

A Survey of Survival Analysis Techniques

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Abstract: Survival analysis is a branch of statistics for analyzing the expected duration of the time until the event of interest happens. It is not only applicable to biomedical problems but it can be widely used in almost every domain since there is a relevant data structure available. Recent studies have shown that it is a powerful approach for risk stratification. Since it is a well established statistical technique, there have been several studies that combine survival analysis with machine learning algorithms in order to obtain better performances. Additionally in the machine learning scientific field the usage of different data modalities has been proven to enhance the performance of predictive models. The majority of the scientific outcomes in the survival analysis domain have focused on modeling survival data and building robust predictive models for time to event estimation. Clustering based on risk-profiles is partly under-explored in machine learning, but is critical in applications domains such as clinical decision making. Clustering in terms of survivability is very useful when there is a need to identify unknown sub-populations in the overall data. Such techniques aim for identification of clusters whose lifetime distributions significantly differs, which is something that is not able to be done by applying traditional clustering techniques. In this survey we present research studies in the aforementioned domain with an emphasis on techniques for clustering censored data and identifying various risk level groups.

1 INTRODUCTION

The pursuit of accuracy is the primary purpose of almost all human field endeavors. A good pursuit of accuracy might be the dominant expectation for the practitioners, especially in the healthcare field, given the extremely small margin of error on the predictions which they make. Survival analysis is a major decision technique in healthcare practices. It can be used for a variety of reasons such as the deeper understanding of the effect of some genetic or proteomic bio markers on prognosis of cancer patients, understanding the impact that risk factors such as diabetes, hypertension and other cardiovascular diseases have on Chronic Kidney Diseases (CKD) or even know the outcome of physical exercises, diets or family health history in understanding cardiac heart problems in patients. In general it is used to predict the time until a particular event of interest happens. Although it was initially created in terms of medical research and the purpose was to model a patient's survival, it can be applied to several other application domains. For instance, survival analysis can be used to estimate

the probability of failure of manufacturing equipment based on the hours of operations. It turns out that the vast majority of the survival analysis research results have focused on time to event prediction either by using statistical methods or by using machine learning algorithms. Despite its importance there is only limited number of research papers focused on survival clustering. For instance in many real world cases practitioner's main pursuit is to discover the various sub-groups of a cohort that corresponds to different risk levels and not necessarily the individual risk estimation of each subject. In such a scenario the target would be the discovery of the clusters which not only have similar traits but simultaneously have different lifetimes. The purpose of this paper is to emerge the importance of this type of methods used for risk stratification. Such methods have been developed for risk-profile based clusters-discovery in a cohort that may have unknown number of clusters. The remainder of this paper is divided into four sections which include the taxonomy of methods that have been used for survival analysis, problem formulation and the definitions of the main survival functions, summary of all the methods used for risk-based clusters discovery and conclusion.

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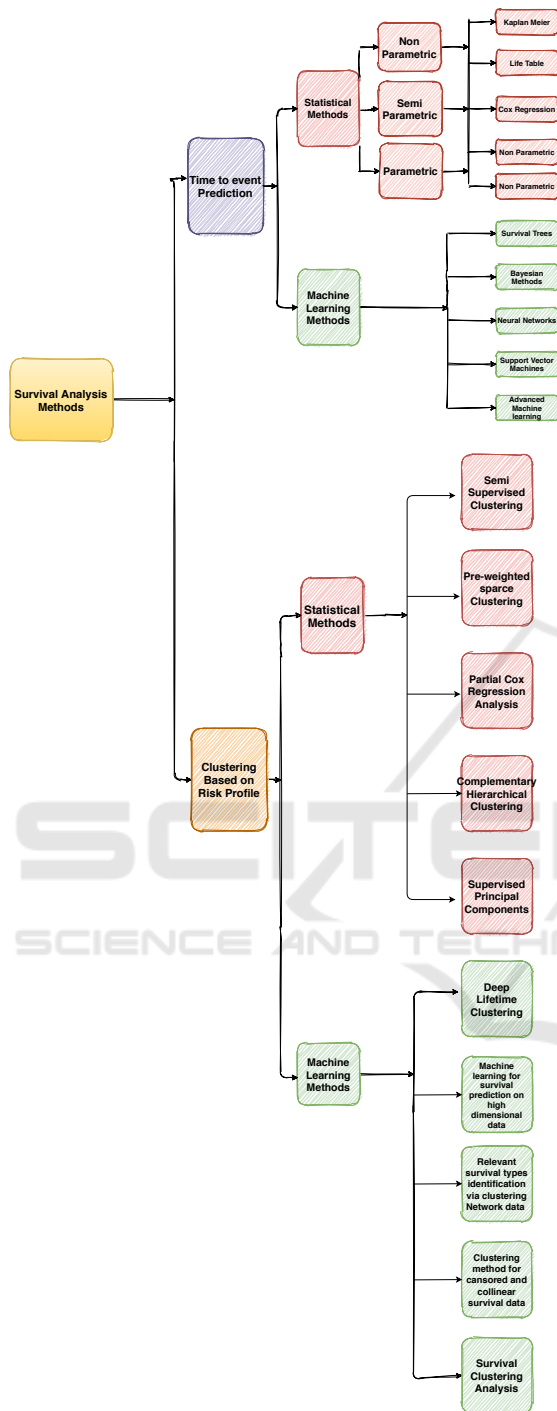


