Practically Efficient Attribute-based Encryption for Compartmented Access Structures

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Abstract: Compartmented access structures (CASs) regulate the access control by requesting the consent of various compartments. Thus, they are particularly useful to Internet of Things or Wireless Sensor Networks applications with cloud support. The construction of practically efficient attribute-based encryption (ABE) schemes for CASs is faced with the fact that these access structures cannot be represented by Boolean formulas. The use of multilinear map based ABE schemes for general Boolean circuits is not only impractical but also suffers from the lack of secure multilinear map candidates. Also, the schemes based on lattice cryptography, even if they are secure, are highly inefficient in practice. We show in this paper that for CASs we can construct practically efficient ABE schemes based on secret sharing and just one bilinear map. The construction can also be applied to multilevel access structures.

1 INTRODUCTION

Recent developments in wireless sensor networks (WSN), internet of things (IoT), and cloud computing are raising increasing problems over access control. Standard access control techniques, such as *discretionary access control* (DAC), *mandatory access control* (MAC), or *role-based access control* (RBAC), prove to be inappropriate in such cases. For instance, DAC is not well suited for large-scale networks with high security requirements mainly because it does not offer any mechanism or method to manage the improper access control: if the software fails to restrict the user from predefined permissions then any hacker can hack into the system, can have access to the confidential files, and can also perform all the actions like read, write, or delete. Neither MAC does better in such cases. For instance, it is difficult to deploy MAC in cloud systems because it does not support separation of duties, delegation, or inheritance. Although RBAC alleviates some of the security issues with DAC and MAC, it is still not very well suited for cloud computing because it does not scale easily to systems with large number of users and roles where the user's roles change frequently. Moreover, it is difficult to extended RBAC across administrative domains as it is difficult to decide a role's privileges.

Attribute based Access Control (ABAC) is one of the techniques that can overcome the shortcomings mentioned above. ABAC uses attributes (of users, objects, actions, environment) and defines policies based on them. Attributes make ABAC a more fine-grained access control model than RBAC. However, when working with encrypted data for example in cloud, it is good to have the access control policy directly embedded in data and the decryption be carried out only by authorized access structures. One of the best methods to do that is by *attributed-based encryption* (ABE) that allows us to define fine-grained access control on the decryption process.

Diversifying the roles of users and managers, increasing the number of resources and their type, subsidiaries and departments of companies, leads to the orientation of the access control more on groups of attributes than on individual objects. For instance, the Oracle Cloud Infrastructure (Jakóbczyk, 2020) uses *compartments* to group related cloud resources. Compartments provide logical isolation, which makes it much easier to govern the management permission policies and track the costs incurred by the related groups of resources.

Often, compartments must be considered in a cer-

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tain (partial or total) order, regardless of whether they consist of users or resources. The supply chain with products grouped in compartments must follow a certain order (which can be partial), the decision process often follows a total order between compartments, etc. The access control structures should then be defined by means of compartments. Existing approaches for such access structures include *multilevel access structures* (MASs) and *compartmented access structures* (CASs) (Simmons, 1988; Ghodosi et al., 1998; Tassa, 2007; Tassa and Dyn, 2008; Yu and Wang, 2011; Pramanik et al., 2018; Ţiplea and Drăgan, 2018).

Contribution. Access control through ABE has proven to be a necessary and important technology in the current context. There are two basic policies in using ABE: the key policy (KP-ABE) and the ciphertext policy (CP-ABE). In a KP-ABE, each message is encrypted together with a set of attributes and the decryption key is computed for the entire access structure; in a CP-ABE, each message is encrypted together with an access structure while the decryption keys are given for specific sets of attributes.

In this paper we focus on the KP-ABE paradigm. The most efficient KP-ABE scheme from the practical point of view is that in (Goyal et al., 2006), which uses linear secret sharing and a single bilinear map. Unfortunately, the access structures supported by this scheme are only those that can be described by Boolean formulas, while compartmented access structures cannot be described by Boolean formulas (Proposition 3.1 in our paper). The extension of KP-ABE to Boolean circuits by means of multi-linear maps as proposed in (Garg et al., 2013) is not secure due to the fact that no candidate multi-linear map proposed so far is secure (Albrecht and Davidson, 2017; Ţiplea, 2018). The extension of KP-ABE to Boolean circuits by means of lattice cryptography as proposed for instance in (Boneh et al., 2013) is rather unpractical due to the large expansion of the ciphertext and the decryption key. The KP-ABE scheme in (Ţiplea and Drăgan, 2015) may accommodate Boolean circuits. However, it is efficient in practice only if the fan-out gates it uses do not lead to an exponential increase of the size of the decryption key.

In this paper we prove first that CASs cannot be described by Boolean formulas. Then, we show that the KP-ABE scheme proposed in (Ţiplea and Drăgan, 2015) can be applied quite efficiently to CASs. The ciphertext has the same size as in the case of the KP-ABE scheme in (Goyal et al., 2006), while the size of the decryption key is four times larger than the one in (Goyal et al., 2006). This is even far more efficient than the scheme in (Garg et al., 2013) if it were to be secure with any multi-linear map candidate.

Due to the particularity of the CASs we show then that the fan-out gates can be removed, leading to an even more efficient KP-ABE scheme in which the size of the decryption key is only two times larger than the one in (Garg et al., 2013). The same technique is also applied to MASs.

Related Work. During the last decade there has been a continuous increase in the use of ABE technology in IoT and WSN with cloud support. One of the most cited papers (Yu et al., 2010) addresses the problem of defining and enforcing access policies based on data attributes and, on the same time, allowing the data owner to delegate most of the computation tasks involved in fine-grained data access control to untrusted cloud servers without disclosing the underlying data contents.

(Touati and Challal, 2016) proposes a KP-ABE scheme for IoT applications based on cloud, with collaborative encryption. When a node with low computational capabilities must encrypt some message, it is assisted by more powerful neighboring nodes.

In order to have an efficient decryption cost and at the same time to offer protection to sensitive data, (Green et al., 2011) came with the solution of outsourcing the decryption process in the case of ABE schemes to the cloud. Since then, this new paradigm presented a high interest, having several extensions and new systems being constructed upon it. A few examples of the extensions include support for verification of the decryption process (Qin et al., 2015), remote auditing (Yu et al., 2016), keyword search (Li et al., 2016), and efficiency improvements (Liao et al., 2020).

