The Power Index at Infinity: Weighted Voting in Sequential Infinite Anonymous Games

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Keywords: Cooperative Games, Weighted Voting, Shapley-Shubik Power Index, Infinite Games, Multi-Agent Systems.

Abstract: After we describe the waiting queue problem, we identify a partially observable $2n+1$-player voting game with only one pivotal player; the player at the $n-1$ order.Given the simplest rule of heterogeneity presented in this paper, we show that for any infinite sequential voting game of size $2n+1$, a power index of size n is a good approximation of the power index at infinity, and it is difficult to achieve. Moreover, we show that the collective utility value of a coalition for a partially observable anonymous game given an equal distribution of weights is n^2+n.This formula is developed for infinite sequential anonymous games using a stochastic process that yields a utility function in terms of the probability of the sequence and voting outcome of the coalition. Evidence from Wikidata editing sequences is presented and the results are compared for 10 coalitions.

1 INTRODUCTION

Cooperative games are used in many applications across various domains such as collective decision making, waste management, economics (such as profit sharing in cooperative e-commerce applications), dynamic robot coalition formation, and utility allocation in open anonymous environments (Skibski et al.,2018; Bachrach and Elind,2008; Eryganov et al.,2020; Smirnov et al.,2019; Zhao et al.,2018). Such cooperative games rely on coalition formation, and in most cases, weighted voting is used to predict the collective payoff gained through cooperation. In weighted voting, allocation of payoff depends on how much each agent is decisive in a sequential voting game. Unlike non-cooperative games which analyze the actions performed by individual players, weighted voting is based on the cooperative game theory which focuses on coalition formation strategies and allocating collective payoff based on group actions.

The Shapley-Shubik power index is the most prominent among weighted voting models and it is used in the majority of cooperative game applications. The shapely value provides interesting properties that make it more suitable for fair payoff allocation in cooperative games such as symmetry (identity of players should not have impact on payoff allocation) and efficiency (all available payoff should be distributed among players) (Skibski et al., 2018). Because the Shapley value is equiprobable (Boratyn et al., 2020), the existence of the grand coalition is necessary at least as a mere assumption for tactical reasons (Elkind et al., 2008). Moreover, equiprobability implies that coalitions are formed such that they represent a perfect sample of the population. The Shapley-Shubik power index measures the value of a coalition based on the hidden (marginal) voting power of each member. This hidden power can be described in terms of the marginal value of a member based on his order in a sequence randomly selected from the set A which includes all agents, i.e. the grand coalition. Given the cardinality of A, assuming that players leave the grand coalition sequentially in a random order ,each coalition is a permutation, and the contribution of each player is the probability of his random order in the coalition he joins averaged over all the permutations of A (Skibski et al.,2018; Benati et al.,2019).

Therefore, studying coalition formation strategies and coalitional structures is paramount and has been an important line of research for many years (Skibski et al.,2018).Some proposals identify a stable coalition structure that embeds all coalitions to optimize resource allocation and allow more than one coalition to win (Elkind et al.,2008). Moreover, discrete-time

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Eid, S.
The Power Index at Infinity: Weighted Voting in Sequential Infinite Anonymous Games.
DOI: 10.5220/0010178504750482
In *Proceedings of the 13th International Conference on Agents and Artificial Intelligence (ICAART 2021) - Volume 2*, pages 475-482
ISBN: 978-989-758-484-8

stochastic processes, such as the Chinese Restaurant Process, have been used to model coalition formation in the Shapley-Shubik model (Skibski et al., 2018). For instance, in the Chinese Restaurant Process, agents leave the grand coalition sequentially and each new agent either forms a new partition or joins an existing one (Skibski et al., 2018).

