# Study of Stability through Lyapunov Theory and Passivity following a FDI on a Velocity Control System*

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Abstract: Ensuring safety and fault tolerant strategies is essential in the development of Advanced Driver Assistance System, such as an automated cruise control.This work presents a study of the stability of switched regulated systems following the reconfiguration of the speed controller due to a fault.

Firstly, the context of these works is presented highlighting the need to have a fault management system with a diagnostic part and a reconfiguration part in order to ensure the operating safety. The reconfiguration part can take the form of a switch thus involving the study of stability. It is in this context that, secondly, the passivity of the plant as well as of both the controllers (CRONE and PI) is demonstrated.

As the switch takes place between two elements of a passive nature, the last point of this work highlights the application of the continuous approach in order to demonstrate the passivity and therefore the stability of the regulated plant despite the presence of the switch.

To address this problem, an augmented model in the form of a generic state space representation of the controllers and the plant is constructed. Then, a Lyapunov candidate function representing the sum of the storage function of the controller and the plant is defined. A sign study of this function as well as its derivative is carried out for the two operational modes (CRONE regulating the plant and PI regulating the plant) in order to demonstrate the passivity of the switched regulated systems.



## 1 INTRODUCTION

Nowadays, research and development of Advanced Driver Assistance Systems (ADAS) in the automotive field focus on the development and integration of increasingly complex autonomous functions.

However, one of the main factors to take into account in the development of these functions is to ensure the safety of the passengers at all times. For this, the good functioning of the various systems present within the Automated Driving (AD) must be ensured.

Tools have therefore been put in place to prevent the presence of any faults or failures that could have disastrous consequences for the system and endanger the passengers of the vehicle. These tools are mainly Fault Detection and Isolation (FDI) methods. These are part of the fault management procedures.

FDI methods are classified in two categories: qualitative methods and quantitative methods (Jones *et al*, 1988). The first one is based on data history. The most known methods are using artificial intelligence or fuzzy logic (Franck et al, 1997), neural network or genetic algorithms (Samanta, 2004) and so on. The second category is based on mathematical model of the system. The main methods are the parity spaces ones (Evans et al, 1970; Potter et al, 1977; Daly et al, 1979), the parametrical estimations ones (Isermann, 1984; Isermann, 2006; Constantinescu et al, 1995) and the state estimations ones (Beard, 1971; Massoumnia, 1986; Edelmayer et al, 1996).

Application on the automotive field focus mainly on mechanical faults such as internal combustion engine (Kim et al, 1998), drive-by-wire (Isermann & al, 2002) or detection of non-aligned wheels or degraded braking (Spooner et al, 1997).

After the fault detection, the important point for

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the safety and the good functioning is to ensure, that despite the presence of the fault, the system continues to operate, either in an operational way or in a degraded way. For this, a reconfiguration is required in order to switch from the defective component or function to the operational or degraded one. It is from this perspective of reconfiguration that switched systems are interesting to set up.

However, from a control engineering perspective and above all regarding stability, switching can cause instabilities within the system and therefore present risks.

To ensure the stability requirements, tools based on Lyapunov's stability have been put in place.

Before presenting a state of the art of the existing methods to guarantee the stability of switched systems, Lyapunov's theory is first recalled. Indeed, the majority of stabilization methods are based on the Lyapunov criterion.

## 1.1 Lyapunov Stability Criterion

By definition, a stable system is a system which, when removed from its position of equilibrium tends to return to it.

One of the major theories in the study of the stability of systems is Lyapunov's theory of stability. The main advantage of this theory is that it has an application to both linear and nonlinear systems.

The Lyapunov stability criterion is based on a candidate state function denoted $V(x)$ representing the energy of the system studied. The latter must be defined positive and its derivative, which is representing the evolution of the energy over time, must be defined negative. This means that the energy of the system is positive but decreases with time. As a result, the system returns to a rest position, so it is stable. These conditions can be written in the form of inequalities as defined below:

$$\forall x \in \mathbb{R}, V(x) > 0 \tag{1}$$

and

$$\forall x \in \mathbb{R}, \dot{V}(x) < 0, \tag{2}$$

where, $x$ represents the state vector.

