Categorical Approach to Swarm Computations

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Abstract: We propose the model approaching the problems of organisation, computing and emergent behavior of certain swarms from category theory point of view. In this model the Yoneda embedding to the category of presheaves spanned over the basic category of partial recursive functions, is activated by external stimuli and the resulting excited domains carry collective self-organising processes. We find that the intuitionistic logic of the presheaf topos becomes a primary logic for a way how swarms act and how they should be described algorithmically.

1 INTRODUCTION

Swarm intelligence (Kennedy and Eberhart, 2001; Bonabeau et al., 1999) is the phenomenon of much concern for information science, computer science, intelligent systems or algorithmic processes but also for cognitive science or consciousness studies. To understand better a more complex social feature of swarms with much degree of complication and autonomy of their members, it is highly desirable to ignore chemical and biological processes of real swarms and to focus on their ability to solve computational tasks in their organisation of transport networks.

There are swarms at the levels of different kingdoms. So, microbial swarms or social bacteria (the level of Kingdom Bacteria) can be exemplified by bacteria *Pseudomonas aeruginosa* which have a quorum sensing allowing them to transmit signals among the cells for their cooperation (Ben-Jacob, 2008). At the level of Kingdom Animalia there are swarms from the level of insects (such as ants *Formicidae*) to mammals (such as naked mole-rats *Heterocephalus glaber* and chimpanzee *Pan troglodytes*). Each swarm, from bacteria to mammals, can solve logistic and transport problems very effectively (Gordon, 2003; Michener, 1969).

Evidently that chemical and biological processes of reacting and motoring for swarms of different levels are different, too. The point is that for different species, biologically active matters are different. There are two main types of biologically active matters: (i) attractants (such as pheromones) which attract and (ii) repellents (such as dangerous conditions) which repel. Computationally these matters can be presented as vertices of graph so that the swarm motion is a computational process along edges among vertices. In this paper, we concentrate on this representation to describe a computational nature of any swarm at different levels.

Recently it was shown that a swarm behavior is detected even at the level of only one cell. So, within one cell there are different proteins (such as microtubules and microfilaments) which are assembled and disassembled under different conditions in response on extracellular stimuli to transform the cell and to transmit the signal. Microtubules and microfilaments react to external stresses to organise different actin filament networks: unstable bunches (parallel unbranched filaments), trees (branched filaments), stable bunches (cross-linked filaments), see (Calderwood et al., 2000; Carlier, 1989; Carlier, 1991; Forgacs, 1995; Hill, 1981; Mooseker and Tilney, 1975). Hence, each filament is a 'swarm agent' that cooperates with other filaments ('agents') in organising some emergent structures from bunches to trees. Also, microtubules and microfilaments are responsible for changing the shape and structure of dendritic spines of neurons so that they play a significant role in the formation of new spines as well as stabilising spines. Thus, due to them the signals are transmitted through neurons (Dillon and Goda, 2005).

Thus, we observe swarms of different scale: from a swarm behavior of some proteins (such as actin fil-

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aments) to social bacteria, social insects, and mammals. In this paper, we analyse a swarm behavior of members (atoms) with emphasis on their emergent effects due to their interconnections. Such a model can abstractly describe swarms of different levels and scales: from actin filaments to chimpanzee *Pan troglodytes*.