Figure 1: Survival Analysis Taxonomies.

2 TAXONOMY OF SURVIVAL ANALYSIS METHODS

Survival Analysis methods can be divided in Statistical and Machine Learning methods. Statistical meth-

ods can be divided into three wide categories: (i) parametric, (ii) non-parametric and (iii) semi-parametric methods. Under the assumption of having a data set that follows a particular distribution, parametric methods can be very efficient and accurate for time to event prediction. For example assuming the time of the examined data set follows a well known theoretical distribution such as exponential it is quite simple to use it for time to event estimation. Although in real life datasets, it is difficult to obtain data that follows a known theoretical distribution. Non-parametric methods can be used in this case since there is no underlying distribution for the event time and there are no assumptions that need to be met. Kaplan Meier (Kaplan and Meier, 1958) method is one of the most popular methods of this category. Third category is a hybrid of the parametric and non-parametric approaches. Like non-parametric methods, at semi-parametric models the knowledge of the underlying distribution of time to event is not required. Cox model (Cox, 1972) is the most widely used semi-parametric survival analysis method in this category and it assumes the attributes have a multiplicative effect in the hazards function that is constant over time.

2.1 Machine Learning Techniques for Time to Event Predictions

Despite statistical techniques which aim to characterize the distribution of the event times as well as the statistical properties of the parameters of each (statistical) model, machine learning methods seek for making prediction of event phenomenon at a given time point. Decision tree algorithm (Bou-Hamad et al., 2011) which is grounded on splitting the data recursively based on a particular splitting criterion adapted for survival analysis (Bou-Hamad et al., 2011). Since the main characteristic of such an algorithm is the splitting criterion there have been some research studies focusing on the finding of a splitting criterion that can be effectively used for survival analysis (Ciampi et al., 1987). Bayesian Analysis is one of the most fundamental principles in statistics which links the posterior probability and the prior probability. Several studies have used such models to predict the probability of the event of interest (Kononenko, 1993) and benefit from the good properties of Bayesian modeling (Ibrahim et al., 2014) such as interpretability. Support vector machines (Van Belle et al., 2007) also are a very important category of machine learning algorithms which can be used both for classification (Shivaswamy et al., 2007) and regression and have been successfully adapted to survival analysis problems.

In addition neural network - based machine learning models have been proposed to predict lifetime of a subject. (Katzman et al., 2018) is based on the semi parametric Cox Proportional Hazard model. Furthermore there have been introduced deep learning techniques for life time prediction such as (Giunchiglia et al., 2018) that is based on a recurrent neural network architecture. The proposed neural network takes as inputs at each time the features characterizing the patient and the identifier of the time step, creates an embedding, and outputs the value of the survival function in that time step. In Figure 1 we demonstrate a graphic illustration for the taxonomy of all the survival methods that have been introduced in literature not only for solving the task of time to event prediction but also techniques that have been proposed for clustering based on risk profile.

3 SURVIVAL ANALYSIS PROBLEM FORMULATION

A given observation i in our dataset represented by a triplet (X_i, y_i, z_i) , where $X_i \in R^{1 \times P}$ is the feature vector, z_i is the binary event indicator which is marked as 1 when the subject has experienced the event of interest, otherwise it is marked as 0. Finally y_i is the observed time which is equal to the survival time T_i if the given observation is uncensored otherwise C_i , if the given observation is censored.

$$y_i = \begin{cases} T_i, & \text{if } z_i = 1 \\ C_i, & \text{if } z_i = 0 \end{cases} \quad (1)$$

The purpose of survival analysis is to estimate the time to the event of interest for a new instance k with feature predictors denoted by a new feature vector X_k .

