Attrition, Promotion, Transfer: Reporting Rates in Personnel Operations Research

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Abstract: Rates of personnel flow, such as attrition, promotion and transfer, are widely reported, compared and modelled in Personnel Operations Research (OR). However, different analysts commonly employ different formulas to define these rates. This paper solidifies the foundation of Personnel OR by presenting a theoretically sound formula for personnel flow rates that we will refer to as the *general formula*. The proposed formula is justified by its properties, but also by analogy with the field of Investment Performance Reporting, where it is known as the Time Weighted Rate of Return. The paper also derives approximation formulas for the rates of personnel flow, and empirically compares them.

1 INTRODUCTION

Personnel Operations Research (OR) is the branch of OR that supports operational decisions through Human Resources (HR) data analysis and workforce modelling. Practitioners of Personnel OR often describe personnel flows using rates, such as attrition rates, promotion rates and transfer rates. These rates are reported, compared, and used within models, or as the basis for forecasts. However, different practitioners compute these rates in a variety of ways. As pointed out by Noble (2011), all agree that the attrition rate is important and that its definition is self-evident, but then go on to give different definitions.

Common ways of defining attrition rates include dividing the count of departing employees by the period's initial population, or alternatively by the period's average population (Bartholomew et al, 1991). Other denominators have also been employed. For example, the Canadian Armed Forces have used the sum of the initial population with half the number of recruits (Okazawa, 2007). In general, different attrition rate definitions attempt to account for the fact that the size of the underlying population varies over the period, but disagree on how to account for that variation. Unfortunately, published work on this subject is scarce. Most authors either report rates without specifying a formula, or when they do present a formula, do not present a theoretical justification.

We aim to solidify the foundation of Personnel OR by introducing a definition for personnel flow rates that we call the *general formula*. As such, we generalize and improve on Okazawa (2007), the only previous attempt to formally justify a personnel flow rate formula of which we are aware. We will also derive practical approximations of the general formula, and empirically measures their accuracy.

2 PROPORTIONAL RATES

Attrition, promotion and transfer rates are *proportional rates*. They represent the proportion of a population that flows in a given direction, over a given time period. For example, the attrition rate is the proportion that leaves the organization entirely, while a promotion rate tracks employees who move to a higher pay grade. The treatment of proportional rates by other disciplines can provide inspiration to Personnel OR.

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Proportional rates are omnipresent in demographics. For example, mortality rates and divorce rates are analogous to the attrition rates of Personnel OR. However, demographic data often come from multiple disparate sources, unlike personnel data which is generally from a single HR system. For example, divorces are promulgated by courts and tracked by justice systems, whereas counts of married couples come from censuses. This makes it impossible to track day-to-day changes in the married population, which would additionally require reconciliation of immigration and mortality data from yet other sources. Divorce rates, therefore, end up being calculated as simple ratios between the numbers of divorces and census population. Demographers also rely on standardized rates, but this is outside our current scope (Statistics Canada, 2017).

Proportional rates are also seen in the reporting of subscription services churn rates, such as the subscriber churn of wireless service providers. Such reporting is widespread, and important to the fair comparison of different carriers' operational prospects, but is not currently standardized. As an example, AT&T reports the average over months, of the number of subscribers who cancel service each month divided by the number of subscribers at the beginning of the respective months (AT&T, 2020). Such a measure is reasonable, but not completely satisfactory, as new subscribers are acquired during the month, and some cancelling subscribers might not have been there at the beginning of the month. This area of proportional rate reporting is not yet mature enough to inform the field of Personnel OR.

The discipline where proportional rates are most mature is finance, where interest rates and rates of return are crucial. In particular, Investment Performance Measurement shares important similarities with the reporting of rates in Personnel OR. It is also highly standardized by regulatory bodies, so as to allow a fair comparison of the returns achieved by different investment firms. The remainder of this section explores the rate formulas used in Investment Performance Measurement.

2.1 Internal Rate of Return

Consider Figure 1, which tracks the value of an investment account over a year. Initially, the account contains investments valued at \$40K, which increase in value to \$50K within three months, representing a 25% increase. At that point, \$200K is transferred into the account. Over the next three months, the value of the account drops to \$150K – a 40% reduction, before \$100K is transferred out of the account. In the last six months of the year, the value of the account grows by 60% from \$50K to \$80K.