It is by now a rather well-established fact that certain biological systems can be considered as computing devices and biological computers based on them are not merely a dream, e.g. (Adamatzky, 2010; Erokhin et al., 2012; Roquet et al., 2016). It is a less known, but rather expected fact, that some such systems exemplify, or interpret, non-classical logic which has been claimed to exist in computing biological systems recently (Schumann, 2019; Schumann, 2017; Ohya and Volovich, 2011). The authors of (Adamatzky and Siccardi, 2015) showed that quantum gates can be interpreted in the actin filament swarms. There were also previous attempts toward modelling an emerging consciousness from quantum computations taking place in some of the systems e.g. (Hameroff, 1998). A special attention has been paid to biological swarms like ant and bee colonies, fish schooling, birds flocking, horse herding, bacterial colonies, multinucleated giant amoebae or the actin filaments. In this context there emerged the new branch of computer science which can be called *swarm intelligence* and is devoted to analysing the collective and decentralised behavior of swarms as reacting on external factors. Defining the factors as stimuli, i.e. attractants and repellents acting on swarms (like low or high temperature, light or darkness), one observes the reaction produced, or calculated, by a swarm which is not just a simple additive reaction of members of the swarms but rather an 'intelligent' emergent reaction of big domains of them without any centralised control. The process can be further used to solve certain mathematical hard problems like Travelling Salesman Problem, the Steiner Tree Problem, the Generalised Assignment Problem and some other (Schumann, 2019). Even though the instances of these tasks are solved only case by case and no general algorithm is certainly known or derivable from the cases, the collective decentralised reaction of the swarms members makes the job.

Hence, one of the goals of the present paper is to analyse to what extent non-classical logic is an inherent feature of certain biological swarms. Connected with this is the attempt to characterise what emergent collective behavior of a swarm could be in order to take part in swarm's computations. Given the possibility to interpret as non-classical (many-valued, quantum) logical gates in the set of the allowed swarm's reactions on external factors, is this merely the interpretation in the fundamentally classical swarms? Or, maybe, swarms are fundamentally non-classical, say, many-valued or even quantum. Stating differently, the swarm's motility and self-organisation are driven by inherently non-classical logical circuits, or they are classical where non-classicality is just an external option due to an interpretation. So the fundamental question in this context would be whether one merely interprets non-classical gates and the following logic within inherently classical biological systems or we are really facing true non-classical realm of the systems which is not reducible to classical one.

We think that this is a highly nontrivial problem to decide the above reducibility at the fundamental level which could help understanding the swarm's intelligence and using it in developing swarm-designed algorithms and software. We motivate our interest in this issue by analogous problem of existence of hidden variables in quantum theory which has marked significantly the development of quantum mechanics and science in general, for years. Another motivation is the special fundamental role of intuitionistic logic in physical world in general which could also have its impact on understanding of the swarm activity considered here, see (Isham, 2000; Król, 2006; Landsman, 2007)

We do not solve the general reducibility problem here, although, we propose a model shedding substantial light on the problem. The method we employ is *categorical* which means that it relies on finding suitable category K where a swarm W would be embedded. All categorical constructions referred to in this paper, like category of sheaves or Yoneda embedding into it, are elementary and a reader can find them in any of many textbooks from category theory, however, we work here with (MacLane and Moerdijk, 1994) which is certainly more than enough. Next, our task is to identify categorical construction responsible for the collective behavior spreading over regions in W. This is achieved by taking the category of presheaves on K, $SET^{K^{op}}$, and Yoneda embedding $y : W \hookrightarrow SET^{K^{op}}$. The model is simplistic since it treats the excited regions of W as very regular, however, we think that filtering of this categorical setup of regular domains through biological realisations should approach the real behavior. Another simplification comes from considering swarms as pseudo atomistic structure where there is a net of atoms-nodes connected by actin fibers. Nevertheless such a presentation is rather generally accepted e.g. (Galkin et al., 2015; Adamatzky, 2018).

The choice of K is already important and should reflect the internal processes of a swarm. For relatively simple swarms like actin filaments or multinucleated giant amoebae one can distinguish nodes connected by links which together constitute the structure of the swarm. The nodes reaction on stimuli is partially responsible for what is called the intelligent behavior of the swarm or its computational power. Thus, we adopt here the point of view that there are nested processes already at nodes (or in between two nodes) which determine the group behavior of swarms without necessity to increase the number of connections. Such an attitude can be seen as the step supporting the orchestrated objective reduction hypothesis in cognitive sciences and neural networks stating that our consciousness is the result of deep processes taking place in neurons rather than due to the myriad of connections (Schumann, 2019). This hypothesis was formulated in (Hameroff and Penrose, 2014) and it is confirmed by our reasoning above that the same computational power of swarms is observed at different levels and scales: from actin filament to mammals. In order to fulfil this requirement we are choosing as K the Turing category of partial computations (Cockett and Hofstra, 2008). Another reason for the choice of such K is the attempt to consider classically computations executed between nodes as morphisms of K. Even though they are classically defined the entire internal structure of $SET^{K^{op}}$ is intuitionistic based on Heyting (not Boolean) algebras.