## 1.2 State of Art of Stability Methods for Switched Systems Methods

Consider a state vector $x(k) \in R^n$, an input vector $u(k) \in R^m$ and an output vector $y(k) \in R^p$ with $k$ being a time index.

Let $\sigma: Z^+ \to \{1,2,3 \dots, S\}$ with $S$, the number of subsystems. $\sigma$ is a piecewise constant function whose value changes at the switching times. This function is called the commutation law.

A switched discrete time system can be described by the following equations:

$$\begin{gathered} x(k+1) = f_{\sigma(k)}\big(x(k), u(k)\big) \\ y(k) = h_{\sigma(k)}(x(k), u(k)) \end{gathered} . \tag{3}$$

In the literature, methods for studying the stability of switched systems, in particular for discrete time systems, have been implemented. The definition of the joint spectral radius presented in (Hetel et al, 2007, Tsitsiklis et al, 1997) is one of these methods and gives a sufficient and necessary condition for the stability of the system by computing the extension of the radius of a set of matrices $A = \{A_1, \dots, A_S\}$, denoted $\rho(A)$. The major difficulty of this method is to compute numerically the joint spectral radius in a generic framework. Several approximations are made in the literature.

Other methods are based directly on the Lyapunov candidate function $V(x)$. In (Shorten *& al*, 2007; King et *al*, 2004; Zhai *& al*, 2002), the principle of a common quadratic Lyapunov function (CQLF) is proposed for continuous second or even third order systems and also give algebraic criteria in order to determine this function. The principle is based on the existence of a Lyapunov function of a quadratic form and common to each subsystem.

However, it is in general very difficult to obtain such a function and its use is restricted to relatively low order systems.

In order to overcome the constraints of a Lyapunov function common to each subsystem, works presented by (Mignone et al, 2000; Daafouz et al, 2002) highlight the use of a multiple Lyapunov functions. In the case of discrete time systems, a poly-quadratic Lyapunov function is presented. In this method, each subsystem has a Lyapunov function $V_i(x)$, which satisfies linear matrix inequalities in order to prove the existence of a poly-quadratic Lyapunov function and therefore the stability of the switched system.

Whatever the methods presented above, they are based on matrix algebra specific to the systems studied. This therefore assumes knowledge of the system model. *However*, in cases that are more complex, obtaining the mathematical model of the plant is very difficult or even impossible, particularly in cases such as a switch between "black box" type systems or AI algorithms.

Hence, the definition of a generic stability criterion for switched systems, which is not necessary based on the knowledge of the mathematical model, is required. It is with this in mind that the notion of passivity and its implication with stability are defined and used for the work presented in this paper. In the next subsection, the notion of passivity is thus presented.

### 1.3 Notion of Passive Systems

Passivity makes it possible to characterize a system based on the notion of energy (McCourt et *al*, 2010).

***Definition 1***: *Let define a causal continuous-time system denoted* $\Sigma$ *, with input vector* $v \in R^m$ *and output vector* $z \in R^p$ *. This system is said to be passive if* $\forall t \geq 0$*, the variation of its stored energy over time noted* $\frac{dV(x)}{dt}$ *is less than the power supplied* $z^T v$ *by its input, i.e.:*

$$\forall t \geq 0, \frac{dV(t)}{dt} - z^T v \leq 0. \quad (4)$$

***Remark 1.1***: The input vector $v$ included all the inputs of the system, i.e. the control inputs and the disturbances.

***Remark 1.2***: Each output is associated with its respective input as part of the power calculation. Otherwise, passivity cannot be guaranteed.

***Remark 1.3:*** For an energy point of view, passivity implies that the energy stored by a system denoted $V(x)$ dissipates and therefore decreases over time. Thus, the Lyapunov stability criteria are verified.
According to (Khalil, 2002), a passive system is therefore a stable system in the sense of Lyapunov but the converse is not true.