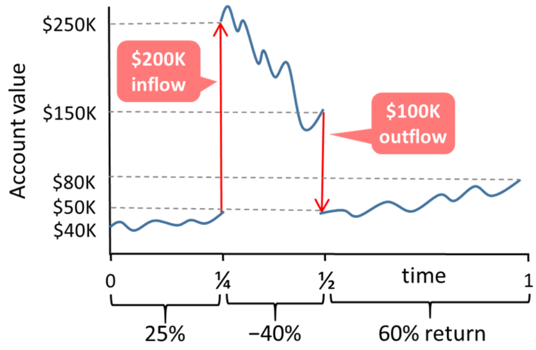

Figure 1: Investment performance example.

The *internal rate of return* (IRR) is the effective rate that achieves the account's final value, when applied to the initial value and the intervening cash transfers. In our example, the IRR is of -42.0%, as obtained by solving

$$80 = 40(1+r) + 200(1+r)^{\frac{3}{4}} - 100(1+r)^{\frac{1}{2}} \quad (1)$$

Regulators, such as the Canadian Securities Administrators (CSA), require that the investment performance of client accounts be reported using the IRR or related metrics (CSA, 2017). However, the IRR is not an appropriate performance measure for investment fund managers. To see this, consider that the IRR varies not only with the outcome of the fund manager's investment decisions, but also with the amounts transferred in and out of the fund. Indeed, with the example in Figure 1, the poor timing of the external transfers is largely responsible for the loss, and likely outside the control of the fund manager.

2.2 Time-weighted Rate of Return

The *time-weighted rate of return* (TWRR) is a measure of investment performance that is invariant with respect to external transfers. It is obtained by compounding rates of return over the sub-periods between each transfer. In our example, the TWRR is of 20%, obtained as

$$\frac{50}{40} \cdot \frac{150}{250} \cdot \frac{80}{50} - 1 \quad (2)$$

This corresponds to the return that would have resulted from a set investment, subject only to changes in the value of the underlying assets (with no external transfers). The TWRR is mandated by the Chartered Financial Analyst (CFA) Institute's Global

Investment Performance Standards for reporting investment fund returns (CFA Institute, 2019).

3 RATES IN PERSONNEL OR

In Personnel OR, rates are often reported in order to compare sub-populations (e.g. women versus men) or to compare different periods (e.g. this year versus last). For this, we need a measure that is invariant with respect to other simultaneous flows (e.g. a measure for promotions that is invariant to recruitment and attrition flows). This is similar to the justification for the TWRR. In the context of Personnel OR, we have taken to referring to the TWRR as the *general formula* for personnel flow rates. We first defined the formula specifically for attrition rates, in a Canadian Department of National Defence internal report (Vincent et al, 2018).

The general formula can be used to measure attrition, promotions, transfers, and all of their variations. Of these, attrition (also called wastage by some authors) is the most commonly reported. It is the departure of employees for any reason (resignation, retirement, dismissal, etc.). To simplify the remainder of this paper, we often describe concepts in terms of attrition rates, but remind the reader that the discussion also applies to other proportional rates of personnel flow.

3.1 The General Formula

Figure 2 shows the headcount, over a year, for a workforce subject to attrition. For illustrative purposes, attrition is atypically high in this example.

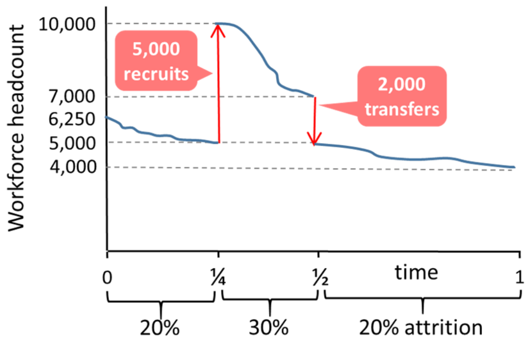

Figure 2: Example of attrition measurement.