Unlike the Chinese Restaurant Process, in this paper, we consider a waiting queue example where each agent at the order n can form a new partition given that $n+1$ anonymous players will form the remaining partition. Moreover, a discrete time stochastic process, played with a fair coin, is modelled to discard contributions made by consecutive voters to model heterogeneity in real-time. Real-time heterogeneity is considered an important prerequisite for effective coalition formation and fair distribution of rewards in cooperative multi-agent systems (Smirnov et al., 2019). Nonetheless, we believe that studying weighted voting in infinite games and creating dependencies on future group actions, especially for open anonymous environments, is important for identifying the threshold beyond which cooperative patterns may need to be refreshed or altered to a certain extent. This means that we highlight the possibility that cooperative patterns in infinite games may reach a certain level of exhaustion at points in time when they achieve the threshold at which task requirements are satisfied.

Fundamentally, the most important condition in the Shapley-Shubik model states that the number of agents in a game is finite and a value function v describes the value of the coalition by mapping subset players to some real number such that $v: 2^N \rightarrow R$. Where v is the collective payoff which the members of a coalition $S \subseteq A$ gain through cooperation. However, the payoff for each player depends heavily on the player's order in the sequence. For instance, the Shapley value of player i in a game of n players described as $G(v, A)$ is :

$$\varphi_i(v) = \frac{1}{n!} \sum_{Q} [v(R_i^Q \cup \{i\}) - v(R_i^Q)] \tag{1}$$

Therefore, the marginal voting power is calculated over the range $n!$, and R_i^Q is the set of players in the sequence Q which precede i. The winning coalition is the one which achieves a certain quota concluded based on either a fuzzy rule or a pre-identified value (for example, fuzzy rules are used to limit cooperation by restricting payoff for some players - see Gallardo and Jiménez-Losada (2017)).Each sequence plays a crucial rule in identifying the value of the coalition, however, when the number of players is large, the Shapley value takes a heuristic form:

$$(S_v)(ds) = \int_0^1 (v(tA + ds) - v(tA)dt \tag{2}$$

Where tA is the proportionality of coalition $S \in A$, and A contains a large number of players. This latter setting matches an infinite game model in which the identity of players does not play a crucial rule in deducing the value of the coalition. In this paper, we redefine the proportionality in terms of the likelihood of occurrence of the sequence. In infinite sequential ordered votes, each player has a minor contribution and the game evolves as a nonlinear game wherein the value of the coalition is more important than the payoff obtained by each agent. In particular, individual contributions are minor with respect to the collective contribution of the coalition. Moreover, in sequential infinite games, it is common to divide the action space into finite sequential games that approximate the utility gain in general situations (Reeves and Wellman, 2012).

Unfortunately, much of the work produced in this field is directed towards studying finite voting games and less work has addressed infinite sequences. In this paper, we obtain the likelihood of the occurrence of any finite sequential partition tA in infinite sequential anonymous games. Firstly, we describe the waiting queue problem, then we identify a $2n+1$-player game with only one pivotal player; the player at the $n-1$ position.We use a stochastic game formula to model an infinite sequence given the simplest rule of heterogeneity in anonymous games identified in our method. We reach an approximate of the power index at infinity using samples of finite sequences, and we show that it is difficult to achieve. Our method is described in the next section. Section 3 analyses the results. Section 4 discusses related work. Finally, section 5 concludes the paper.

2 METHOD

Infinite sequences disproportionately influence the voting power of players because at any position in a sequential game the number of remaining voters is undefined or possibly large. In this case, predicting the proportionality of a coalition is more viable than computing the collective payoff based on the shapely value of each player. For instance, a voting game $G(v, A)$ can be represented as a vector of values $W =$

$(w_1, w_2, \dots w_n; q)$ that represents the weights of players and q is the quota (Bachrach and Elind, 2008). If W represents the weights of an infinite number of players, the shapely value is undefined. To reduce this problem, we assume that the weight of each position in an infinite sequence is dependent on the number of players yet to vote, and moreover, we assume that every infinite game is partially observable. By "position" we mean the order of the player regardless of the identity of neighboring players.

This problem resembles a waiting queue problem in which the weight of any agent i is dependent on the number of agents that precede i in the queue. However, to deal with the problem of having an infinite number of players, we assume that every queue of size n is only a partition of a larger queue of size $2n+1$. if we consider a queue with size $2n+1$, and the function $f(n) = k$ identifies the waiting time of any member i_{k-1} at the $k-1$ position (assuming that each member corresponds to a single unit of time), we find that $f_k(2n+1) = 0$, $f_k(2n) = 2n+1-2n = 1$, and we always find that $f_k(n) = n+1$.This is observable for any sequence of size $2n+1$. However, algebraically we can conclude that $f_k(n-1) = (n-1)+1 = n$.