The authors of (Wang et al., 2017) combine searchable encryption along with ABE, resulting in a multiple keyword search ABE scheme. Their system also supports attribute revocation without changing the ciphertext. The system uses a linear secret sharing scheme as access structure.

Another recent work on ABE in cloud system is presented in (Wang et al., 2018), where an ABE scheme with multiple functionalities is proposed: the scheme provides malicious user traceability, attribute revocation for malicious users, and updates over secret key and ciphertext in order to provide security against collusion attack between users. However, the system is limited to Boolean formulas as access structures.

In (Chatterjee and Das, 2015), a password-based user access control scheme with ABE support has been proposed to provide access control over WSNs.

Because most ABE schemes are computationally heavy, sensor requests are grouped under cluster heads, where they are encrypted under some set of attributes, specific for the sensor information. Then each user is assigned a smart card, which stores an access tree. If a user's access structure is satisfied by the attributes from some ciphertext, then it can decrypt the information from the sensor. However, complex real life situations such as healthcare systems where the sensors must contain medical data, may require advanced access control structures that could be unable to express with Boolean formulas.

The necessity of using compartments to define access control policies has already been advocated by many researchers. (Brenner et al., 2017) proposes a secure platform, called *TrApps*, to offer solutions for secure execution in untrusted cloud systems. The applications are divided in small trusted compartments in order to protect sensitive data.

We have already mentioned that the Oracle cloud infrastructure (Jakóbczyk, 2020) uses a compartmented hierarchical form of grouping together related applications. One of the reasons for this is to facilitate access control granting of the resources. Such a system could benefit from an ABE scheme with compartmented access structure: each resource is assigned some attributes, and each application has its own access structure, based upon which it is granted access to resources.

(Hyla et al., 2014) have proposed *MobInfoSec*, a system to protect sensitive information on mobile devices. The authors recognize the importance of multilevel access structures in order to have a good access control system, and tries to combine certificate public-key cryptography with general access structure.

One of the papers that tries to design ABE schemes for multi-level access control policies is (Kaaniche and Laurent, 2017). The access control policy is based on a Boolean tree whose root is a threshold gate and the set of its children is partitioned into sets called security levels. A message is viewed as a vector of components, each of which being encrypted by a standard CP-ABE scheme. So, message components can be obtained by decryption depending on the security level of the user. The scheme does not offer a threshold multi-level access on security levels. So, the decryption entity has to satisfy a precise set of attributes to be able to decrypt a given sub-sets of data blocks with respect to a given security level. This aspect is somehow fixed in (Kaaniche and Laurent, 2018) by using two bilinear maps. We emphasize that the access control structures in these papers are different from MASs and CASs. A similar idea is used in (Li, Zhao and Huan, Shuiyuan, 2018), where the access control architecture is built on five entities: data owner, cloud service provider, center authority, department user, and user. Nor does this scheme use policies such as MAS or CAS.

Paper Structure. The paper is divided into five sections. The next one fixes the basic terminology and notation used throughout the paper. Section 3 includes our main contribution. It starts by motivating our work on practically efficient ABE schemes for CASs and shows that CASs cannot be represented by Boolean formulas. Then, it proves efficiency of the KP-ABE scheme in (Ţiplea and Drăgan, 2015) when applied to CASs. An improvement of this scheme is also proposed and it is shown that the technique can also be applied to the case of multilevel access structures. Section 4 discusses on the implementation of our scheme, and the last section concludes the paper.

2 PRELIMINARIES

We recall in this section the basic terminology and notation that is to be used throughout this paper.

The set of integers is denoted by $\mathbb{Z}$. A positive integer $a > 1$ is a *prime* number if the only positive divisors of it are 1 and a. Two integers a and b are called *congruent modulo n*, denoted $a \equiv b \mod n$ or $a \equiv_n b$, if n divides $a - b$ (n is an integer too). The notation $a = b \mod n$ means that a is the *remainder* of the integer division of b by n. The set of all congruence classes modulo n is denoted $\mathbb{Z}_n$. For a set A, $a \leftarrow A$ means that a is uniformly at random chosen from A.

Access Control Structures. Given a non-empty finite set $\mathcal{U}$ whose elements are called *attributes* in our paper, an *access structure* over $\mathcal{U}$ is any set $\mathcal{S}$ of non-empty subsets of $\mathcal{U}$ (Stinson, 2005). $\mathcal{S}$ is called *monotone* if it contains all subsets $B \subseteq \mathcal{U}$ with $A \subseteq B$, whenever $A \in \mathcal{S}$. The subsets (of $\mathcal{U}$) that are in $\mathcal{S}$ are called *authorized sets*, while those not in $\mathcal{S}$, *unauthorized sets*.

It is customary to represent (monotone) access structures by (monotone) Boolean circuits (for more details about Boolean circuits the reader is referred to (Bellare et al., 2012)). We only consider monotone Boolean circuits that have a number of input wires (which are not output wires of any other gates), just one output wire (which is not input wire of any gate), and arbitrarily many logic (k,n)-gates. A (k,n)-gate, where $1 \leq k \leq n$, has n input wires and one or more output wires. That is, the fan-in of the gate is n, while the fan-out may be arbitrarily large but at least one. If the input wires of a (k,n)-gate are assigned Boolean values and at least k of them are 1, the output wires of the gate will get the value 1; otherwise they will get 0. $(1,2)$- and $(2,2)$-gates are usually referred to as OR- and AND-gates, respectively.

The fan-out of a Boolean circuit is the maximum fan-out of its gates. Boolean circuits of fan-out one correspond to *Boolean formulas*.

If the input wires of a Boolean circuit $\mathcal{C}$ are in a one-to-one correspondence with the elements of $\mathcal{U}$, we will say that $\mathcal{C}$ is a Boolean circuit over $\mathcal{U}$. Each $A \subseteq \mathcal{U}$ evaluates the circuit $\mathcal{C}$ to one of the Boolean values 0 or 1 by simply assigning 1 to all input wires associated to elements in A, and 0 otherwise; then the Boolean values are propagated bottom-up to all gate output wires in a standard way. $\mathcal{C}(A)$ stands for the Boolean value obtained by evaluating $\mathcal{C}$ for A. The access structure defined by $\mathcal{C}$ is the set of all A with $\mathcal{C}(A) = 1$.