2 SWARM AS A CATEGORY

Following the pseudo-atomic model of actin filaments (Galkin et al., 2015; Adamatzky, 2018), it is represented by a graph $\langle \mathbf{V}, \mathbf{E}, \mathbf{Q}, f \rangle$ where $\mathbf{V}$ is a set of vertices, $\mathbf{E}$ is a set of edges, $\mathbf{Q}$ is a set of allowed states, and f is a transition function switching the nodes states. $f : \mathbf{Q} \times [0,1] \to \mathbf{Q}$ calculates the new state depending on the fraction of excited nodes in around the given node. Here we adopt the point of view that the change of the state is accompanied by the change of a node which mathematically corresponds to a computation process realised in principle by a classical Turing machine. We do not distinguish here the one computation over another, this will be left as option and implemented at further stage via introducing phenomenological parameters. Thus, the state space $\mathbf{Q}$ is eventually extended for the infinite many allowed states such that they correspond to recursively computable functions (partial recursive) $f : \mathbb{N} \to \mathbb{N}$ (or more generally $f : \mathbb{N}^k \to \mathbb{N}^m$).

The category K representing a swarm W is thus $K = Comp(\mathbb{N})$ the Turing category (Cockett and Hofstra, 2008) which objects are k-tuple products $\mathbb{N}^k$ and morphisms partial recursive functions $f : \mathbb{N}^k \to \mathbb{N}^m$. Any Turing category is equipped with a Turing object (which here is $\mathbb{N}$) and realises a notion of computability which, in a general Turing category, can be not necessary SET-based. The basic categorical construct behind Turing categories is a partial category which realises partial concepts of Cartesian closedness or powers (Mulry, 1994). Such a partial category is Turing if there exists a Turing object in it.

There are defining features of $Comp(\mathbb{N})$ relevant to the process of representing W by it. Let us discuss that point more closely. The particular purpose of the categorification of the simplest biological swarms (like actin filaments) is

1. considering W as computing system composed of computing nodes,
2. the computations by nodes are elementary – it is represented by partially recursive functions $f : \mathbb{N}^k \to \mathbb{N}^m$ – resulting in changing the states of nodes,
3. the partiality enables to take into account algorithms which do not halt on some data and thus giving no definite result,
4. different algorithms at different nodes can be used,
5. the collective computations in W leads to the emergent behavior and swarms computability and 'intelligence'.

The space of natural numbers $\mathbb{N}$ is the common domain for all programs and all data on which programs compute. The reason is Gödel numbering which encodes programs as natural numbers and data as natural numbers and the result of a computation is again a code which is a natural number. We can effectively enumerate all partial recursive functions (PR). PR functions are generated by three operations: constant (assigning a constant value a to a set of n variables), successor (assigning the value $x+1$ to a variable x and projection on i-th variable of a set of k variables). The point is that all Turing computable functions are represented by PR functions. Hence, coding PR, we code all Turing programs executed on natural numbers. We do not need to leave the realm of natural numbers when talking about all possible Turing computations on all Turing machines.

Let e be a code for some Turing machine (a natural number). There exists the enumeration of PR functions, $\{\phi_i\}_{i \in \mathbb{N}}$, such that (Cockett and Hofstra, 2008; Baez, 2019)

T1. The functions $\phi_e(x_1, x_2, \ldots, x_n) =: \Phi^{(n)}(e, x_1, x_2, \ldots, x_n)$, $n > 0$ are PR.