***Remark 1.4:*** In addition, the advantage of using passivity is that the interconnection of passive systems (in parallel and in feedback) is passive. The proofs are demonstrated in (McCourt et *al*, 2012). This characteristic is very interesting specially in the case of hybrid systems.

The state of the art on the methods on the stability of switched systems are mainly based on the knowledge of a mathematical model, which can be difficult or even impossible to obtain, hence the need to focus on another approach. Passivity and its link with stability as well as its application for interconnected systems offer a good alternative for the stability of switched systems.

The work is therefore presented as follows. Section 2 recalls the study framework, in particular the detection of the fault on one of the controller in the velocity control, which is at the origin of the switch. The passivity of the plant, modelled by a longitudinal bicycle model is then studied as well as the passivity of both of the controllers: CRONE and PI. The different proofs of passivity lead to the conclusion that the switched system switches between two passive subsystems. Section 3 then, introduce the principle of continuous approach and allows to conclude of the stability of the switch.

## 2 STUDY OF PASSIVITY

This section mainly focuses on the analysis of the passivity of the plant, modelled by a longitudinal bicycle model, as well as of the two controllers present within the speed controller.

First, the study framework is recalled in order to determine the origin of the switch. Then, an analysis of the passivity of the plant through the analysis of the analytical expressions of the nonlinear model is made. The linear case is presented in order to introduce the different expressions necessary for the study of stability in Section 3.

Finally, this section presents the analysis of passivity for a 2nd generation CRONE controller and a PI controller.

### 2.1 Study Framework

The work presented in this paper follows the development of a fault-tolerant strategy for an automotive cruise control detailed in (Ruhnke *& al*, 2020).

As a reminder, this work consists of regulating the longitudinal speed around a reference value using the CRONE controller. This latter undergoes at an arbitrary time $t_d$ a sampling fault, which forces its output value to an erroneous value. This has the consequence to fault the speed regulation.

The objective is therefore to design a supervisor, which makes it possible both to detect the fault on the CRONE controller and, following the detection, to switch to a functional PI controller in order to ensure the good functioning of the velocity control system. The block diagram of the system is illustrated Figure 1.

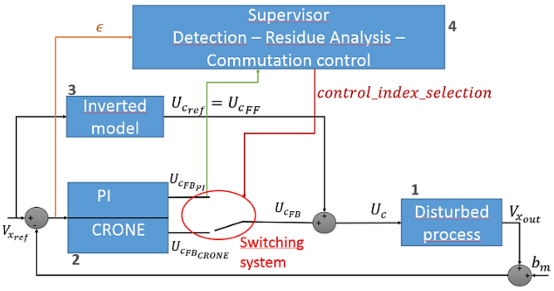


Figure 1: Block diagram of the studied system.

## 2.2 Plant Analysis

In order to analyse the passivity of the plant, the nonlinear model is first presented as well as the simplifying hypotheses linked to the study framework. The analytical expression of the variation of the storage function is then defined followed by sign study.

### 2.2.1 Context

For this work, it is assumed that the longitudinal speed is regulated around a reference speed $V_{x_{ref}}$.

The vehicle is an urban electrical vehicle with two in-wheel motors in the front wheel. This vehicle is supposed to drive in a straight line on a horizontal dry road. The total mass of the vehicle $M_t$ is evenly distributed throughout the vehicle.

In this paper, a simplified driving scenario is studied. Thus, the plant does not undergo disturbance.

In the following notations, index $i$ indicates whether if it is the front wheels $i = 1$ or the rear wheels $i = 2$, which are considered, and the index $j$ indicates whether if it is the left wheels $j = 1$ or the right wheels $j = 2$, which are considered.

For the modelling of the vehicle, the longitudinal bicycle model is used and is illustrated Figure 2.

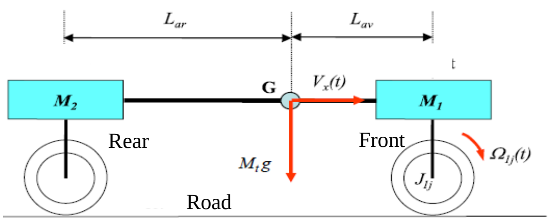


Figure 2: Longitudinal model of the vehicle.