Key-policy Attribute based Encryption. A *key-policy attribute based encryption* (KP-ABE) scheme consists of four probabilistic polynomial-time (PPT) algorithms (Goyal et al., 2006):

$Setup(\lambda)$**:** this is a PPT algorithm that takes as input the security parameter λ and outputs a set of public parameters PP and a master key MSK;

$Enc(m,A,PP)$**:** this is a PPT algorithm that takes as input a message m, a non-empty set of attributes $A \subseteq \mathcal{U}$, and the public parameters, and outputs a ciphertext E;

$KeyGen(\mathcal{C},MSK)$**:** this is a PPT algorithm that takes as input an access structure $\mathcal{C}$ (given as a Boolean circuit) and the master key MSK, and outputs a decryption key D (for the entire Boolean circuit $\mathcal{C}$);

$Dec(E,D)$**:** this is a deterministic polynomial-time algorithm that takes as input a ciphertext E and a decryption key D, and outputs a message m or the special symbol $\perp$.

The following correctness property is required to be satisfied by any KP-ABE scheme: for any $(PP,MSK) \leftarrow Setup(\lambda)$, any Boolean circuit $\mathcal{C}$ over a set $\mathcal{U}$ of attributes, any message m, any $A \subseteq \mathcal{U}$, and any $E \leftarrow Enc(m,A,PP)$, if $\mathcal{C}(A) = 1$ then $m = Dec(E,D)$, for any $D \leftarrow KeyGen(\mathcal{C},MSK)$.

We consider the standard notion of selective security for KP-ABE (Goyal et al., 2006). Specifically, in the *Init* phase the *adversary* (PPT algorithm) announces the set A of attributes that he wishes to be challenged upon, then in the *Setup* phase he receives the public parameters PP of the scheme, and in *Phase 1* oracle access to the decryption key generation oracle is granted for the adversary. In this phase, the adversary issues queries for decryption keys for access structures defined by Boolean circuits $\mathcal{C}$, provided that $\mathcal{C}(A) = 0$. In the *Challenge* phase the adversary submits two equal length messages m_0 and m_1 and receives the ciphertext associated to A and one of the two messages, say m_b, where $b \leftarrow \{0,1\}$. The adversary may receive again oracle access to the decryption key generation oracle (with the same constraint as above); this is *Phase 2*. Eventually, the adversary outputs a guess $b' \leftarrow \{0,1\}$ in the *Guess* phase.

The *advantage* of the adversary in this game is $P(b' = b) - 1/2$. The KP-ABE scheme is *secure* (in the selective model) if any adversary has only a negligible advantage in the selective game described above.

Bilinear Maps and the Decisional BDH Assumption. Given G_1 and G_2 two multiplicative cyclic groups of prime order p, a map $e : G_1 \times G_1 \to G_2$ is called *bilinear* if it satisfies:

- $e(x^a, y^b) = e(x,y)^{ab}$, for any $x,y \in G_1$ and $a,b \in \mathbb{Z}_p$;
- $e(g,g)$ is a generator of G_2, for any generator g of G_1.

G_1 is called a *bilinear group* if the operation in G_1 and e are both efficiently computable.

The *Decisional Bilinear Diffie-Hellman* (DBDH) problem in the bilinear group G_2 is the problem to distinguish between $e(g,g)^{abc}$ and $e(g,g)^z$ given g, g^a, g^b, and g^c, where g is a generator of G_1 and a, b, c, and z are randomly chosen from $\mathbb{Z}_p$. The *DBDH assumption* for G_2 states that no PPT algorithm $\mathcal{A}$ can solve the DBDH problem in G_2 with more than a negligible advantage.

3 OUR CONTRIBUTION

3.1 Motivation and Main Goal

Compartmented Access Structures. Threshold access structures (Barzu et al., 2013; Ţiplea and Drăgan, 2014; Drăgan and Ţiplea, 2018) are suitable when participants have the same degree of trust. However, many real-world applications such as cloud storage, healthcare systems, wireless sensor networks and so on need more complex access structures based on different degrees of trust and privileges associated to participants. Compartmented access structures can cope with this problem. Within such structures the set of participants is partitioned into groups called compartments, and thresholds are assigned on whose basis authorized sets are defined.

A *compartmented access structure* (Simmons, 1988) over a finite set $U = \{1,\dots,n\}$ of attributes is a tuple $(\overline{U},\overline{c},t,\mathcal{S})$, where:

- $\overline{U} = (U_1,\dots,U_k)$ is a partition of U into $k \geq 1$ nonempty subsets called *compartments* (the number of participants in U_i is n_i, for all $1 \leq i \leq k$);
- $\overline{c} = (t_1,\dots,t_k)$ is a vector of strictly positive integers called *thresholds* that satisfy $t_i \leq n_i$ for all $1 \leq i \leq k$;
- t is a global threshold satisfying $\sum_{i=1}^{k} t_i \leq t \leq n$;
- $\mathcal{S}$ is the set of all *authorized sets* defined by

$$\mathcal{S} = \{A \subseteq U | (\forall 1 \leq i \leq k)(|A \cap U_i| \geq t_i) \wedge (|A| \geq t)\}.$$

That is, an authorized set in such an access structure should include enough attributes from each compartment and should also be large enough (please see (Ţiplea and Drăgan, 2018) for more details.)

The importance of CASs has been recognized by many researchers, as we have already mentioned in the first section of the paper.

CASs and Boolean Formulas. The ABE scheme in (Goyal et al., 2006) is the most practically efficient scheme known so far when it comes to access structures defined Boolean formulas. Unfortunately, CASs cannot be described by Boolean formulas, as the following proposition shows.

Proposition 3.1. *Compartmented access structures cannot be defined by Boolean formulas.*

Proof. Assume that CASs can be represented by Boolean formulas (that is, by Boolean circuits with fan-out of one). Consider then the following CAS:

- $\mathcal{U} = \{1,2,3,4,5\}$;
- $\mathcal{U}_1 = \{1,2,3\}$, $\mathcal{U}_2 = \{4,5\}$;
- $t_1 = 1$, $t_2 = 1$, and $t = 3$.