The headcount starts at 6,250 and gradually decreases. After the third month, 5,000 recruits show up. Then, at the six month mark, 2,000 employees are transferred out, perhaps the result of a spin-off – this loss of personnel does not "count" as attrition. In the end, 4,000 employees are left. In this example, the general formula rate of attrition is of 55.2%, as obtained by compounding the rates from the three sub-periods that are free of other flows:

$$1-\frac{5{,}000}{6{,}250}\cdot\frac{7{,}000}{10{,}000}\cdot\frac{4{,}000}{5{,}000} \tag{3}$$

As with the TWRR, the general formula rate does not vary with the timing of non-attrition flows. Typically, HR data is captured at a daily resolution, with inflows and outflows occurring between work days. It is thus practical to express the general formula as a compounding of daily rates:

$$1-\alpha=\prod_{i=1}^{n}\frac{p(i)}{p(i)+\mathrm{a}(i)} \tag{4}$$

where α is the rate being measured, n the number of days in the period of interest, $p(i)$ the headcount at the end of the i^{th} day, and $\mathrm{a}(i)$ the magnitude of the relevant personnel flow on that day.

The general formula has two properties worth highlighting. The first is that when $\mathrm{a}(i)$ is the only flow affecting $p(i)$, $p(i)+\mathrm{a}(i)=p(i-1)$ for all i, which leads to:

$$1-\alpha=\frac{p(n)}{p(0)} \tag{5}$$

If at the same time, a denotes the magnitude of the flow over the entire period, we also have $p(n)=p(0)-\mathrm{a}$, such that Equation (5) can be re-cast as

$$\alpha=\frac{\mathrm{a}}{p(0)} \tag{6}$$

The common definition of attrition rate as the number of employees who depart divided by the starting population is thus seen as a special case of the general formula applicable when attrition is the only flow.

The second property to highlight is that given sub-period rates α_i, the general formula rate can simply be obtained by compounding:

$$1-\alpha=\prod_{i}(1-\alpha_i) \tag{7}$$

This multiplicative property directly follows from Equation (4).

If the two desirable properties described by Equations (5) and (7) are instead taken as a starting point, we will notice that they are sufficient to derive the general formula (Equation (4)). This is in fact how we first identified the general formula as our preferred method for reporting attrition in (Vincent et al, 2018). We only later drew parallels with investment performance measurement.

3.2 Internal Rate of Personnel Flow

We now look at how the general formula must be adapted in order to be applicable to the naïve forecasting of future flows. This will lead us to the internal rate of personnel flow – the analogue of the IRR.

Figure 3 tracks a workforce undergoing a single non-attrition flow of magnitude x, perhaps the arrival of a cohort of new hires, occurring at time k.

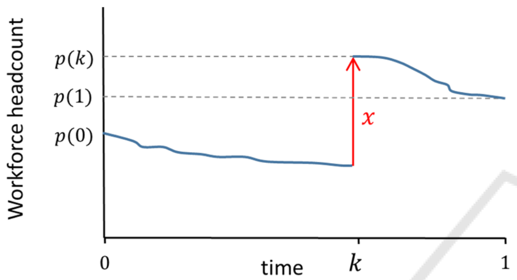

Figure 3: Workforce with a single non-attrition flow.

Per Equation (5), the attrition rate over the sub-periods without external flows are obtained from the ratios of start/end headcounts as

$$1 - \alpha_{[0,k]} = \frac{p(k) - x}{p(0)} \tag{8}$$

$$1 - \alpha_{[k,1]} = \frac{p(1)}{p(k)} \tag{9}$$

with $p(k)$ denoting the population immediately before the non-attrition flow of magnitude x. Per Equation (7), attrition over the entire period $[0,1]$ is

$$1 - \alpha = \frac{p(k) - x}{p(0)} \cdot \frac{p(1)}{p(k)} \tag{10}$$

Substituting Equation (10) into the following easily verifiable identity

$$p(1) = p(0) \cdot \left[\frac{p(k) - x}{p(0)} \cdot \frac{p(1)}{p(k)}\right] + x \cdot \frac{p(1)}{p(k)} \tag{11}$$

we obtain

$$p(1) = p(0) \cdot (1 - \alpha) + x \cdot (1 - \alpha_{[k,1]}) \tag{12}$$

Equation (12) separates the effect of attrition on $p(0)$ from its effect on x, and may be generalized to multiple non-attrition flows. Thus, given $p(0)$ and knowledge of planned future non-attrition flows (i.e. planned recruitment), Equation (12) can be used to naïvely forecast attrition. However, doing this also requires foreknowledge of $\alpha_{[k,1]}$.