$L_{ar}$ and $L_{av}$ represents respectively the rear and the front wheelbase. $M_1$ and $M_2$ are respectively the front and rear masses. $G$ is the center of gravity of the vehicle and $g$ the gravitational force. $J_{ij}$ , $\Omega_{ij}(t)$ and $V_x(t)$ are defined below in the expression of the equations of the model.

Following the various simplifying assumptions, only the longitudinal dynamics are considered. By applying the fundamental principle of dynamics, the plant can be modelled through the expressions of the longitudinal velocity $V_x(t)$ and the wheel rotation speeds $\Omega_{ij}(t)$. The two quantities are expressed in the absolute reference, such as:

$$V_x(t) = \frac{1}{M_t}\int_0^t F_{\Sigma_x}(\tau)d\tau + V_x(0), \quad (5)$$

where, $M_t$ represents the total mass of the vehicle and $F_{\Sigma_x(t)}$ is the sum of the longitudinal forces, which is expressed in its general form as follows:

$$F_{\Sigma_x}(t) = 2\,F_x(t) - F_{0_x}(t) - F_a(V_x) - F_{rr}(V_x). \quad (6)$$

$F_x(t)$ are the longitudinal nonlinear forces expressed by the Pacejka model (Morand & *al*, 2015) for one wheel. The forces developed by the left front tire $F_{x_{11}}(t)$ are equal to the forces developed by the right front tire $F_{x_{12}}(t)$, so the following notation is applied: $F_x(t) = F_{x_{11}}(t) = F_{x_{12}}(t)$.

$F_a(V_x)$ and $F_{rr}(V_x)$ are the aerodynamic and the rolling resistance forces and $F_{0_x}(t)$ represents the force associated with the gust of wind.

As the disturbance $F_{0_x}(t)$ is not considered for the analysis of passivity in the nonlinear case, expression (6) can be rewritten as follows:

$$F_{\Sigma_x}(t) = 2\,F_x(t) - F_a(V_x) - F_{rr}(V_x). \quad (7)$$

Regarding the wheel rotation dynamics, only the two front driving wheels are considered. The rotation speed of the wheels can therefore be written as follows:

$$\Omega_{ij}(t) = \frac{1}{J_{ij}}\int_0^t C_\Sigma(\tau)d\tau + \Omega_{ij}(0)\,. \quad (8)$$

$J_{ij}$ is the moment of inertia and $C_\Sigma(t)$ is the sum of the momentum applied on the front wheels. The sum is expressed as follows:

$$C_{\Sigma_{1j}}(t) = C_{r_{1j}}(t) - r_0\,F_x(t) - C_{f1j}(t), \quad (9)$$

where, $C_{r_{1j}}(t)$ represents the motor torque and the control command, $C_{f1j}(t)$, the viscous friction and $r_0\,F_x(t)$ , the resistant momentum. For more information, (Morand & *al*, 2015) provides a more detailed model.

### 2.2.2 Nonlinear Case without Disturbance

For simplifying the notation, temporal index will not be written in the following expressions and $\Omega_{ij}(t)$ is rewritten as $\Omega(t)$.

In order to analyse the passivity of the plant, the storage function $V(x)$ must be defined. In this case, it is equal to the sum of the kinetic energies of the system and the potential energy of gravity, which is here constant and denoted $K$, that is:

$$V(x) = \frac{1}{2} M_t\, V_x^2 + 2 * \frac{1}{2} J_{ij}\, \Omega^2 + K. \tag{10}$$

By deriving equation (10), $\dot{V}(x)$ is equal to:

$$\dot{V}(x) = M_t\, V_x\, \dot{V}_x + 2 J_r\, \Omega\, \dot{\Omega}, \tag{11}$$

where,

$$\dot{V}_x = \frac{1}{M_t}\,(2\, F_x(V_x, \Omega) - F_a(V_x) - F_{rr}(V_x)) \tag{12}$$

and

$$\dot{\Omega} = \frac{1}{J_{1_j}}(C_{r_{1j}} - r_0\, F_x(V_x, \Omega) - b_r\, \Omega). \tag{13}$$