Let $\mathcal{C}$ be a Boolean circuit of fan-out one that represents this CAS. This circuit has five input gates, namely the attributes 1, 2, 3, 4, and 5. We remark that at least two input gates must be directly connected. We have then the following cases:

1. There is a gate Γ that connects directly inputs only from the same compartment. Let us assume that Γ connects directly 1 and 2 and, moreover, Γ is evaluated to 1 whenever one of these two inputs is assigned to 1. Remark that $\{1,4,5\}$ and $\{2,4,5\}$ are authorized, but $\{1,4\}$, $\{1,5\}$, $\{2,4\}$, and $\{2,5\}$ are not. As the circuit is of fan-out one, the gates 1 and 2 cannot be connected to other logic gates. Therefore, the gates 4 and 5 must be connected to logic gates in such a way that the circuit is evaluated to one only if these two gates are simultaneously assigned to one. But then, $\mathcal{C}(1,2,4) = 0 = \mathcal{C}(1,2,5)$ because it does not matter that 1 or 2 or both evaluate Γ to 1 as long as these inputs have fan-out one. Therefore, we have arrived at a contradiction;
2. There is a gate Γ that connects directly inputs only from the same compartment. Let us assume that Γ connects directly 1 and 2 and, moreover, Γ is evaluated to 1 whenever at least two of its inputs are assigned to 1. As $\mathcal{C}(1,4,5) = 1$, $\mathcal{C}(2,4,5) = 1$, and in both cases Γ is evaluated to 0, the fact that the circuit is of fan-out one leads to the conclusion that the evaluation of $\mathcal{C}$ to 1 does not depend on the value of Γ. But then we get $\mathcal{C}(4,5) = 1$, which is a contradiction;
3. The other cases, when Γ connects input only from the second compartment or when it connects inputs from both compartments, are treated in a similar way.

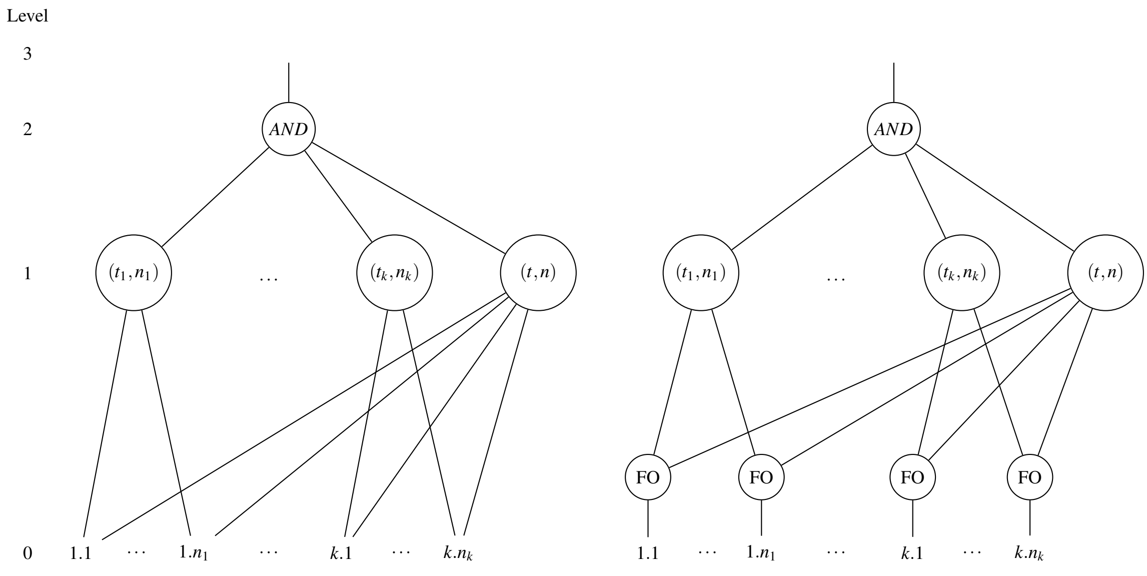
As in all possible cases we were led to a contradiction, we conclude that CASs cannot be represented by Boolean formulas. □

CASs can however be described by Boolean circuits of fan-out two. Before showing this let us adopt the following notation for CASs, which will be used throughout our paper. If the set of attributes is $\mathcal{U} = \{1,\dots,n\}$ and there are k compartments, then these will be denoted by $\mathcal{U}_i = \{i.1,\dots,i.n_i\}$, for all $1 \leq i \leq k$. All the compartments' attributes are taken in order from 1 to n and, therefore, $i.j$ refers to the attribute $i.j = j + \sum_{\ell=1}^{i-1} n_\ell$, for all $1 \leq j \leq n_i$. The threshold for $\mathcal{U}_i$ is denoted t_i, and the global threshold is t.

Given a CAS as above, it can be described by a Boolean circuit with input gates of fan-out two, as it is shown in Figure 1(a). For the sake of readability, we have used a generalized AND-gate with more than two input wires; it can be regarded as the threshold $(k+1,k+1)$-gate.

Our Main Goal. Due to Proposition 3.1, the ABE scheme in (Goyal et al., 2006) cannot be applied to CASs. The options we have then are the following:

1. Use the ABE scheme in (Garg et al., 2013) or even the more efficient one in (Drăgan and Ţiplea, 2016b). Unfortunately, both of them are



(a) Boolean circuit for CASs

(b) Boolean circuit with FO-gates for CASs

Figure 1: Boolean circuit representation of compartmented access structure.

based on multi-linear maps and the recent results show clearly that no candidate multi-linear map proposed so far is secure (Albrecht and Davidson, 2017; Ţiplea, 2018);

2. Use ABE schemes based on lattice cryptography, such as (Boneh et al., 2013). They are secure schemes but, unfortunately, they generate very large ciphertexts and keys;

3. Use the ABE scheme in (Ţiplea and Drăgan, 2015). This is a solution based on secret sharing and just one bilinear map. Under this scheme, the sharing process produces multiple shares on the gates with fan-out greater than two. If such gates are chained, then the input gates may get too many shares, which means a large increase in the size of the decryption key. However, for certain access structures, the gates with fan-out greater than two are in a limited number and without overlapping. In such cases, the decryption key might have reasonable size and the scheme becomes, in terms of efficiency, comparable to that in (Goyal et al., 2006).

In this paper we will show that the ABE scheme in (Ţiplea and Drăgan, 2015) can efficiently be used for CASs. In this context, the scheme produces a decryption key of size $2n\log p$, together with a public key of the same size, where n is the number of attributes and p is a prime. Recall that the scheme in (Goyal et al., 2006) produces a smaller decryption key of size $n\log p$, but it is limited to Boolean formulas.