A reasonable assumption for $\alpha_{[k,1]}$ is that attrition will advance at the same pace over $[k, 1]$, as over the entire period:

$$1 - \alpha_{[k,1]} \cong (1 - \alpha)^{1-k} \tag{13}$$

Then, Equation (12) becomes

$$p(1) \cong p(0) \cdot (1 - \alpha) + x \cdot (1 - \alpha)^{1-k} \tag{14}$$

which, for an arbitrary number of external flows x_i occurring at times k_i, generalizes as

$$p(1) \cong p(0) \cdot (1 - \alpha) + \sum_{i=1}^{n} x_i \cdot (1 - \alpha)^{1-k_i} \tag{15}$$

We call Equation (15) the *internal rate of personnel flow*. It can be understood as a model derived from the general formula under an assumption of fixed paced attrition throughout the period. This assumption is unlikely to be strictly true in practice, but is reasonable when nothing is known about the actual attrition pattern, such as when forward-projecting a rate in order to predict future attrition.

In Investment Performance Measurement, the TWRR and IRR can give very different values, as was seen with the example from Figure 1. In Personnel OR, on the other hand, the rate from the general formula and the internal rate of personnel flow are typically much closer. This is because personnel flows tend to occur at a steadier pace, and tend to be small relative to the headcount.

4 RATE APPROXIMATIONS

The general formula defined by Equation (4) provides a sound basis for measuring, reporting and comparing personnel flows. However, applying it directly can prove cumbersome in practice.

Say that we wanted to compare the attrition rates of infantry and artillery captains. Applying the general formula requires daily attrition counts, which are easily extracted from HR System logs. It also requires the daily population size for infantry and artillery captains, which are typically harder to obtain. That is because they must typically be derived from transaction logs that track hiring, attrition, promotions in and out of the rank of captain, and occupation changes to and from infantry and artillery. Coordinating all of these transactions can be delicate, especially when some occur simultaneously, and when the logs contain inconsistencies. Nevertheless, code can be developed to solve the problem. However, we might then want to compare the same population segments, but only on a given military base. Then, the code that determined daily populations must be revised to consider posting transactions to and from that base. If we are then interested in further segmenting based on sex, age, education or qualifications, the task of developing code to derive accurate daily population sizes quickly becomes overwhelming.

Approximation formulas that do not require daily population sizes make easier the task of measuring and comparing rates. This section derives such formulas, while the next evaluates them empirically.

In general, we seek formulas for estimating α from $p(0)$: the initial headcount, $p(1)$: the headcount at the end of the period (usually a year) and a: the total attrition volume over the period. In practice, when implementing such approximations, a is easily extracted from the attrition transaction log, whereas $p(0)$ and $p(1)$ are taken from precomputed annual workforce snapshots that list all employees along with their relevant attributes (e.g. rank, occupation, location, age, sex, etc.).

4.1 Half-intake Approximation

First, we set

$$x = p(1) - p(0) + \mathrm{a} \tag{16}$$

as the net non-attrition flow in or out of the workforce. For a given total attrition volume (a), the value of α given by Equation (4) varies with how a and x vary with respect to each other over the period in question. To simplify Equation (4), we assume that half of x occurs before all of a, itself followed by the other half of x, as shown in Figure 4.

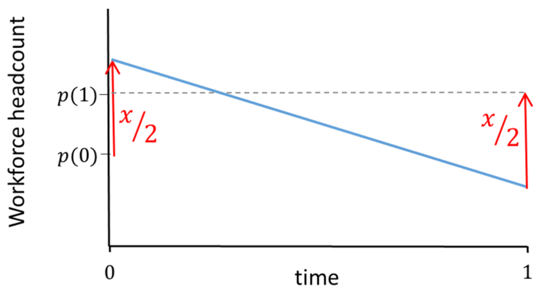

Figure 4: Flows resulting in the half-intake approximation.