T2. There exists a set of PR functions $S^n_m, n, m \in \mathbb{N}$ such that

$$\Phi^{(n+m)}(e, x_1, \ldots, x_m, v_1, \ldots, v_n) =$$
$$\Phi^{(m)}(S^n_m(e, x_1, \ldots, x_m), v_1, \ldots, v_n).$$

for any Turing machine e and any natural $m, n > 0$. T1. is known as an universality property and in fact it states that $\mathbb{N}$ is a universal Turing machine. T2. is the so -called parameterisation theorem.

A swarm W is now interpreted such that its members correspond to objects while morphisms to computations resulting by executing algorithms on data in nodes which the process computes the change of a state in the nearby node. The rules for composition of morphisms in $Comp(\mathbb{N})$ makes the computations spreading over the entire swarm. This makes that the point 1. of the categorification procedure is fulfilled. The point 2. is fulfilled as well. Regarding 3., the partiality of computations reflects the fact that not all computations halt – we do not have general algorithms to predict this fact. Thus, a nature of computability refers to unpredictable halting which on the level of category theory is encapsulated in the notion of partial functions and is defined within restriction category. One example of a restriction category crucial for this paper is our category of $\mathbb{N}^k$ with partial recursive functions. So, already PR functions contain the halting indeterminacy which reflects the fundamental fact that primitive recursive functions are nontrivially extended by PR ones and PR represent all Turing computations. Thus, 3. is realised by the model based on $Comp(\mathbb{N})$ which is also true for 4. Namely, different algorithms and Turing machines are indeed involved in the computations realised in a swarm represented by $Comp(\mathbb{N})$, since it obviously holds (Cockett and Hofstra, 2008)

Lemma 1. *T1. and T2. hold true in* $Comp(\mathbb{N})$.

In the next section we present analysis of the point 5. related to the collective computability.

3 EMERGENT PHENOMENA IN THE CATEGORICAL SWARMS

The reaction of a swarm to external stimuli can be a source for collective and possibly emergent behavior. We want to understand this from the category theory point of view. Let A be an atractant and R a repellent. They lead to the appearance of certain excitations of the swarm which spread over its domains. It seems natural to think about these excitations as originated in certain morphisms/computations/change of the state of a node in W. However, such stimuli are not in W so there is also another possibility to describe the following excitations of W as being categorically external to $Comp(\mathbb{N})$. We follow the relation of presheaf category to the base category in this respect.

Given a category, say K, one can create the category of presheaves on it, $SET^{K^{op}}$, where SET is the category of sets and functions between them and K^{op} is the category opposite to K, i.e. morphisms are taken with opposite direction than in K and objects are the same as in K (MacLane and Moerdijk, 1994). The objects of $SET^{K^{op}}$ are thus contravariant functors $F : K \to SET$ and morphisms are natural transformations between F's. Understanding the presheaves level of swarms refers to the concept of varying sets and to the internal logic of toposes (MacLane and Moerdijk, 1994).

Given the swarm W spanned on objects-nodes of $Comp(\mathbb{N})$ and an external stimuli which leads to the excitations restricted to certain node(s) and connections, the excited region is travelling within W with the effect of eventual increasing, deforming or inhibitting the excitation area. The regions can then meet themselves, interact, dissipate etc. (Adamatzky, 2018). We want to understand this process categorically, especially its computational aspect.

There is a natural embedding of $K = Comp(\mathbb{N})$ into $SET^{K^{op}}$, the Yoneda embedding

$$y : K \to SET^{K^{op}}.$$

Let $a_s \in Ob(K)$, $s \in I$ be objects and $h_p \in Hom(K), p \in J$ be morphisms of K. An object a in K is sent to the functor $F_a \in SET^{K^{op}}$ such that $F_a(b) = Hom(b, a)$ – the set of all arrows (morphisms) from b ending at a. We also say that the functor F_a is at the stage $b \in K$ meaning that we consider the set $F_a b$. Thus, by changing the stage we have different sets of arrows within a single presheaf F_a.