In light of the previous queue problem, we define a game $G = (A, v)$ is a voting game with A: the set that contains all agents (grand coalition), and v is a value function that maps a coalition to some real number ; $2^N \to R$.Now consider a finite game with size $2n+1$ and quota $q = n$,i.e. $G: \{a_1, a_2, a_n \dots . a_{2n+1}; n\}$ where n is any small number. In addition, the collective utility (value) function is identified by $\varphi_i(v) = w_i t_i$ where $\in \{0,1\}$, i.e. voters can only conduct a 'yes' or no 'vote' and $w = f_k(n) = n+1$ for any i. In this case, we find that the only pivotal player is the player at the $n-1$ position.

Lemma 1: Given a partially observable sequential voting game of size $2n+1$ of which only an n-player partition is observable, the player at the order $k = n-1$ is pivotal for the grand coalition.

Proof: For any quota $q = \mathrm{n}$ and Q^{2n+1} is the sequence identified by $G = \{a_1, a_2, a_n \dots . a_{2n+1}\}$, we split G into $G' = \{a_1, a_2, .. a_n\}$ and $G'' = \{a_{n+1}, a_{n+2}, .. a_{2n}, a_{2n+1}\}$ .We the assume that only $G'$is observed with the assumption that $\exists t_{i>n} \in T = 1$and $i \in G''$,and therefore, the quota q' for G' is $n-1$ .Given that weights are distributed equally among voters, we find that the player at the order $k = n-1$ is pivotal for G' .Given that G'' is not observable, we conclude that player $a_{\mathrm{n}-1}$ is pivotal for an infinite sequence of size $2n+1$.

Example 1: Consider the game $G = \{1,1,1,1,1,0,0,0,0; 4)$,this is a $2n+1$ game with $n = 4$. If all players up to the n^{th} position vote 'yes', the quota is achieved assuming that there is at least one player at any position $k > n$ will vote 'yes'. In the above example, the outcome for $G' \in G$, where G' contains all players up to the n^{th} position, is 4.Any player in the remaining sequence, let's say the one at the order $k = n+1$, can guarantee achieving the quota if he conducts a 'yes' vote. In the above game, the player at the $n+1$ position is the one at the 5th position, therefore, the collective outcome of G is 5. Therefore, given a $2n+1$ game and an n-player partition , at the order $k = n$, we find that there are still $n+1$ players yet to vote , and assuming that at least one player in the remaining sequence will conduct a 'yes' vote , we conclude that the pivotal player in any infinite sequential game of size $2n+1$ is the one at the $n-1$ position. Although the player at the n^{th} order guarantees achieving the quota if he conducts a positive vote, he is not pivotal to the coalition. Our interest here is identifying the pivotal player that first guarantees achieving the quota, regardless of whether the quota is exceeded, and given that at least one player in G'' will conduct a positive vote.

Lemma 2: For an infinite sequential voting game and a coalition of size $2n+1$ players, and quota $q = n$, the value of the coalition $\varphi(v)$ is $n^2 + n$.

Proof: Consider the coalitional utility value function $2^N \to R$ represented by $\varphi(v) = \sum w_i t_i$ and $t \in \{0,1\}$, and moreover, the function $f_k(n) = n+1$ represents the weight w_i of player i at any order k.For simplicity, we assume that $\forall\, (i) \in A, t_i = 1$. For an n-player partition we get :

$$\sum_{k=1}^{n} w_i t_i = n(n+1) = n^2 + n \tag{3}$$

By revisiting example 1 above, we notice that the collective value of the coalition should be $(4)^2 + 4 = 20$.Moreover, the collective value of $G' \in G$ is 14, where G' contains all players up to the n^{th} position. In this case, the first player at the least $n+1$ order, i.e. the player at the 5th position, has sufficient weight to achieve the coalitional value, however, there is uncertainty about the state of all players at $k > n$ since G is an infinite game.