Then, we will also show how to simplify the scheme above to one that does not need any public key. In this way, we probably get the most efficient ABE scheme, based on secret sharing and just one bilinear maps, for compartmented access structures.

3.2 Our Scheme

The aim of this section is to show that the ABE scheme in (Ţiplea and Drăgan, 2015) can efficiently be used to accommodate CASs. Then, a more efficient ABE scheme will be derived.

Recall first the scheme in (Ţiplea and Drăgan, 2015) adapted to CASs. The Boolean circuit for a CAS, as it is required in (Ţiplea and Drăgan, 2015), looks like in Figure 1(b). As one can see, the Boolean circuit uses FO-gates that simply multiple the output of the gates to which they are associated. These gates are just a technical ingredient used to help us better understand the secret sharing process.

The ABE scheme uses a secret sharing procedure $Share(y,\mathcal{C})$ that on a Boolean circuit $\mathcal{C}$ as above and a value $y\in\mathbb{Z}_p$, where p is a prime, shares y top-down on $\mathcal{C}$ as follows:

- Initially, y is assigned to the output wire of the circuit (the output wire of the AND-gate);
- Uniformly at random choose $y_1,\ldots,y_k \leftarrow \mathbb{Z}_p$, compute $y_{k+1}=y-(y_1+\cdots+y_k) \bmod p$, and assign y_i to the i-th input wire of the AND-gate, for all $1\leq i\leq k+1$ (from left to right);
- Share y_1 at the (t_1,n_1)-gate as follows. If $t_1=1$, then y_1 is "copied" at all its input

wires. Otherwise, choose uniformly at random $a_{1,1},\dots,a_{1,t_1-1} \leftarrow \mathbb{Z}_p$ and define the polynomial

$$f_1(x) = y_1 + a_{1,1}x + \cdots + a_{1,t_1-1}x^{t_1-1} \bmod p$$

Then, assign to the input wires of the gate the shares $f_1(1),\dots,f_1(n_1)$ (from left to right).

Share then $y_2,\dots,y_k$ in the same way y_1 was shared. For y_{k+1} we choose uniformly at random $a_1,\dots,a_{t-1} \leftarrow \mathbb{Z}_p$ and define the polynomial

$$f_{k+1}(x) = y_{k+1} + a_1x + \cdots + a_{t-1}x^{t-1} \bmod p$$

Then, assign to the input wires of the gate the shares $f_{k+1}(i.j)$ in lexicographic order on i and j (please remark that y_{k+1} has to be shared at the (t,n)-gate which has n input wires);

- The FO-gate that splits the output of the input gate $i.j$ ($1 \le i \le k$, $1 \le j \le n_i$) gets two shares: $f_i(j)$ and $f_{k+1}(i.j)$. Each of them is shared down as follows. Uniformly at random choose $a^1_{i,j}, a^2_{i,j} \leftarrow \mathbb{Z}_p$ and compute $b^1_{i,j} = f_i(j) - a^1_{i,j} \bmod p$, $b^2_{i,j} = f_{k+1}(i.j) - a^2_{i,j} \bmod p$, $g^{b^1_{i,j}}$, and $g^{b^2_{i,j}}$. The values $a^1_{i,j}$ and $a^2_{i,j}$ are passed down to the input gate $i.j$ as shares, while $g^{b^1_{i,j}}$ and $g^{b^2_{i,j}}$ are public keys associated to the FO-gate.

At the end of the sharing procedure, each gate $i.j$ gets two shares denoted $S(i.j,1)$ and $S(i.j,2)$, while its associated FO-gate is assigned two public values denoted $P(i.j,1)$ and $P(i.j,2)$ (please see Figure 2).

Now, the ABE scheme can be described as follows (we will name it SSBM_1 as an acronym for *secret sharing and bilinear map based ABE scheme*).

SSBM_1 ABE Scheme

Setup(λ,n)**:** the setup algorithm uses the security parameter λ to choose a prime p, two multiplicative groups G_1 and G_2 of prime order p, a generator g of G_1, and a bilinear map $e : G_1 \times G_1 \to G_2$. Then, it chooses $y \in \mathbb{Z}_p$ and, for each attribute $i.j$, chooses $r_{i,j} \leftarrow \mathbb{Z}_p$ (please see the notation above). Finally, the algorithm outputs the public parameters

$$PP = (p, G_1, G_2, g, e, n, Y = e(g,g)^y, \\ (T_{i,j} = g^{r_{i,j}} | i, j))$$

and the master key $MSK = (y, r_{i,j} \mid i, j)$;

Encrypt(m,A,PP)**:** the encryption algorithm encrypts a message $m \in G_2$ by a non-empty set A of attributes as follows:

- $s \leftarrow \mathbb{Z}_p$;
- output $E = (A, E' = mY^s, (E_{i,j} = T^s_{i,j} = g^{r_{i,j}s} | i.j \in A), g^s)$;

KeyGen$(\mathcal{C},MSK)$**:** the decryption key generation algorithm generates a decryption key (D,P) for the CAS defined by the Boolean circuit $\mathcal{C}$ as follows:

- $(S,P) \leftarrow$ *Share*$(y,\mathcal{C})$ (please see the notation above);
- output (D,P), where $D(i.j,\ell) = g^{S(i.j,\ell)/r_{i,j}}$ for all $1 \le i \le k$, $1 \le j \le n_i$, and $\ell = 1,2$;

Decrypt$(E,(D,P))$**:** given E and (D,P) as above, the decryption works as follows:

- Compute $V_A(i.j,\ell)$ for all attributes $i.j$ and $\ell = 1,2$ by

$$V_A(i.j,\ell) = \begin{cases} e(g,g)^{S(i.j,\ell)s}, & \text{if } i.j \in A \\ \perp, & \text{otherwise} \end{cases}$$

  where $e(g,g)^{S(i.j,\ell)s} = e(g^{r_{i,j}s}, g^{S(i.j,\ell)/r_{i,j}})$ and $\perp$ means “undefined”;

- For each attribute $i.j$ use the public key $P(i.j,1)$ to compute $F_A(i.j,1) = e(g,g)^{f_i(j)s}$ by

$$F(i.j,1) = V_A(i.j,1) \cdot e(P(i.j,1), g^s).$$

  In a similar way, $F_A(i.j,2) = e(g,g)^{f_{k+1}(i.j)s}$ is computed by means of $P(i.j,2)$. Remark that $F_A(i.j,\ell) = \perp$, whenever $i.j \notin A$;

- If (t_1,n_1)-gate is satisfied (i.e., at least t_1 attributes from the first compartment are in A), then use the Lagrange interpolation formula to derive $e(g,g)^{y_1s}$ from the corresponding attributes’ F_A-values (as computed before). If the gate is not satisfied, then the value will be $\perp$. Do the same for all gates on the first level;
- If the values for the gates on the first level are all different than $\perp$, then multiply them and get $O = e(g,g)^{ys}$ as the value of the output wire of the AND-gate. Otherwise, $O = \perp$;
- $m := E'/O$.