3.1 Concept of Censorship/ Censored Data

In real world scenarios the collection of complete data sets may be a challenging expectation due to various reasons. Especially in clinical studies the data collection period may last for several years and would require consistency of the participants of the study in order to keep tracking their data. It is possible that the event of interest can not be observed for some instances. Censorship is related to the problem of missing data, consequently this is the main reason which a traditional regression model cannot be fitted for making predictions in such kind of data (Prinja et al., 2010). Censorship can generally be observed in three variations (Clark et al., 2003): (i) right-censoring,

which is the most common type of censoring and it occurs when the observed survival time is less than or equal to the true survival time, (ii) left-censoring, for which the observed survival time is greater than or equal to the true survival time, and (iii) interval-censoring, for which we only know that the event occurs during a given time interval. In survival analysis when utilizing censored data the time to the event of interest is the target variable. This is only known for those instances who have experienced the event during the study period. In Figure 2 we cite an example of a cohort that may be used in the context of survival analysis. The cohort is made up of four subjects each one of them belongs to a different category of censorship and one not censored subject. Subject 1 is not censored since we have monitored the duration of its life from the early start until the event occurs. Subject 2 and subject 3 are considered to be right censored because we have track them from the early start of our study but it had not been experienced the event of interest until the end of study so we do not have precisely information about the duration of its lifetime. Subject 4 is considered to be left censored since it is unknown when it entered the study and subject 5 is considered to be interval censored since data collectors lost its signals in the middle of the study.

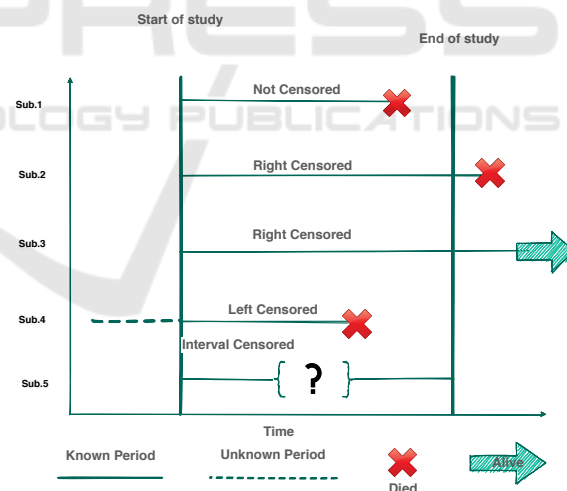


Figure 2: Concept of Censorship.

3.2 Survival Analysis Formulas and Definitions

The survival function (Lee and Wang, 2003) represents the probability that the time to the event of interest is not earlier than a specified time t . Often survival function is referred as: the survivor function or survivorship function in problems of biological survival, and as reliability function in mechanical sur-

vival problems. Survival function is represented as follows:

$$S(t) = P(T > t) \tag{2}$$

The function above denotes an individual that survives longer than t . Survival function decreases when the t increases. Its starting value is 1 for $t = 0$ which represents that in the beginning of the observation all subjects survive. From the definition of cumulative death distribution function $F(t)$,

$$S(t) = 1 - F(t) \tag{3}$$

Cumulative death function represents the probability that the event of interest occurs earlier than time t . The survival function is therefore associated with a continuous probability density function by

$$S(t) = P(T > t) = \int_t^{\infty} P(x)dx \tag{4}$$

Similarly the survival function is related to a discrete probability $P(t)$ by

$$S(t) = P(T > t) = \sum_{T>t} P(x) \tag{5}$$

3.3 Survival Hazard Function

In survival analysis, another commonly used function is the hazard function $h(t)$, which is also called the force of mortality, the instantaneous death rate or the conditional failure rate (Dunn and Clark, 2009). The hazard function t (Lee and Wang, 2003) does not indicate the prospect or probability of the event of interest, but it is the rate of event at time t as long as no event occurred before time t . In this sense, the hazard is a measure of risk. The hazard function is defined as:

$$h(t) = \frac{f(t)}{S(t)} \tag{6}$$

Specifically is the ratio of the probability density function to the survival function. In particular since by definition the probability density function is:

$$f(t) = \lim_{dt \rightarrow 0} \frac{F(t + dt) - F(t)}{dt} \tag{7}$$

where dt denotes the time interval we can write the hazard function as:

$$h(t) = \lim_{dt \rightarrow 0} \frac{Pr(t \leq T < t + dt | T \geq t)}{dt} \tag{8}$$