Note that for any game of size $2n+1$ there is a partition of size n, i.e an n-player game that contains at least 2 players. This rule is violated for $n < 2$. Consider a game with $n = 1$ and 3 players, for example $G = \{0,0,1;1\}$, at the n^{th} position there should be a number of $n-1$ players that guarantee achieving the quota if they all conduct a "yes' vote. Therefore, for a $2n+1$ player game with 3 players there is no pivotal player, and the quota is not achievable in this case.

For infinite sequential voting games, we model a sequence of infinite games each with $n = 1$ using a discrete-time stochastic Bernoulli process. Note that given a Bernoulli process with $n = 1$, a_1 does not affect the state transition in all cases because it is the initial input to the process. Moreover, in anonymous environments, heterogeneity is not observed, and therefore, the weight is identical for any two players i and j. Therefore, in order to model heterogeneity in its simplest form, we assume that anonymous players cannot conduct two consecutive votes. This rule is described in definition 1 below:

Definition 1: In anonymous games, the simplest Rule of Heterogeneity (RoH) states that $i \neq j$ where i and j are the players at the positions $k-1$ and k respectively.

Follows from definition 1 that the total number of votes in any fair game with an ordered sequence Q is bounded according to Lemma 3 below:

Lemma 3: Given an infinite anonymous game with fair distribution of votes, the sequence Q^{2n+1} with size $2n+1$ has n number of votes.

Proof: Given a Bernoulli stochastic sequence of games, each with a state space {0, 1}, and assuming that all games are played using a fair coin, a sequence of size Z contains $Z-1$ coin tosses. A sequence of size $2n+1$, therefore, contains $2n$ coin tosses with probability $\frac{1}{2}$ for any selection.

To model heterogeneity, we use a discrete-time Bernoulli stochastic game with state space $\{a = 0, b = 1\}$ where a and b are Boolean variables that represent the satisfaction and dissatisfaction of RoH.

Moreover, $Strategy_{WIN} = \{a\}$ is the winning strategy which occurs when RoH is satisfied, and $Strategy_{LOSS} = \{b\}$ is vice versa. Thus, given that the random variable X has a Bernoulli distribution, the probability of obtaining a win is P and the probability of obtaining a loss is $1-P$. The game starts at state T_1 with applying the biased function $f'(X)= x$; if the result is $f'(X) = b$ then there is a loss and the process stays at T_1, if the result is $f'(X) = a$ then the result is a win and there should be a state transition to T_2. Note that the function $f'(X)$ does not guarantee collecting n votes because $f'(X)$ is not a fair function yet it represents the likelihood of occurrence of the best possible sample of the population. More precisely, $f'(X) = a$ is the degree to which the voting sequence is heterogeneous. Given an infinite random game, intuitively, the probability of obtaining n votes should be extremely low.

Moreover, in a sequence of size $2n+1$, the probability of obtaining l losses is P^l and the probability of obtaining w wins is $(1-P)^w$, and $w = 2n+1-l$. The higher the value of P, the less likely two consecutive wins will occur. Now considering the sequence Q, given that the discrete random variable X has a Bernoulli distribution, the probability of obtaining Q is given by $P(Q) = P(X_1, X_2, X_3, X_4, \ldots . X_n) = P^l(1-P)^w$. The reason Q is observed is due to the collective measure of fairness it can provide; P(Q) is the likelihood of occurrence of the partition tA which is a perfect sample of the population of agents. To calculate the power index for the k[th] player in the game, equation 1 is not valid because the sequence can be large enough such that equation 1 cannot be solved in polynomial time. Equation 2 satisfies a voting game with large number of participants, however, it does not satisfy an infinite game in which the identity of agents cannot be resolved in real time. The probability of occurrence of any sequence Q is $P(Q)$ and it has been concluded as described above. Therefore, given that the game is anonymous and with a large number of players, it is reasonable to replace tA with P(Q),the probability of the sequence, in equation 2 above, and thus, the index of the k^{th} player in the game is calculated as follows:

$$(S_v)(ds) = \int_0^1 \left(v\left(\frac{P(Q^{2k})}{1-P(Q^{2k})}\right) + v\left(\frac{P(Q^{2k+1})}{1-P(Q^{2k+1})}\right)\right) - \frac{P(Q^{2k})}{1-P(Q^{2k})} \tag{4}$$