Then, attrition takes the headcount from $p(0) + x/2$ down to $p(1) - x/2$ without intervening non-attrition flows. Using Equation (6), we get

$$\alpha \cong \frac{\mathrm{a}}{p(0) + \frac{x}{2}} \tag{17}$$

This formula is known as the Simple Dietz method in the Investment Performance Measurement literature, where it was originally derived in the context of uncompounded returns (Dietz, 1966). In Personnel OR, we have taken to calling it the *half-intake* formula, as it is obtained by adding half of the non-attrition flow (which often consists of new recruits, or intake) to the denominator.

To obtain an expression based only on $p(0)$, $p(1)$ and a, we use Equation (16) and get

$$\alpha \cong \alpha_{HI} = \frac{2\mathrm{a}}{p(0) + p(1) + \mathrm{a}} \tag{18}$$

The half-intake formula was introduced to Personnel OR by Okazawa (2007), based on a derivation that was not tied to the general formula. It has since become the most-often used attrition rate measurement for the Canadian Armed Forces.

4.2 Uniform Taylor Approximation

Figure 4 yields a useful approximation formula, but is a very artificial attrition pattern. A more realistic assumption for many personnel flows is to distribute them evenly across time. We have obtained a good approximation when assuming uniformly distributed net non-attrition flows, along with a constant pace of attrition across the period, which amounts to assuming that attrition behaves according to the internal rate of personnel flow formula. When distributing x uniformly across time into Equation (15), we get

$$p(1) \cong p(0) \cdot (1 - \alpha) + \sum_{i=1}^{x} (1 - \alpha)^{1 - \frac{i}{x+1}} \tag{19}$$

In order to benefit from classical numerical approximations for continuous functions, we map Equation (19) to a continuous flow model:

$$p(1) \cong p(0) \cdot (1 - \alpha) + x \int_0^1 (1 - \alpha)^{1-t}\, \mathrm{d}t \tag{20}$$

which, through integration, becomes

$$p(1) \cong p(0) \cdot (1 - \alpha) + x \cdot \frac{-\alpha}{\ln(1 - \alpha)} \tag{21}$$

In order to avoid a numerical solution of Equation (21) for α, we use the Taylor series

$$\frac{-\alpha}{\ln(1-\alpha)} \cong 1 - \frac{\alpha}{2} - \frac{\alpha^2}{12} - \frac{\alpha^3}{24} - \cdots \tag{22}$$

Then, to avoid having to numerically solve for α, we only keep the quadratic terms. When substituted into Equation (21), we obtain the quadratic polynomial

$$p(1) \cong p(0) \cdot (1-\alpha) + x \cdot \left(1 - \frac{\alpha}{2} - \frac{\alpha^2}{12}\right) \tag{23}$$

which solves for α as

$$\frac{p(0)+\frac{x}{2}-\sqrt{\left(p(0)+\frac{x}{2}\right)^2 + \frac{x}{3}\left(p(0)+x-p(1)\right)}}{-x/6} \tag{24}$$

when $x \neq 0$, and $[p(0)-p(1)]/p(0)$ otherwise. We refer to this as the *uniform Taylor* approximation (denoted α_{UT}).

Notice that if only the linear terms of Equation (22) are kept, we get and alternative derivation of the half-intake formula (Okazawa, 2007).

4.3 Mean Continuous Approximation

Like the uniform Taylor approximation, this one will also be based on the internal rate of personnel flow. First, we convert the periodically compounding rate (α) to a continuously compounding rate (γ), as is often done with rates of return in finance, by defining

$$\gamma = -\ln(1-\alpha) \tag{25}$$

When the conversion is applied to Equation (15), the internal rate of personnel flow formula becomes

$$p(1) \cong p(0) \cdot e^{-\gamma} + \sum_{i=1}^{n} x_i \cdot e^{-\gamma(1-k_i)} \tag{26}$$

The attrition volume over [0,1] defined by Equation (26) can be derived from Equations (16) as

$$\begin{aligned} \mathrm{a} &= p(0) + x - p(1) \\ &\cong p(0) \cdot (1-e^{-\gamma}) + \sum_{i=1}^{n} x_i \cdot \left(1-e^{-\gamma(1-k_i)}\right) \end{aligned} \tag{27}$$