It is defined as of failure during a very small time interval assuming that the individual has survived to the beginning of the interval. The hazard function can also be defined in terms of the cumulative distribution function $F(t)$ and the probability density function $f(t)$ as:

$$h(t) = \frac{f(t)}{1 - F(t)} = \frac{f(t)}{S(t)} \tag{9}$$

and finally the cumulative hazard function describes the accumulated risk up to time t given by

$$H(t) = \int_0^t h(x)dx \tag{10}$$

In addition to the above relations, there is another important connection between $h(t)$ (or $H(t)$) and $S(t)$ given by

$$S(t) = \exp\left(-\int_0^t h(x)dx\right) = \exp(-H(t)) \tag{11}$$

4 CLUSTERING BASED ON RISK PROFILE

Li and Gui proposed a different extension of partial least squares (PLS) regression to the censored survival data in the framework of the Cox model by providing a parallel algorithm for constructing the latent components (Li and Gui, 2004). The proposed algorithm involves constructing predictive components by iterated least square fitting of residuals and Cox regression fitting. These components can then be used in the Cox model for building a useful predictive model for survival. Although the purpose of constructing such a method is not towards survival clustering it can also be used for clustering survival data since the principal components are constructed as well.

In a relevant study, researchers noticed the importance of cancer subtype discovery using genes expression data and clinical data together (Bair and Tibshirani, 2004). Discovered subtypes appeared to have significant differences in terms of patients survival when the semi supervised proposed technique was used. The authors addressed the problem of cancer subtypes identification without having any prior knowledge of the existence or the number of cancer subtypes in the dataset. The whole process of this approach has two parts. Firstly only genes expression data are utilized using Cox regression in order to assign each of them a (“Cox”) score and then select only those genes with high score. After that procedure only significant genes have been chosen for the dataset. Having chosen a subset of genes expression they apply traditional clustering techniques e.g. K-means only on genes expression data and they obtain the desirable number of clusters. At the second part of the proposed approach they test the cluster assignment using only the clinical data. Utilizing clinical data they set cluster assignment as the dependent variable and apply classification algorithms. Finally the classification algorithm performs well which means

that clusters assignments have been correctly identified.

Bair (Bair et al., 2006) and Tibshirani (Bair and Tibshirani, 2004) mentioned the drawback of the usage of principal components for regression and survival model which is the fact that few principal components may summarise a large proportion of the variance present in the data in this way there is no guarantee that these principal components are associated with the outcome of interest. Therefore they proposed (Bair et al., 2006) a semi-supervised approach, which they called supervised principal components (SPC). In this method univariate Cox scores are computed for each feature and the choice of the most significant features is done by picking only the features with the best Cox scores obtained. Mainly supervised principal components method is similar to conventional principal components analysis except that it uses a subset of the predictors selected based on their association with the outcome. An improved variation of previous method “pre-weighted sparse clustering” has also been proposed (Gaynor and Bair, 2017). As mentioned before Sparse clustering method and also semi-supervised clustering method have significant limitations mainly because they are heavily depend on the number of features that have been characterized as “significant”. Pre-weighted sparse clustering aims to overcome the limitations of sparse clustering by performing conventional sparse clustering. It identifies features whose mean values differ across the clusters. Then the sparse clustering algorithm is run a second time, but rather than giving equal weights to all features as in the first step, this pre-weighted version of sparse clustering assigns a weight of 0 to all features that differed across the first set of clusters. The motivation is that this procedure will identify secondary clusters that would otherwise be obscured by clusters that have a larger dissimilarity measure. Moreover in this study it is proposed the supervised version of pre-weighted sparse clustering which assigns the initial weights of the chosen clustering algorithm by giving non zero weights to the features that are most strongly associated with the outcome variable.