Since the contribution of each player is minor, we have assumed that weights are distributed equally among voters. Therefore, $(S_v)(ds)$ now represents the power index of agent a_k in terms of the marginality (hidden power) over the range $1-P(Q)$. Due to the minority of individual contributions, the value ds now is very small and can be ignored, hence, the value function at position k can be reduced to represent the power index of the k^{th} position. This is calculated in equation 5.

Table 1: Sample of the results of 10 coalition obtained by analysing Wikidata's editing sequences.

	P	$1-P$	$P(Q)$	$1-P(Q)$	$w_k(v)$	n'	$\varphi_k(v)$	$F(\lambda)_k$
e_1	0.55465587	0.44534413	0.247012736	0.752987264	0.32804371	73	27	0.037037037
e_2	0.58552631	0.414473684	0.242685249	0.757314751	0.320454935	65	35	0.028571429
e_3	0.64734299	0.352657005	0.228290042	0.771709958	0.295823631	61	39	0.025641026
e_4	0.29003021	0.709969789	0.205912688	0.794087312	0.259307364	56	44	0.022727273
e_5	0.57142857	0.428571429	0.244897959	0.755102041	0.324324324	55	45	0.022222222
e_6	0.27118644	0.728813559	0.197644355	0.802355645	0.246330111	38	62	0.016129032
e_7	0.37804878	0.62195122	0.2351279	0.7648721	0.307408128	54	46	0.02173913
e_8	0.28179551	0.718204489	0.202386801	0.797613199	0.253740537	72	28	0.035714286
e_9	0.61940298	0.380597015	0.235742927	0.764257073	0.308460249	64	36	0.027777778
e_{10}	0.178	0.822	0.14631	0.853684	0.17139363	40	60	0.016666667

$$w_k(v) = v\left(\frac{P(Q^{2k+1})}{1-P(Q^{2k+1})}\right) \tag{5}$$

Indeed, the quota for any sequence $\{a_1, a_2, a_n, \ldots\ldots a_{2n+1}\}$ is $n' = n$ .Where n' is the number of votes obtained by using the biased function $f'(X)$ at the pivotal position where $k = n - 1$ and $w_k(v)$ is the marginal power at the k^{th} position. Moreover, there is a proximity function that measures the degree to which the outcome of any sequence Q is close to the quota:

$$F(\lambda)_k = \frac{1}{|n - n'|} \tag{6}$$

Note that as n' gets closer to n, $F(\lambda) \to \infty$ and $F(\lambda)_k \in [0, \infty)$. The value of the coaltion in this case can be concluded by updating equation 3 as follows:

$$\varphi_k(v) = w_k(v) * n' \tag{7}$$

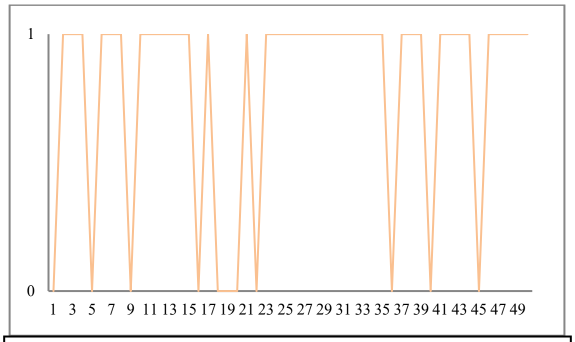
Where $\varphi_k(v)$ represents the collective utility value of the coalition, i.e. the payoff members of the coalition gain through cooperation. In the next section, we use equations 6 and 7 to derive the coalitional values for 10 Wikidata coalitions. Furthermore, equation 6 is used to calculate the proximity value for each coalition with respect to the predicted output at infinity (which should precisely be equal to ∞).