$\dot{V}_x$ and $\dot{\Omega}$ are replaced in (11) by their respective expressions:

$$\dot{V}(x) = V_x\, [2\, F_x(V_x, \Omega) - F_a(V_x) - F_{rr}(V_x)] + 2\Omega\, (C_{r_{1_j}} - r_0 F_x(V_x, \Omega) - b_r \Omega). \tag{14}$$

By expanding equation (14), $\dot{V}(x)$ can be expressed as follows:

$$\dot{V}(x) = 2\, \Omega\, C_{r_{1j}} - b_r\, \Omega^2 - 2\, \Omega\, r_0\, F_x(V_x, \Omega) + V_x\, 2\, F_x(V_x, \Omega) - \mathrm{V_x}\, (F_a(V_x) + F_{rr}(V_x)), \tag{15}$$

where, $2\, \Omega\, C_{r_{1j}}$ represents the input/output product associated with the power calculation, $-b_r\, \Omega^2$ is the power lost to rotation and $V_x\, (F_a(V_x) + F_{rr}(V_x))$ is the power lost to translation.

According to equation (4), the plant is considered as passive if the following inequality holds:

$$\forall t \geq 0, \dot{V}(x) - 2\, \Omega\, C_{r_{1j}} \leq 0. \tag{16}$$

As the vehicle moves forward and the variables are expressed in the absolute coordinate system, the sign of these latter is known.

Thus, for the driving scenario under study, $V_x$, $\Omega$, $C_{r_{1j}}$ and $F_x(V_x, \Omega)$ are positive as well as the module of $F_a(V_x)$ and $F_{rr}(V_x)$. Since $r_0$ and $b_r$ are positive, this implies that $b_r\, \Omega^2$, $\mathrm{V_x}\, (F_a(V_x) + F_{rr}(V_x))$ are thus positive.

Therefore, validation of inequality (16) depends on the sign of:

$$V_x\, 2\, F_x(V_x, \Omega) - 2\, \Omega\, \mathrm{r}_0\, F_x(V_x, \Omega), \tag{17}$$

which can be rewritten as:

$$2\, F_x(V_x, \Omega)\, [V_x - r_0\, \Omega]. \tag{18}$$

In order to define the sign of (18), the expression of the slip rate in traction, $\tau_{tract}$, is recalled.

$$\tau_{tract} = \frac{r_0\, \Omega - V_x}{r_0\, \Omega}. \tag{19}$$

As the vehicle moves forward, $\tau_{tract}$ is positive.

By isolating the numerator of (19), the following equation is as follows:

$$r_0\, \Omega - V_x = \tau_{tract}\, r_0\, \Omega > 0. \tag{20}$$

As a result, $V_x - r_0\, \Omega$ is negative as well as equation (18).

Inequality (16) is therefore respected and the plant is passive.

### 2.2.3 Linearized Model of the Plant

In this subsection, the expression of the linearized model of the plant as well as the expression of the storage function and its derivative are presented.

The objective is not to demonstrate the passivity of the plant in the linear case, but to introduce the matrices of the linearized model and the expression of the storage function of the plant, which are necessary for the approach developed in Section 3 for the proof of the stability of the switched system.

For this purpose, a linearization of equations (12) and (13) around an operational point is made.

A linear state space representation is obtained by linearization of equations (12) and (13) around an equilibrium point denoted $X_e = (V_{x_{ref}}\, \Omega_{ref})$, where $V_{x_{ref}}$ represents the reference longitudinal speed and $\Omega_{ref}$, the wheel rotation speed of reference associated with $V_{ref}$. Thus, the matrices A, B, C and D of the state space representation are as follows (Morand & *al*, 2015):

$$A = \left.\begin{bmatrix} a_{11} = \frac{\partial h_1}{\partial X_1} & a_{12} = \frac{\partial h_1}{\partial X_2} \\ a_{21} = \frac{\partial h_2}{\partial X_1} & a_{22} = \frac{\partial h_2}{\partial X_2} \end{bmatrix}\right|_{\mathrm{X}=X_e} ;\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{21}$$

and

$$C = [0 \quad 1];\, D = 0, \tag{22}$$

with $x = (v_x\ \omega)$, $v = c_m$ and $z = \omega$, representing respectively the small variations around the equilibrium point for the state, the input and the output vectors of the linearized model.