Given two presheaves P, F in $SET^{K^{op}}$ and a morphism $\eta : F \to P$ in $SET^{K^{op}}$ the sets on stages a_s in both functors are related as in the graph in Fig. 1

$$\begin{tikzcd} Fa_1 \arrow[r, "\eta_{a1}"] \arrow[d, "F(h)"'] & Pa_1 \arrow[d, "P(h)"] \\ Fa_2 \arrow[r, "\eta_{a2}"'] & Pa_2 \end{tikzcd}$$

Figure 1: The natural transformation η on stages a.

Hence, the natural transformation η between 2 functors F and P in $SET^{K^{op}}$ is a family of maps between sets on every stage a: $\{\eta_a\}_{a \in K} : Fa \to Pa$ such that for every morphism $h : a_1 \to a_2$ in K the square above commutes, i.e. $\eta_{a2} \circ F(h) = P(h) \circ \eta_{a1}$. Given two natural transformations $\eta_1 : F \to P, \eta_2 : P \to G$ there emerges the composition $\eta_2 \circ \eta_1 : F \to G$ which is also a natural transformation as a morphism in $SET^{K^{op}}$. Thus, we have indeed $SET^{K^{op}}$ as a category which, however, is very rich from the categorical point of view, namely it is a topos. In particular there always exist exponential objects F^P to every pair (F,P) of objects (functors or presheaves) in presheaves category $SET^{K^{op}}$ and there always exists the object of all subobjects (monics) of F, $\mathcal{P}(F)$, (here $\mathcal{P}$ means the operation of taking the power object). Both, exponential and power objects are presheaves, hence functors, in $SET^{K^{op}}$. Moreover, finite products and coproducts exist as does the subobject classifier (MacLane and Moerdijk, 1994).

The sets $Hom(b,a)$, when restricted to W, are $Hom^W(b,a) \subset W$ – the sets of connections in W. $Hom^W(b,a) \subset W$ is not just a set of physical connections in actin filaments originated in the node a it is rather a set of possible signals (computations) which can be sent to a from b through the physical connections. The physical connections constitute a kind of skeleton for categorified W.

The functor $R_a : K \to SET$ in $SET^{K^{op}}$ sending an object b to the set in SET of arrows from b ending at a, $Hom(b,a)$, is particularly important both in category theory and for swarms. It is the representable functor and in fact for any a in K there is such representable F_a such that the entire collection of $F_a, a \in K$ defines the Yoneda embedding. Further this representability allows for considering excited regions in $W \subset K$ as sets of morphisms ending at a which could define the representable subfunctor in $SET^{K^{op}}$. Then the logic of emergent collective behavior of W follows the dynamics of excited domains in W which are grouped in subpresheaves on stages. Stated differently, the Yoneda embedding

$$y : W \hookrightarrow K \xrightarrow{y} SET^{K^{op}}$$

defines the representable (sub)functors as governing excited domains of W. In particular, the intersections of two colliding domains is given by pullback in $SET^{K^{op}}$. Let F,P be two presheaves in $SET^{K^{op}}$ and their corresponding two excited domains at the stage $c \in K$ (for a and the representable $F = R_a$ this is the set of arrows from c ending at a), $Fc \subset R_a c, P_a \subset R_a c$. R_a is the representable functor which on stage c reads $R_a c = Hom_K(c,a)$. Then the intersection (pullback) of F and P, $F \cap P$, exists in $SET^{K^{op}}$, i.e. the pullback square below commutes

Now, given excited domain $D \subset W$ such that $R_a b, b \in D$ in W (which is the set of excited arrows in W ending at a in D) we have bijection of Fa with the set of natural transformations of R_a and F. It holds

Theorem 1 (The Yoneda lemma)**.** *For an arbitrary functor $F : K^{op} \to SET$ and the representable functor*

$$\begin{tikzcd} F \cap P \arrow[r] \arrow[d] & P \arrow[d, "m2"] \\ F \arrow[r, "m2"'] & R_a \end{tikzcd}$$

Figure 2: Pullback of F and P in $SET^{K^{op}}$ as subobject of R_a.