3 ANALYSIS

In this analysis we exploit the nonlinearity of Wikidata, the largest structured crowdsourced knowledgebase that currently exists. Wikidata depends on a peer production service in which a large sample of the general population, called the crowd, is hired to perform editing tasks. Editing tasks merely depend on individual agent preferences. Therefore, selecting a targeted knowledge resource (entity) is a random process, as well as selecting the time to execute edits.

The editing process in Wikidata is a sequential process with no bound on the amount of edits required to complete the knowledge. Thus, editing events can be modelled as an infinite sequential voting game where each edit is equivalent to a single vote. Moreover, identity of editors does not affect the editing sequence, while in particular many of the editing events are already performed by anonymous users or automated group programs (bots).Therefore, Wikidata editing events can ideally be modelled as an infinite anonymous voting game. In this context, editing events of many entities were analyzed. A sample of the result is shown in table 1 above. The editing sequence of each entity represents a coalitional game for which the value of the coalition at the k^{th} position is calculated as in equation 5 and each entity is represented by a coalition $S \subset A$.

Moreover, entities are chosen such that the assumption can be made that the progress of different coalitions is not necessarily competitive, but distinct games should at least be comparable. Concerning the targeted quota, the progress of 1000 coalitions over five iterations, was analysed, but no coalition achieved the target quota. As expected, n is difficult to achieve in an infinite game. As shown in table 1 above, some coalitions came very close to n with proximity as low as ~ 0.016 , but no coalition achieved $F(\lambda)_k = \infty$ which means that n can be considered as the power index at infinity.



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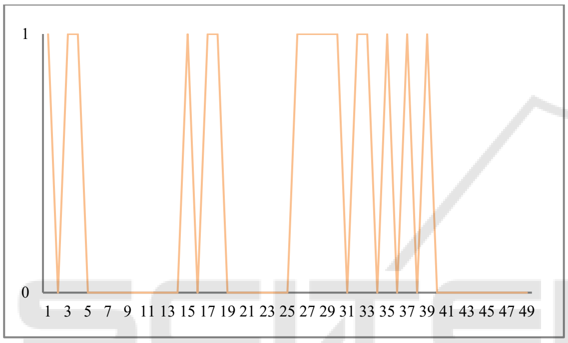
1011110111110101101110010000111011111111
0111011111011111010001

Figure 1: Voting pattern and BF for e_1.



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0001010101110011011011

Figure 2: Voting pattern and BF for e_6.

Equation 7 is used to obtain the utility value of each coalition as shown in table 1.By examining many entities, we noticed that there is a strong relation between the utility function $\varphi_k(v)$ and the publicity of entities. For example, the above entities represent a selection of higher educational institutions, the two entities with the highest coalitional values 62 and 60 are the most prominent in this list. Similarly in other coalitions that belong to comparable categories, in many cases, the value of the coalition is related to the publicity of entities.

Because $\varphi_k(v)$ also represents the quality of cooperative patterns in terms of achieving real-time heterogeneity (represented by the probability of the sequence captured at certain points in time), this result suggests that entities with higher public awareness may have higher quality cooperative patterns. However, the quality of cooperative patterns should not directly relate to the quality of the content of the corresponding Wikidata articles.

Moreover, the sequential voting scheme for Wikidata yields a Boolean fingerprint (BF) as shown in figures 1 and 2 above. Both figures show the BF for the coalition with the largest gain (e_1) and the coalition with the lowest gain (e_6) respectively at the pivotal position $n-1$ for $n=100$ (the complete BF for each entity is shown below each graph). However, it should be noted that $P(Q)$ does not represent the number of aggregated votes, but it represents the probability of the sequence, and it depends on the degree to which the voting sequence is heterogeneous. Heterogeneity in real-time is not the total number of players in the coalition, but it is the number of consecutive actions performed by different players. Therefore, on some occasions, coalitions with lower number of players may have highly heterogeneous patterns, hence higher quality cooperative patterns, than coalitions with higher number of players.