The identification of “secondary” clusters that may be “covered” by the primary clusters involving large numbers of high variance features has been a case of research (Nowak and Tibshirani, 2008). At the step of this method traditional hierarchical clustering is performed on a data matrix X . A new data matrix X results from this hierarchical clustering procedure and is defined to be the expected value of the residuals when each row of X is regressed on the group labels when the hierarchical clustering tree is cut at a given height. The expected value is taken over all pos-

sible cuts. This has the effect of removing high variance features that may be obscuring secondary clusters. Traditional hierarchical clustering is then performed on this modified matrix X , yielding secondary clusters.

Zhang (Zhang et al., 2016) following the Bair approach (Bair and Tibshirani, 2004) used a mixed methodology composed by statistical and machine learning methods. The proposed method was focused on clusters discovery over clinical and genes expression data. Authors proposed a semi supervised pipeline for survival clustering discovery. Authors initially used only the clinical part of the data in order to estimate the censored lifetimes. Actually they utilized penalized logistic regression and penalized proportional hazard model with the Expectation minimization algorithm in order to select only the most significant clinical features which are correlated with the event of interest. After a list of significant clinical variables have been identified they used the K neighbors based method (with 10 neighbors) on the filtered data set for the survival time estimation for the patients with censored survival time. After they applied silhouette method on the filtered data set in order to identify the optimal number of clusters. Then Fast correlation based filter method is applied on genes expression data in order to select the most significant features. With this method redundant features with lower relevancy are removed from the list until the number of last features reaches a targeted low bound or there are no more features to be removed. Finally a classifier is used in the selected genes in order to predict the label identified from the clinical data set. The performance of the classifier can be considered as a measure of the robustness of the performed clustering and also if the identified groups share the same survival distribution.

Mouli (Mouli et al., 2017) proposed a decision tree based approach which aim for survival clustering. The final purpose of this research paper was to cluster censored data and identify two or more populations with different risk level. Paper’s objective was the survival distributions identified to be different across clusters. The initial step of this method is to concretely break the data set in two populations and based on attribute - values test to observe the identified populations survival distribution. This is done by using the Kaplan-Meier estimates. Kuiper (Kuiper, 1960) statistics is used after in order to quantify the the significance of the difference across survival distributions. The procedure is repeated for all attribute - value pairs in order to choose the one with the best results is used as a node in the constructed decision tree. In this step of the algorithm best result is consid-

ered to be the lowest p-value of the aforementioned Kuiper statistic measure where the significance level of the algorithm can be specified by the user. Authors describe that the process of performing many statistical tests at each node conduct a multiple hypothesis problem which can be corrected using Bonferroni (Rupert Jr et al., 2012) correction as proposed in the paper. The suggested methodology results in a tree where each leaf node has an associated population of users and thus clusters can be observed at leaf nodes. Despite the fact that in his way, subjects with similar survival distributions will be placed closer in the tree diagram, the degree of dissimilarity between identified clusters may not necessarily be significant. Hence the target is to identify clusters with different survival distributions authors propose the usage of complete graph. The graph consists of leaf nodes as vertices and p-values as edge weights. Each nodes will be connected to the other with the edges (normalized p-values) which will denote a significant or non significant relationship between them. Markov clustering algorithm (Van Dongen, 2001) is applied to the final graph .

A recent research study (Mouli et al., 2019) introduced DeepCLife an inductive neural network based clustering model architecture. This framework aims for the observation of empirical lifetime distribution of underlying clusters. The final purpose of this framework is to conduct clusters which have different lifetime distributions whereas the subjects of the same cluster share the same lifetime distribution. The proposed model does not assume proportional hazards. A very important asset of this research paper is that it addresses the issue of unobservability of termination signals, meaning that it can be applied on data sets that termination signals have not been recorded. The main contribution of this work is the proposal of a novel clustering loss function which is based on Kuiper two-sample test. Authors provide a tight upper bound of the Kuiper p-value, without computationally expensive gradients which until then was the main difficulty of its usage as a loss function (due to the test's infinite sum - this is not going to be included). In this paper authors describe the usage of a feedforward neural network although the proposed model is not restricted to a feedforward architecture, which makes this approach flexible and worthy to try different models in order to observe differences in the performance. The proposed model aims to identify clusters with maximizing the divergence between the empirical lifetime distributions of each cluster.