At the same time, the mean headcount over [0,1] defined by Equation (26) can be obtained by separating the effect of attrition on the initial headcount $p(0)$ from its effect on each non-attrition flows x_i as

$$\begin{aligned} \bar{p} &\cong \int_0^1 p(0) \cdot e^{-\gamma t} dt + \sum_{i=1}^{n} \int_{k_i}^{1} x_i \cdot e^{-\gamma(1-t)} dt \\ &= \frac{p(0) \cdot (1-e^{-\gamma})}{\gamma} + \sum_{i=1}^{n} \frac{x_i \cdot \left(1-e^{-\gamma(1-k_i)}\right)}{\gamma} \end{aligned} \tag{28}$$

Dividing Equation (27) by Equation (28), much cancels out:

$$\frac{\mathrm{a}}{\bar{p}} \cong \gamma \tag{29}$$

Because Equation (29) is a continuously compounded rate obtained by dividing by the mean headcount, we call it the *mean continuous* approximation formula. Using Equation (25), we can convert γ back to an annually compounding rate:

$$\alpha \cong 1 - e^{-\mathrm{a}/\bar{p}} \tag{30}$$

Notice that no assumption was thus far made about the pattern of non-attrition flows. Equation (30) is essentially a reformulation of the internal rate (Equation (15)). This is interesting because, Equation (15) and the IRR are generally thought of as requiring a numerical solution. The drawback of Equation (30) is however that it relies on $\bar{p}$, which is not readily available.

We want an approximation based only on $p(0)$, $p(1)$ and a. The straightforward estimate of $\bar{p}$ from these values is $(p(0)+p(1))/2$, which gives

$$\alpha \cong \alpha_{MC} = 1 - \exp\left(\frac{-2\mathrm{a}}{p(0)+p(1)}\right) \tag{31}$$

It is interesting to note that Equation (29) is an estimate of the attrition rate that is obtained by dividing the attrition volume by the mean population – a common definition of attrition used in Personnel OR. However, as we have shown, this rate is correctly understood as continuously compounding, and must be converted to Equation (30), in order to represent an annually compounding rate.

5 EMPIRICAL COMPARISON

The previous section derived three approximation formulas. We now apply them to real-world data, in order to find out how closely they approximate the exact rates produced by the general formula.

We used Canadian Armed Forces Regular Force data covering fiscal years 2009/10 to 2018/19. We measured five different rates: overall attrition, medical releases, Component Transfers (CT) from the Regular Force to the Primary Reserve Force, promotions, and Occupational Transfers (OT) – transfers from one occupation to another, such as from infantry to artillery. Each of the five rates was calculated for 32 different workforce segments. The segments were defined according to the following attributes: age (older/younger than 40), Occupational Authority (Army, Navy, Air Force and Assistant Chief Military Personnel – which covers joint trades, such as medical and logistics) and rank (junior and senior segments for officers and for non-commissioned members). Junior recruits, Generals Officers, Chief Warrant Officers and Special Forces were excluded. At the end of 2018/19, the largest of the segments comprised 8,251 members, and the smallest 183. In total, we thus conducted 1,600 tests for each approximation formula: 5 rates × 10 years × 32 segments.

For each test, the exact general formula rate was calculated using Equation (4), based on daily flow volumes and headcounts. The rate approximations were obtained using only annual figures: a, $p(0)$ and $p(1)$. α_{HI} was calculated with Equation (18), α_{UT} with Equation (24) and α_{MC} with Equation (31).

5.1 Results

Table 1 shows the mean absolute differences between the exact rates and corresponding approximations over all conducted tests. It includes a row for each type of rate investigated and the overall mean for the 1,600 tests of each approximation formula.

Table 1: Mean absolute difference between exact rates and approximations.

	α_{HI}	α_{UT}	α_{MC}
Attrition	0.068931%	0.069231%	0.069488%
Medical	0.022225%	0.022386%	0.022508%
CT	0.008456%	0.008464%	0.008459%
Promotion	0.038723%	0.039441%	0.039422%
OT	0.027445%	0.027347%	0.027390%
Overall	0.032670%	0.032885%	0.032969%

All three formulas provide very close approximations, especially given that personnel flow rates are rarely reported with more than a tenth of a percent precision. The best-performing approximation formula in each row of Table 1 is highlighted in green. We see that the half-intake formula most often outperformed the others.