The expression of the storage energy can be rewritten in the linear case, such as:

$$V(x) = \frac{1}{2} M_t v_x^2 + 2\frac{1}{2} J_r \omega^2 + K. \qquad (23)$$

By deriving expression (23) and replacing the expressions of $\dot{v_x}$ and $\dot{\omega}$, the derivative of the storage function is equal to:

$$\dot{V}(x) = M_t\, v_x\, [a_{11}\, v_x + a_{12}\, \omega] + 2J_{1_j}\omega \left[a_{21} v_x + a_{22}\, \omega + \frac{c_m}{J_{1_j}}\right]. \qquad (24)$$

By expanding the expression (24),

$$\dot{V}(x) = M_t\, a_{11}\, v_x^2 + \left[M_t\, a_{12} + 2\, J_{1_j}\, a_{21}\right] v_x\, \omega + 2J_{1_j} a_{22}\, \omega^2 + 2\, \omega\, c_m, \qquad (25)$$

where $2\, \omega\, c_m$ represents the input/output product.

To simplify the notations, equation (25) is rewritten as follow:

$$\dot{V}(x) = a\, v_x^2 + b\, v_x\, \omega + c\, \omega^2 + 2\omega\, c_m, \qquad (26)$$

with, $a = M_t\, a_{11}$, $b = M_t\, a_{12} + 2J_{1_j}\, a_{21}$ and $c = 2J_{1_j}\, a_{22}$.

The next step is to verify the passive nature of the CRONE controller and the PI controller. This analysis will lead to the conclusion that the switch takes place between two passive subsystems.

## 2.3 Passivity of the Controllers

In order to study the passivity of CRONE and PI controllers, Definition 1 is applied.

For both controllers, the expression of the transfer function as well as the Partial Fraction Decomposition (PFD) are defined. A causal diagram of the PI controller and of one of the cell of the CRONE controller are illustrated. Finally, the passivity of each controller is studied.

As a reminder, a system is passive if the following inequality is respected:

$$\forall t \geq 0, \frac{dV(t)}{dt} - z^T v \leq 0. \qquad (27)$$

where $z^T v$ represents the input/output product associated with the power calculation.

### 2.3.1 Passivity of the PI Controller

The transfer function of the PI controller is of the following form:

$$PI(s) = C_0 \frac{1+\frac{s}{\omega_i}}{\frac{s}{\omega_i}}. \qquad (28)$$

The PFD of transfer function (28) can be written as follows:

$$PI_{PFD}(s) = \frac{r_{PI}}{s} + k_{PI}, \qquad (29)$$

with $r_{PI}$, $k_{PI} > 0$.

The input of the controller is the error signal $\epsilon(t)$ and the output is $U_{PI}(t)$ associated with a voltage and proportional to the motor torque $C_m(t)$ through a factor $M$.

The causal diagram associated with the parallel form of the PI is illustrated Figure 3.

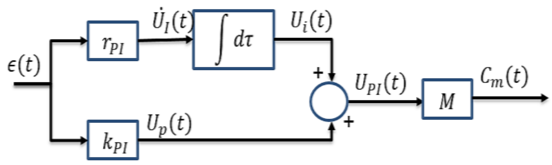


Figure 3: Causal diagram associated with the parallel form of the PI controller.

In order to study the controller's passivity, the storage function must be defined. In the case of the PI, the energy is stored only in the integral element, so $V(x)$ is expressed as follows:

$$V(x) = \frac{1}{2} \frac{1}{r_{PI}} U_I(t)^2. \qquad (30)$$

The next step is to derive equation (30) to check if inequality (27) is respected.