Liverani (Liverani et al., 2020) proposed a Dirichlet process mixture model for censored survival data with covariates, inspired predominantly by (Molitor

et al., 2011) and (Molitor et al., 2010). The proposed model is a mixture of Weibull distributions and also distributions suitable for the data set covariates which non parametrically assigning data to clusters. In this approach the response variable (the presence of absence of the event) and the covariates are modeled independently which allows the exploration of the complex relationship between them. Despite the fact of independently modelling this approach can uncover linear and non-linear relationships between covariates and response.

Unlike DeepCLife which aims to identify clusters by maximizing pairwise differences between the survival function of all cluster pairs Chapfuwa's (Chapfuwa et al., 2020) study focuses on characterization of time-to-event predictive distributions from a clustered latent space conditioned on covariates. In particular authors at Survival Cluster Analysis produced a complex model which provides not only risk profile based clustering but also acts as a deterministic encoder that maps covariates into a latent representation and on top of that a stochastic survival predictor feeds from the latent representation. In this paper a Bayesian non parametric approach was used for the clustering process. The Bayesian approach emboldened the latent representation to act like a mixture of distributions while distribution matching approach (in this study) follows a Dirichlet Process. Table 1 summarize all the algorithms developed in the literature that aim to address the problem of distinguish subjects in a dataset based on their risk profile taking into account also the censorship which is usually met in real world datasets.

5 CONCLUSIONS

Survival analysis can be used widely in almost every domain since there is a relevant data structure available. Notwithstanding the fact that there have been published many scientific reports applying machine learning techniques for time to event predictions which have been obtaining very good performances, published scientific researches dedicated to survival clustering techniques were significantly fewer. Clustering in terms of survivability is very useful when there is a need to identify subpopulations in the overall dataset who are unknown. Such techniques aim for identification of clusters whose lifetime distributions significantly differs which is something that is not able to be done by applying traditional clustering techniques.

Table 1: This is a summary of all methods and algorithms that have been used in literature for clustering based on risk profile while using censored data.

Survival Clustering Algorithms		
Title	Application Domain	Characteristics / Attributes / Features
Partial Cox regression analysis for high-dimensional microarray gene expression data	Applicable at every Domain	Reduces the number of features in a high-dimensional dataset since it aims to identify the principal components and parallel apply partial least squares method. It has not computational limitations in terms of number of variables in contrast to other statistical methods. Proportional Hazard assumption must be met
Semi-Supervised Methods to Predict Patient Survival from Gene Expression Data	HealthCare	Restricted to be used for Medical data (clinical and genes expression). Results may depend heavily on the features chosen. Proportional Hazard assumption must be met. It is dedicated to clinical and genes expression data even though it can be perfectly fitted in medical applications
Complementary hierarchical clustering	HealthCare	Uncovers structures arising from the weaker genes. It is an automatic procedure and after performing the initial clustering, there is no need to decide how many groups should be considered or where to cut the dendrogram. The backfitting algorithm used somewhat complicates the interpretation of the initial, complementary clustering
Supervised principal components (SPC)	Applicable at every Domain	Reduces the number of features in a high-dimensional dataset. Proportional Hazard assumption must be met
Using the machine learning approach to predict patient survival from high-dimensional survival data	Healthcare	It is dedicated to clinical and genes expression data though it can be perfectly fitted in medical applications. Restricted to be used for Medical data (clinical and genes expression). Results may depend heavily on the features chosen. Proportional Hazard assumption must be met.
Identifying User Survival Types via Clustering of Censored Social Network Data	Applicable at every Domain	The model is based on decision trees and thus can handle both categorical and numerical features with ease and without encoding variables.
Identification of relevant sub-types via pre-weighted sparse clustering	HealthCare	It is not essential to choose an “optimal” set of initial weights since the procedure tends to correct itself. Tends to self-correct so that relevant features get nonzero weight (even if their initial weight was zero) and irrelevant features get 1 zero weight (even if their initial weight was nonzero)
Deep Lifetime Clustering	Applicable at any Domain	Does not assume Proportional Hazard. It can smoothly handle the absence of termination signals. There is not a way to find the optimal number of clusters apart from making many different trials and keep the best results.
Clustering method for censored and col-linear survival data	HealthCare	Deals with data collinearity, therefore model can achieve a good performance without to be necessary strongly correlated covariates to be removed before. The independent response - covariates modelling allows the exploration of the complex relationship between the response and the covariates.
Survival Cluster Analysis	Applicable at any Domain	It is not necessary for the user to specify the number of clusters since they are identified automatically. The model identifies interpretable populations

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