4 RELATED WORK

Skibski et al. (2018) derive the “stochastic shapely value” by applying the Chinese restaurant process to games with externalities .In the Chinese Restaurant Process, each player can either form a new group with probability $1/n$ or join an existing group with probability b/n where b is the number of existing players. The stochastic shapely value takes into consideration , not only the marginal power of each player, but also the weighted average over all partitions , which is based on the probability that the coalition will form using the Chinese Restaurant Process.

Benati et al. (2019) Show that stochastic approximation can produce effective sampling of additive exponential values in cooperative games. This is achieved by applying probability concepts only to the sample and reduced sum. They show that there stochastic approximation method produces accurate predictions equivalent to the actual value of the game with minor standard error.

de Keijzer et al.(2010) addressed the possibility of manipulating voting games by designing a weighted voting game that yields a target power index, or at least a power index that is close enough to a certain threshold in an n-player game. This problem is a difficult one because identifying a priori for agent i’s position such that the coalition yields a certain value requires examining an infinite set of weights and calculating all permutations over the finite set of players, and moreover, this applies to each weighed voting game. Our method escapes this restriction for two reasons: 1. there is an infinite number of players which implies that each player has a minor contribution to the value of the coalition.2.players are anonymous, and given the simplest RoH, the maximum contribution of each player is bounded by uniformity.

However, the method represented by de Keijzer et al. (2010) requires excluding dummy players and identifying a superset, i.e. a super coalition that is always winning, then iteratively identifying the maximum-weight losing coalition. The previous approach is close to identifying the best response profiles in partially observable infinite games (Reeves and Wellman, 2012). Usually, computing the linear best response strategies in infinite games, such as the work of Reeves and Wellman (2012), needs a number of steps: 1.developing an algebraic formula for predicting the utility given a strategy and a set of parameters. 2. The action space of agents is then partitioned, and the maximum utility action, i.e. the action with the potential to maximize the utility expression, is identified. This method depends heavily on effectively identifying partitions such that the maximum utility action is the best one for generalization.

To study false name manipulation, Bachrach and Elind (2008) studied a game that consists of all players in the grand coalition, and weights are distributed fairly among players. In addition, a value function $\varphi_i(v) \leq 2$ describes the gain and each player can split his weight fairly between two false identities. For example ,in a game $G: \{2,2,2, \ldots; 2n\}$,a player can split his weight equally among 2 identities resulting in the game $G': \{2,2, \ldots 2,1,1; 2n\}$. Indeed, in such game the shapely value for any player is $1/n$ (given that this is the only permutation that exists).Therefore, the maximum gain for the last two players with false identities is $2n/(n+1)$.Due to our RoH we have considered the game $\{2,1,1; n\}$,and given our stochastic process , the maximum gain of this coalition is $n+1$.Moreover, to model an infinite game of this sample we have generalized our formula to model any sequence of size $2n+1$.

5 CONCLUSIONS

Infinite sequential games are now important in internet open anonymous environments, such as the Wikidata case study presented in this paper. We have reached the best approximation of the voting power index at infinity for a coalition. In light of the waiting queue problem described in this paper, we identified a voting game with only one pivotal player. In particular, for a partially observable sequence of size $2n+1$, we have found that the only pivotal player is the one at the order $n-1$ given a utility function that identifies the coalitional value gained through cooperation. Moreover, we have found that n is a good approximation of the power index at infinity, and it is difficult to achieve. This result is based on the simplest rule of heterogeneity which prevents anonymous players from conducting consecutive votes.

We have applied our game formula to Wikidata editing sequences of many entities and a sample of the results has been presented. Using a discrete-time stochastic process, we have modelled Wikidata editing events as infinite sequential voting games. The quality of cooperative patterns, identified in terms of the degree of real-time heterogeneity, has played a key role in yielding higher coalitional values. An interesting line of research for future work is identifying the threshold beyond which cooperative patterns need to be refreshed to yield higher coalitional values or to maintain a winning strategy.

ACKNOWLEDGEMENTS

The author would like to thank the anonymous reviewers for their constructive comments which contributed to improving the final version of the paper.

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