$R_a : K^{op} \to SET$, there exists a bijective correspondence at every a (an object in K)

$$\Theta_{F,R} : \textit{ Nat. trans. } (R_a, F) \xrightarrow{\simeq} Fa$$

where Fa is a set which is assigned to a by functor F.

So, we can approach the emergent logic of excited domains in W based on Yoneda embedding and internal logic of $SET^{K^{op}}$. Given two excited domains, D_1, D_2 by certain external stimuli s_1, s_2, the domains can interact, e.g. they collide, dissipate, inhibit etc. which leads to further deformations and generating a resulting domain D_3 such that the logical functions are defined on $D_i, i = 1,2,3$. For example, one can define conjunction of D_1 and D_2 which is the pullback $D_1 \cap D_2$. We consider the domains as sets of arrows which are precisely subfunctors in the presheaves category evaluated at certain stages a,b. The point is that the category $SET^{K^{op}}$ is usually much more complete than K itself and, e.g., the pullbacks for sure exist in $SET^{K^{op}}$ as there exist exponents and finite limits and colimits. Moreover, usually the sheaf category contains also richer partiality structure that the base category K (Mulry, 1994) which means that the computability space of $SET^{K^{op}}$ is also richer. As a consequence, the general conclusion is that whenever external stimuli cause the excitations of D_1, D_2 such that they have components in $SET^{K^{op}}$, the logic of such collective processes is the internal logic of $SET^{K^{op}}$ which is intuitionistic, i.e. neither using set theoretic axiom of choice nor the logical principle of the excluded middle are allowed. The model presented here indicates also that the emergent collective computability of swarms can differ from the set based computability in K. Both the properties above will be more thoroughly analysed in a separate publication.

4 CONCLUSION

There are many algorithms developed for explicating the swarm behavior from the swarm motility of birds and horses within the Particle Swarm Optimization (Kennedy and Eberhart, 1995; Wang et al., 2011) to coworking of ants within the Ant Colony Optimization (Dorigo and Stutzle, 2004), bees within the Artificial Bee Colony Algorithm (Karaboga and Akay,

2009) and within the Bees Algorithm for Generalized Assignment Problem (Ozbakir et al., 2010), and many others. There are defined different logic circuits on the basis of different swarms: ants (Coello Coello et al., 2000), bees (Mollabakhshi and Eshghi, 2013), slime mould (Adamatzky, 2010; Schumann, 2019), etc. Nevertheless, there is no general theory of swarm computation which would summarise all the approaches. In other words, there is no 'metamathematics' or 'foundations' of swarm intelligence. In the research programme of (Aczel et al., 2013) in the foundations of mathematics, there was proposed homotopic type theory as ultimate mathematical foundations. In our approach, we assume that within this programme we can also identify isomorphic computational structures to define types and their hierarchies of different chemical and biological systems as substrates of swarm computing. In order to fulfill this task, we have started with defining categories on swarms. Presumably in the course of defining suitable mathematical structures behind various phenomena realised by intelligent swarms we need certain modifications of toposes, e.g. (Asselmeyer-Maluga and Król, 2019). It is our preliminary result and rather draft in developing 'foundations' of swarm intelligence.

The proposed categorical model for swarm computability and collective behavior indicates that the intrinsic logic of such swarm phenomena has to be intuitionistic. The particular case of intuitionistic logic is Boolean logic encompassing multivalued (also infinite many) Boolean logic, since Heyting algebras on which toposes are built on are generalisations of Boolean algebras. Deciding up to what extent the appearance of the intuitionistic logic is generic for swarm intelligence in general, requires further studies which would contain also the detail development of the scenario proposed here.

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