The correctness of the SSBM_1 scheme simply follows from its description (one may also consult (Ţiplea and Drăgan, 2015) for the general case), and its security follows from the general approach in (Ţiplea and Drăgan, 2015) (the scheme we have described above is just an instantiation of the general case in (Ţiplea and Drăgan, 2015)).

As with respect to its efficiency, we may say that the SSBM_1 scheme is quite efficient:

1. The size of the secret key is $2n\log p$, and so is the size of the public key;
2. The secret sharing phase needs to randomly split y in $k+1$ shares, to apply Shamir’s secret sharing for each of them, and to split $2k$ secrets at the FO-gates, each in exactly two shares;

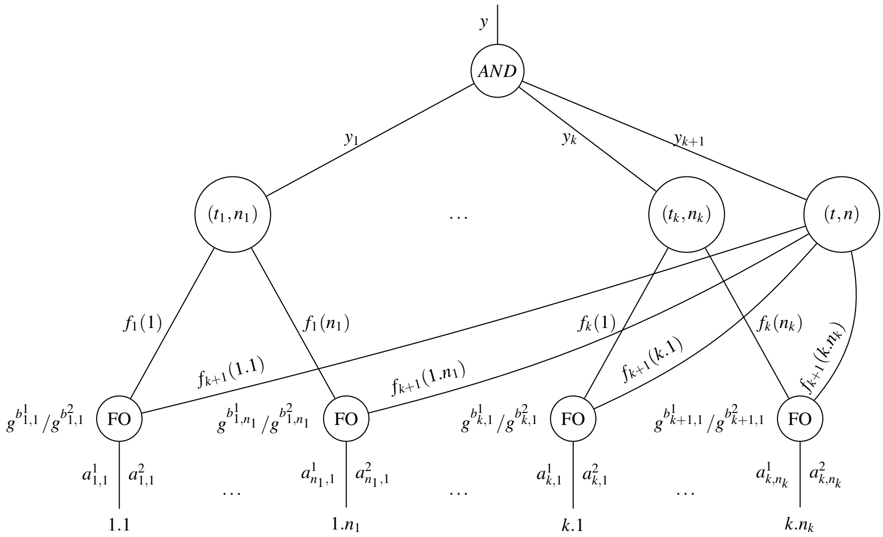


Figure 2: Sharing procedure.

3. The decryption phase needs $4n$ computations of the map e, $k+1$ secret reconstruction by polynomial interpolation, and $2n+k$ multiplications.

Even if the SSBM_1 scheme is quite efficient for CASs, we may still improve its efficiency. The fundamental observation is that the FO-gates only duplicate the outputs of the input gates. As a result, it is no longer necessary for the shares coming to them from top to bottom be once again shared (this statement will be rigorously argued a little bit later). Therefore, the secret sharing scheme can be simplified as it is illustrated in Figure 3. That is, the FO-gates are removed and the shares from the (t_i,n_i)-gate and from the (t,n)-gate come directly to the attribute $i.j$. In this way, the public keys are completely removed and each attribute $i.j$ gets the shares $S(i.j,1) = f_i(j)$ and $S(i.j,2) = f_{k+1}(i.j)$. We denote this new secret sharing procedure by $Share'(y,\mathcal{C})$.

Thus, we arrive at the following ABE scheme.

SSBM_2 ABE Scheme

$Setup(\lambda,n)$**:** the same as in SSBM_1 scheme;

$Encrypt(m,A,PP)$**:** the same as in SSBM_1 scheme;

$KeyGen(\mathcal{C},MSK)$**:** the decryption key generation algorithm generates a decryption key D for the CAS defined by the Boolean circuit $\mathcal{C}$ as follows:

- $S \leftarrow Share'(y,\mathcal{C})$ (please see the notation above);
- output D, where $D(i.j,\ell) = g^{S(i.j,\ell/r_{i,j}}$ for all $1 \le i \le k$, $1 \le j \le n_i$, and $\ell = 1,2$;

$Decrypt(E,D)$**:** given E and D as above, the decryption works as follows:

- Compute $F_A(i.j,\ell)$ for all attributes $i.j$ and $\ell = 1,2$ by
$$F_A(i.j,\ell) = \begin{cases} e(g,g)^{S(i.j,\ell)s}, & \text{if } i.j \in A \\ \perp, & \text{otherwise} \end{cases}$$
where $e(g,g)^{S(i.j,\ell)s} = e(g^{r_{i,j}s}, g^{S(i.j,\ell)/r_{i,j}})$ and $\perp$ means "undefined";
- If (t_1,n_1)-gate is satisfied (i.e., at least t_1 attributes from the first compartment are in A), then use the Lagrange interpolation formula to derive $e(g,g)^{y_1 s}$ from the corresponding attributes' F_A-values (as computed before). If the gate is not satisfied, then the value will be $\perp$. Do the same for all gates on the first level;
- If the values for the gates on the first level are all different than $\perp$, then multiply them and get $O = e(g,g)^{ys}$ as the value of the output wire of the AND-gate. Otherwise, $O = \perp$;
- $m := E'/O$.

It is straightforward to prove the correctness of this new scheme. Just remark that the recovering procedure produces the same result at the threshold gates on the first level as in the case of the SSBM_1 scheme.

The security of the SSBM_2 scheme is subject of the following theorem.

Theorem 3.1. *The SSBM_2 ABE scheme is secure in the selective model under the decisional bilinear Diffie-Hellman assumption.*

Proof. Assume that the SSBM_2 ABE scheme is not secure in the selective model, and let $\mathcal{A}$ be an adversary that has a non-negligible advantage against this scheme (when applied to CASs).

