_{ei} \mathbf{e} \quad (9)$$

After solving (9) for every measured-nominal point pair, all introduced error parameters can be calculated.

2.4 Indistinguishable Error Parameters

Some generalized errors from link $i - 1$ contribute to the same EE pose errors as errors from link i and their individual influence can not be distinguished. To remove this redundancy from the model an analytical approach described in (Meggiolaro and Dubowsky, 2000) can be used. For each link i following combinations have the same effect on the position or orientation of the end-effector:

$$\begin{aligned} e_{i2} &= e_{i-13} s \alpha_i = e_{i-16} a_i c \alpha_i \\ e_{i3} &= e_{i-13} c \alpha_i = -e_{i-16} a_i s \alpha_i \\ e_{i5} &= e_{i-16} s \alpha_i \end{aligned}$$

$$e_{i6} = e_{i-16} c \alpha_i. \quad (10)$$

If joint i is prismatic additional combinations are present:

$$\begin{aligned} e_{i1} &= e_{i-11} \\ e_{i2} &= e_{i-12} c \alpha_i = e_{i-13} s \alpha_i = e_{i-16} a_i c \alpha_i \\ e_{i3} &= -e_{i-12} s \alpha_i = e_{i-13} c \alpha_i = -e_{i-16} a_i s \alpha_i \\ e_{i5} &= e_{i-16} s \alpha_i \\ e_{i6} &= e_{i-16} c \alpha_i. \end{aligned} \quad (11)$$

When only positional part of the end-effector pose is considered and the last joint n is revolute and $a_n = 0$:

$$\begin{aligned} e_{(n-1)1} &= e_{(n-1)5} d_n \\ e_{(n-1)2} &= -e_{(n-1)4} d_n. \end{aligned} \quad (12)$$

The above equations can now be used to eliminate the redundant error parameters depending on the structure of the manipulator. It should be noted that apart from this approach that studies the structural properties of the manipulator a numerical approach can also be used. But considering that it needs to be carried out for every data point pair, an analytical method is preferred.

2.5 Error Parameters Modeling

After calculating the values of the error parameters for every measurement and eliminating the redundant parameters, an estimator based on the obtained data can be constructed and error parameters can be modeled as functions of joint variables. For this, the available experimental measurements of the position of EE are first divided into training and testing sets. The training set is used to determine the regression coefficients of the model and for tuning model parameters, and performance evaluation is done on the test set.

2.5.1 Regression Analysis

As input parameters, the joint configurations of the manipulator were chosen. In order to model nonlinear relationships, they were also extended with combinations of different joint configurations with a polynomial degree of up to 4. Even if higher polynomial degrees can better model some functions we should avoid overfitting. To prevent features that have big variance from dominating the objective function, all input variable were centered and scaled. Ridge regression was chosen for modeling error parameters of the manipulator, because it uses regularization by minimizing a penalized residual sum of squares. This approach is effective in case of highly correlated input variables.

To determine the regularization parameters of the models m -fold cross-validation (CV), during which the training set is split into m smaller sets, is used. Then for each of these sets a model is trained using $m - 1$ of the remaining sets, and the resulting model is validated on the remaining part of the set. The resulting performance measure of the m -fold CV is calculated as the average of the values computed in the loop. Those parameters that result in the smallest CV error are chosen for model training. To evaluate the performance of regression models, R^2 metric (the coefficient of determination) was used.

It should also be noted that for the resulting model the rotational part of the estimated resulting matrix can no longer be a rotation matrix. Therefore, the obtained values should go through an additional orthogonalization procedure to project the rotational part to the closest rotation matrix. If the unprojected rotational part of the matrix is denoted as $\tilde{\mathbf{R}}$ it can be singular-value-decomposed as:

$$\tilde{\mathbf{R}} = \mathbf{U}\mathbf{S}\mathbf{V}^T \quad (13)$$

Then the closest rotation matrix to $\tilde{\mathbf{R}}$ would be given by (Horn et al., 1988):

$$\mathbf{R} = \text{sign}(\det(\mathbf{S}))\mathbf{U}\mathbf{V}^T \quad (14)$$

3 EVALUATION

To show the validity of the proposed approach the modeled values were compared with values measured from a real robot. A 7DOF LBR iiwa, a lightweight robot with non-linear influences in the structure was used in the experiments.

3.1 Experimental Setup

In order to compare considered input parameters, resulting models are evaluated on an example of KUKA LBR iiwa 7 robot. The position of the EE was measured using a FARO laser tracking system. The accuracy of the laser tracker is up to 0.015 mm . Because only positional data of the manipulators end-effector could be recorded no comparison of the rotational results would be made. The measured dataset includes position data from 800 uniformly distributed points, measured in the working volume of the robot. Considering the redundancy of the 7-DOF manipulator regarding the task space, we need additional parameter to uniquely specify a configuration for a given EE pose. For a $S - R - S$ (spherical-rotational-spherical) manipulator structure, the first and last three joints can

Table 1: LBR IIWA R800 nominal D-H parameters.