Table 2 looks at the worst cases among all tests, rather than the mean. None of the three approximation formulas clearly outperforms the others in Figure 2. In all of the 1,600 tests conducted, none of the three approximations were off by much more than 0.5%.

Table 2: Maximum absolute difference between exact rates and approximations.

	α_{HI}	α_{UT}	α_{MC}
Attrition	0.504028%	0.506517%	0.502548%
Medical	0.159302%	0.162011%	0.163096%
CT	0.097457%	0.097575%	0.097451%
Promotion	0.330949%	0.354450%	0.355539%
OT	0.471317%	0.467894%	0.469390%
Overall	0.504028%	0.506517%	0.502548%

The differences between approximations and the general formula were highest for those tests where the population varied most, and when the personnel flow being measured occurred near a population extremum. For example, the worst differences from Table 2 are for a segment where the headcount dropped from 207 to 177, but not before peaking at 224 in July. Furthermore, most of the attrition that year occurred in July near the peak.

Small errors in the measurement of a personnel flow rate will rarely alter the conclusions of an analysis. For example, if the goal of a study is to highlight differences in the flows observed between two segments (e.g. men and women), one would likely only want to draw conclusions from flows that differ by more than a percent. In our worst case, the difference of 0.5% in a population of roughly 200 individuals amounts to a single person.

5.2 Using Monthly Snapshots

When a closer approximation is needed than can be obtained from the methods investigated thus far, an option is to use higher resolution data (e.g. monthly headcounts and attrition volumes). If time is measured in months rather than years, the previous formulas can be reinterpreted as yielding monthly (rather than annual) rates. To obtain an annual rate from twelve monthly rates, the monthly rates need only be compounded as follows:

$$1 - \alpha = \prod_{i=Jan}^{Dec} 1 - \alpha_i \qquad (32)$$

Thus, Equation (32) can be combined with any rate approximation formula applied to monthly data to yield better approximations. Table 3 presents the absolute differences between exact rates and approximations now derived from monthly data, over the same 1,600 tests as before.

Table 3: Absolute difference between exact rates and rate approximations derived from monthly data.

	α_{HI}	α_{UT}	α_{MC}
Mean Difference	0.008974%	0.008957%	0.008959%
Maximum Difference	0.241489%	0.240426%	0.241214%

In practice, the authors often use this approach. We maintain database tables of monthly snapshots that track relevant employee attributes, along with transaction logs for attrition, promotions and transfers. To measure a rate, we obtain monthly headcounts from the snapshots, count the relevant logged monthly transactions, estimate monthly rates, and finally apply Equation (32).

Table 3 shows the uniform Taylor approximation as marginally more accurate. However, it is harder to communicate and less intuitive than the other two. Before completing the present research, the authors had used the half-intake formula for many years, and Tables 1, 2 and 3 confirm that it produces accurate estimates. We will thus continue to use the half-intake formula for approximating reported rates.

Our results are based on Canadian Armed Forces personnel data, which might not be representative of other workforces. Personnel data is not generally shared externally, for privacy reasons, but we would like to invite others to replicate our tests within their own organizations, so as to confirm of our results.

6 CONCLUSIONS

The goal of this paper was to lay a foundation for the study of proportional rates in Personnel OR. We have proposed the general formula for personnel flow rates as that foundation, based on its properties. In addition, we showed how the internal rate of personnel flow can be derived from the general formula, and how it offers a tool for the naïve forecasting of flows. Finally, we justified the need for approximation methods and provided options to obtain such approximations. We showed empirically how our proposed approximations are sufficiently accurate in most cases, especially when computed from monthly personnel data.

This paper addressed the need to appropriately describe proportional rates in Personnel OR. We were able to find inspiration from Investment Performance Measurement, a field where the understanding of proportional growth rates is fairly mature. However, other fields still lack that depth of understanding. One example is the reporting of churn rates for subscription services. The specific requirements and constraints of each field warrant their own investigation, but the results of this paper can hopefully inspire such investigation.

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