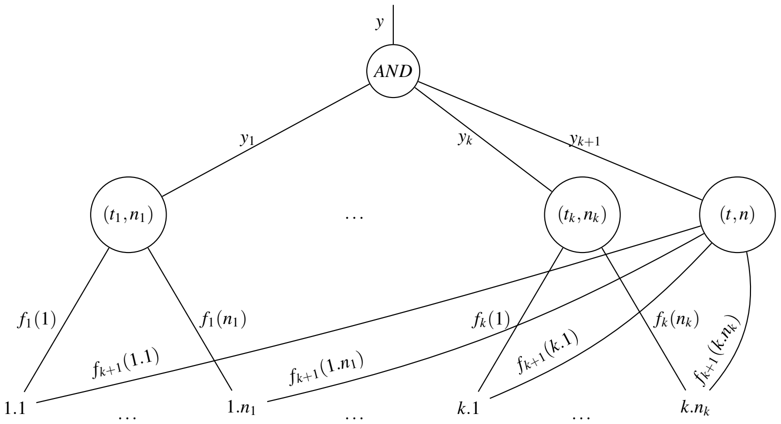


Figure 3: Simplified secret sharing procedure.

We define an adversary $\mathcal{A}'$ against the SSBM_1 ABE scheme and we show that it has a non-negligible advantage against this scheme, which is a contradiction (this scheme is an instance of the scheme in (Ţiplea and Drăgan, 2015), which is secure in this security model). From a technical point of view, if $\mathcal{C}$ denotes a Boolean circuit in the SSBM_2 scheme (as it is, for instance, the circuit in Figure 1(a)), then the corresponding circuit with FO-gates in the SSBM_1 scheme (as it is the one in Figure 1(b)) is denoted by $\mathcal{C}'$. The adversary $\mathcal{A}'$ does as follows:

- $\mathcal{A}'$ announces the set A of attributes that he wishes to be challenged upon;
- In the Setup phase $\mathcal{A}'$ receives the public parameters PP of the scheme;
- In Phases 1 and 2 oracle access to the decryption key generation oracle is granted for the adversary $\mathcal{A}'$. Querying this oracle for a Boolean circuit $\bar{\mathcal{C}}$ (that represents some CAS) with $\bar{\mathcal{C}}(A) = 0$, the adversary gets the decryption key (D',P') as it is given in the description of the SSBM_1 ABE scheme. Using the notation in this scheme, $\mathcal{A}'$ computes $F_A(i.j,1) = e(g,g)^{f_i(j)s}$ and $F_A(i.j,2) = e(g,g)^{f_{k+1}(i.j)s}$, for all attributes $i.j$.
  If we look now to the SSBM_2 scheme, we see that the F_A-values computed by $\mathcal{A}'$ are exactly the F_A-values computed by $\mathcal{A}$ if $\mathcal{A}$ had interrogated the key decryption oracle of the SSBM_2 scheme with the circuit $\mathcal{C}$ and the secret sharing would have been done in the same way at the logic gates (remark that $\mathcal{C}(A) = 0$);
- In the Challenge phase the adversary $\mathcal{A}'$ submits two equally length messages m_0 and m_1 and receives the ciphertext associated to A and one of the two messages, say m_b, where $b \leftarrow \{0,1\}$.

It is clear that $\mathcal{A}'$ can compute at least the same information as $\mathcal{A}$. Therefore, $\mathcal{A}'$ may guess b with at least the same probability as $\mathcal{A}$. This shows that $\mathcal{A}'$ has a non-negligible advantage against the SSBM_1 scheme, which is a contradiction. □

3.3 Variations on the Same Theme

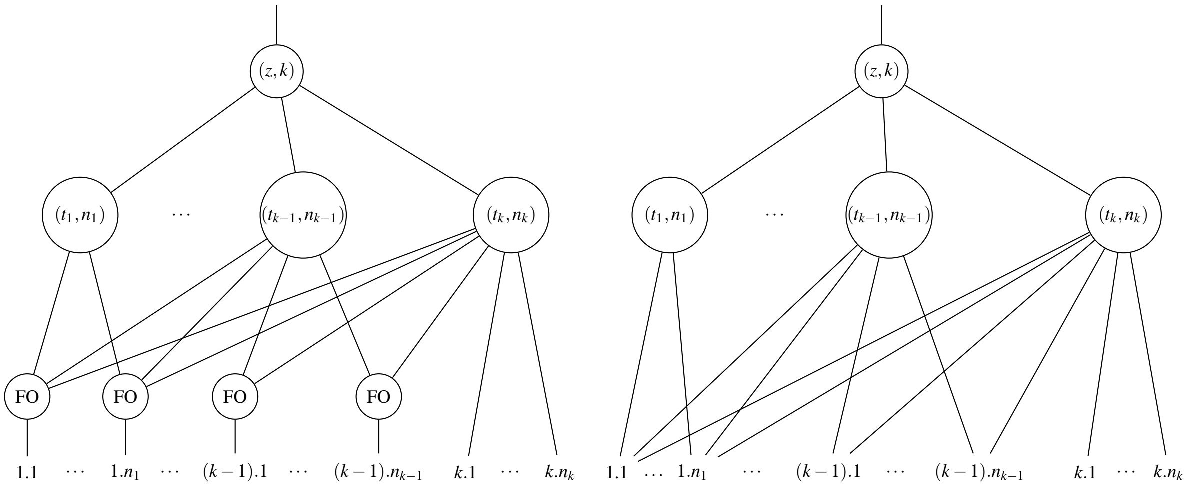
Multilevel Access Structures. are another example of access structures that cannot be described by Boolean formulas. This was shown in (Ţiplea and Drăgan, 2015), where the most efficient ABE scheme (at that time) for such access structures was proposed. However, the SSBM_2 scheme can also be adapted for multilevel access structures, leading to an even more efficient solution than the one in (Ţiplea and Drăgan, 2015). But, let us first recall the multilevel access structures.