Frame	d (mm)	a (mm)	α (rad)	θ (rad)
A_1	340	0	$-\frac{\pi}{2}$	θ_1
A_2	0	0	$\frac{\pi}{2}$	θ_2
A_3	400	0	$\frac{\pi}{2}$	θ_3
A_4	0	0	$-\frac{\pi}{2}$	θ_4
A_5	400	0	$-\frac{\pi}{2}$	θ_5
A_6	0	0	$\frac{\pi}{2}$	θ_6
A_7	126	0	0	θ_7

be represented as a shoulder (S) and wrist (W) spherical joints while θ_4 is then called the elbow (E). Using this notation, one of the additional parameters can be an arm angle, which corresponds to the angle between the plane spanned by S, E , and W , and a reference plane of a virtual non-redundant manipulator. This reference plane can be chosen for the case when joint angle θ_3 is fixed to zero. All of the experimental measurements were taken for the same value of the arm angle.

The D-H parameters of LBR IIWA R800 are obtained from nominal data according to the official manufacturer specification and are listed in Table 1.

Considering that LBR IIWA has all revolute joints, (10) and (12) result in the following combinations of error parameters:

$$\begin{aligned} e_{22} &= e_{13}, e_{25} = e_{16}, e_{32} = e_{23}, e_{35} = e_{26} \\ e_{42} &= -e_{33}, e_{45} = -e_{36}, e_{52} = -e_{43}, e_{55} = -e_{46} \\ e_{56} &= e_{65} = d_7 e_{61}, e_{62} = e_{53}, e_{64} = -e_{62} d_7 \\ e_{73} &= e_{63}, e_{66} = e_{76}. \end{aligned} \quad (15)$$

This reduces the number of error parameter functions to 12 for the $RotXYZ$ -error model, and 25 for the $RotXYZTransXYZ$ -error model.

The available dataset was split into training (100 points) and testing (600 points) sets. First the two models were compared with different polynomial degrees. The models were trained using joint angles and 5-fold CV to choose the hyper-parameters. Results presented in Table 2 show that there is no substantial difference in resulting accuracy between the $RotXYZ$ and $RotXYZTransXYZ$ -error models. In the following the $RotXYZ$ -error model with 3rd polynomial degree was chosen for the further analysis. To illustrate the results of the modeling, the frequency plot of the resulting positional errors, measured as the Cartesian distance between the nominal and measured position (red) alongside with the distance from modeled to measured position (blue) is presented in Figure 2.

To investigate how the size of the training set influences the performance of the modeling approaches,

Table 2: Distance error (mm) comparison for different error models.

Error model	degree 2			degree 3			degree 4		
	mean	std	max	mean	std	max	mean	std	max
<i>RotXYZTransXYZ</i>	0.1658	0.0972	0.8147	0.1583	0.0889	0.5612	0.1570	0.0878	0.5242
<i>RotXYZ</i>	0.1759	0.1134	0.8953	0.1648	0.0975	0.6241	0.1563	0.0885	0.5102

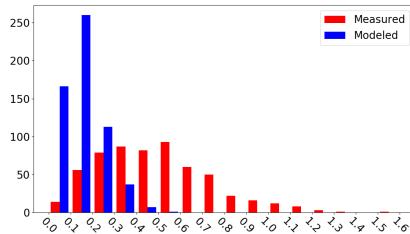


Figure 2: Frequency histogram of the distance error, mm, for *RotXYZ* model of 3rd degree.

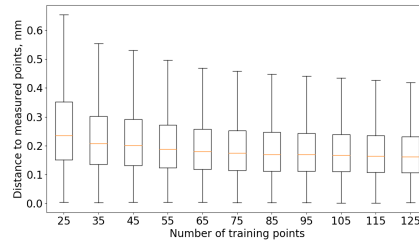


Figure 3: Distance error for the test set for different number of training points.

error models were trained using [25, 35, . . . , 125] points from the available dataset. To ensure that each point was used for training and testing at least once 5-fold CV was used. To account for the variance in the algorithm itself the cross validation procedure was run 25 times, giving an estimate of the performance of the algorithm on the dataset and an estimation of how robust its performance is.

The box plots of the resulting positional errors for *RotXYZ* (Figure 3) show that the mean value stays almost the same and only the maximum values change. From the Table 3, that presents the statistical information of the regression models comparison, we can see the steady decline of mean, standard deviation and maximum values as the number of training points increases.

Table 3: Distance error comparison for different number of training points.

Training points	mean, mm	std, mm	max, mm
25	0.2979	0.2965	8.5011
35	0.2469	0.2047	6.5084
45	0.2316	0.2047	4.4081
55	0.2163	0.1634	4.8526
65	0.2039	0.1553	5.2877
75	0.2013	0.1456	5.3608
85	0.1933	0.1579	3.4861
95	0.1911	0.1279	3.7388
105	0.1879	0.1265	3.8411
115	0.1841	0.1162	3.5205
125	0.1799	0.1044	2.4463

4 CONCLUSIONS

In this paper, a full *RotXYZTransXYZ* and only rotational *RotXYZ*-error model for learning-based calibration were compared. Both models showed significant improvement in positional error. And it was shown that reduced model performed comparably with the full model, but with half of the needed error functions. The *RotXYZ* model was then used to analyze the performance of modeling based on the number of training points. Of course as with any model-based approach the bigger number of training set resulted in improved performance, but starting from 85 points the rate of improvement declined. Because the measurements were uniformly distributed in the working volume of the robot and the training points were chosen randomly, the results contain some outliers. In the future work we will consider finding optimal training points distribution to prevent such cases.

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