A *disjunctive multilevel access structure* (DMAS) (Simmons, 1988) over a set $\mathcal{U} = \{1,\ldots,n\}$ of attributes is a tuple $(\bar{t},\overline{\mathcal{U}},\mathcal{S})$, where $\bar{t} = (t_1,\ldots,t_k)$ is a vector of positive integers satisfying $0 < t_1 < \cdots < t_k$, $\overline{\mathcal{U}} = (\mathcal{U}_1,\ldots,\mathcal{U}_k)$ is a partition of $\mathcal{U}$ ($\mathcal{U}_i$ is the i-th level of $\mathcal{U}$), and $\mathcal{S}$ is defined by:

$$\mathcal{S} = \{A \subseteq \mathcal{U} | (\exists 1 \leq i \leq k)(|A \cap (\cup_{j=1}^{i} \mathcal{U}_j)| \geq t_i)\}.$$

If we replace "∃" by "∀" in the above definition, we obtain the concept of *conjunctive multilevel access structure* (CMAS) (Tassa, 2007).

If we adopt the same notations for attributes as in the case of CASs, then the Boolean circuit used in (Ţiplea and Drăgan, 2015) to represent multilevel access structures looks like the one in Figure 4(a). If we remove the FO-gates we get the Boolean circuit in Figure 4(b) to which we can apply *Share*′. In this case, the attribute $i.j$ will get $k-i+1$ shares denoted $S(i.j,\ell)$, with $1 \leq \ell \leq k-i+1$ (remark that the number of shares only depends on the level i). The ABE scheme SSBM_2 can be applied in this case too, with


(z,k)
(t₁,n₁)
···
(tₖ₋₁,nₖ₋₁)
(tₖ,nₖ)
FO
FO
FO
FO
1.1 ··· 1.n₁ ··· (k−1).1 ··· (k−1).nₖ₋₁ k.1 ··· k.nₖ
(z,k)
(t₁,n₁)
···
(tₖ₋₁,nₖ₋₁)
(tₖ,nₖ)
1.1 ··· 1.n₁ ··· (k−1).1 ··· (k−1).nₖ₋₁ k.1 ··· k.nₖ

(a) Boolean circuit with FO-gates

(b) Boolean circuit without FO-gates

Figure 4: Boolean circuit representation of multilevel access structure: $z = 1$ for the disjunctive case, and $z = k$ for the conjunctive case.

the only difference that ℓ takes values as above. Remark also that for DMAS, $z = 1$ means that the (z,k)-gate is a generalized OR-gate and, therefore, the secret y is "copied" on all its input wires. When $z = k$, the (z,k)-gate is a generalized AND-gate as in the case of CASs.

The decryption key size produced by the ABE scheme SSBM_2 for multilevel access structures is

$$k \cdot n_1 + (k-1) \cdot n_2 + \cdots + n_k \cdot 1$$

which gives on average $n(k+1)/2$ (just take all levels of the same size). The approach in (Ţiplea and Drăgan, 2015) generates a public key too of the same size. Therefore, we get a substantial improvement over the approach in (Ţiplea and Drăgan, 2015) which, besides the one proposed in this paper, was the most efficient one (please see (Ţiplea and Drăgan, 2015) for details).

Can FO Gates Always be Removed? FO-gates have been used in (Ţiplea and Drăgan, 2015) to specify a kind of secret sharing at the gates of fan-out greater than two, in order to defeat the backtracking attack (Goyal et al., 2006). As we have seen so far, we were able to successfully remove the FO-gates whenever they were attached to the circuit's input gates (where the secret sharing process is completed). However, if FO-gates are attached to gates inside the circuit, removing them may be more complex. An idea of how we could do it would be to duplicate the sub-circuits whose roots are logic gates with fan-out greater than two, as suggested in Figure 5.

As one can see, the sub-circuit with the OR-gate as root (in the left hand side picture) must be duplicated. Clearly, this can lead to a very large increase of the final circuit compared to the original one (just think that the duplicated sub-circuits contain another gates with fan-out greater than two, which means that the duplicating process must be repeated).

4 IMPLEMENTATION

An implementation of our SSBM_2 ABE scheme can be found at https://github.com/Juve45/abe-cas. The programs are written in C for better portability. For bilinear map support we have used the PBC library (Lynn, 2007) and also the GMP library for multi precision arithmetic. Thus, our system should run în any operating system that supports GMP and PBC. The implementation was tested în Linux (Linux 4.9, Debian 9.12) and Windows 10.

5 CONCLUSIONS

Building access control policies based on compartments plays an important role in today's technologies, such as IoT and WSN with cloud support. In addition, the need to work with encrypted data in the cloud requires that such access policies be integrated with encryption techniques. Attribute-based Encryption (ABE) is an encryption technique that integrates access control policies defined in the most general way, namely, through Boolean circuits. However, ABE schemes developed to date are practically efficient

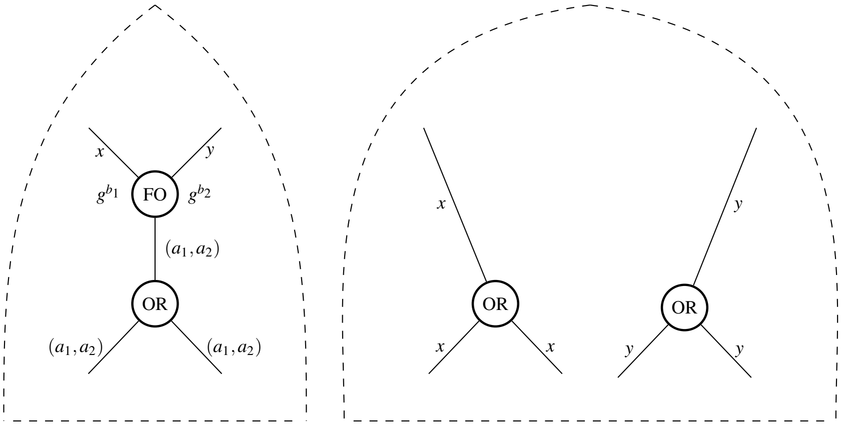


Figure 5: Removing FO-gates.

only for Boolean formulas, while compartmented access structures cannot be expressed by Boolean formulas.

In this paper we have shown that for the case of compartmented access structures we can construct practically efficient ABE schemes. We started from the scheme in (Ţiplea and Drăgan, 2015) and we have refined it to a new scheme that is likely to achieve the maximum possible efficiency. Also, by applying the same technique, the case of multilevel access structures presented in (Ţiplea and Drăgan, 2015) has been made more efficient.

We believe that multilevel and compartmented access structures, along with access structures that can be defined by Boolean formulas, cover most of the practical needs. Possibly, there could be a certain interest for weighted or distributed access structures such as those of (Drăgan and Ţiplea, 2016a).

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