Thus,

$$\dot{V}(x) = \frac{1}{r_{PI}} U_I(t)\, \dot{U}_I(t), \qquad (31)$$

where,

$$\dot{U}_I(t) = r_{PI}\, \epsilon(t) \qquad (32)$$

and

$$U_I(t) = U_{PI}(t) - U_P(t). \qquad (33)$$

By replacing $\dot{U}_I(t)$ and $U_I(t)$ in (33), $\dot{V}(x)$ can be rewritten as,

$$\dot{V}(x) = \frac{1}{r_{PI}}\, r_{PI}\, \epsilon(t)\, (U_{PI}(t) - U_P(t)). \qquad (34)$$

As $U_P(t) = k_{PI}\, \epsilon(t)$, by developing and rewriting (34), $\dot{V}(x)$ is equal to:

$$\dot{V}(x) = U_{PI}(t)\, \epsilon(t) - k_{PI}\, \epsilon(t)^2, \qquad (35)$$

where, $U_{PI}(t)\, \epsilon(t) = \frac{1}{M} C_m(t)\, \epsilon(t)$ represents the input/output product associated with the power calculus;

The PI is passive if the following inequality holds:

$$\dot{V}(x) - U_{PI}(t)\, \epsilon(t) \leq 0. \qquad (36)$$

As $\dot{V}(x) - U_{PI}\epsilon(t) = -k_{PI}\, \epsilon(t)^2$ $k_{PI}\, \epsilon(t)^2$is always positive, inequality (36) is always respected and therefore the PI controller is passive.

### 2.3.2 Passivity of the CRONE Controller

The CRONE controller is a 2nd generation CRONE. It was calculated from the loop-shaping of the open loop. For more information about the design of the CRONE controller (Morand et *al*, 2015) presents the different stages of the design.

The rational form of the controller has two parts: an integer part and a part, which represents the rationalization of the phase lead cell of non integer order rewritten as a recursive product of zeros and poles. The expression of the transfer function of the rational form of the controller is the following:

$$CRONE(s) = C_0 \frac{\left(\left(1+\frac{s}{\omega_1}\right)*\left(1+\frac{s}{\omega_2}\right)\right)}{\left(\frac{s}{\omega_l}\right)\left(1+\frac{s}{\omega_h}\right)} \left(\frac{1}{1+\frac{s}{\omega_h}}\right) \prod_{i=1}^{N} \frac{1+s/\omega_i'}{1+s/\omega_i}. \quad (37)$$

The PFD of the transfer function (37) is expressed as follows:

$$CRONE_{PDF}(s) = \sum_{i=1}^{7} \frac{r_i}{s-p_i}, \quad (38)$$

with $r_i > 0$.

By posing $\omega_i = -p_i$, with $p_i < 0$, the expression (41) can be rewritten such as,

$$CRONE_{PFD}(s) = \sum_{i=1}^{7} \frac{r_i}{s+\omega_i}. \quad (39)$$

The input of the controller is the error signal $\epsilon(t)$ and the output is $U_{CRONE}(t)$ associated with a voltage and proportional to the motor torque $C_m(t)$ through a factor $M$.

The causal diagram associated with the parallel form of the rational form of the CRONE is illustrated Figure 4.

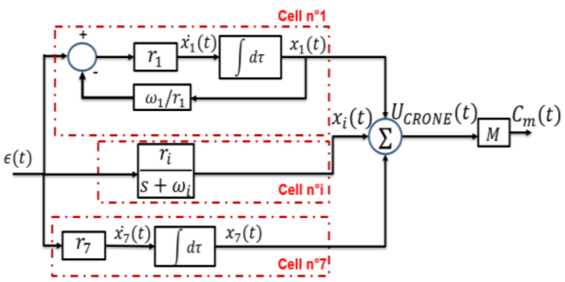


Figure 4: Causal diagram of the parallel form of the rational form of the CRONE controller.

For the study of the passivity of the controller, the storage function is defined. As the energy is only stored in the integral elements, $V(x)$ is defined as follows:

$$V(x) = \frac{1}{2}\sum_{i=1}^{7} \frac{1}{r_i} x_i(t)^2. \quad (40)$$

The derivative of the storage function is then equal to:

$$\dot{V}(x) = \sum_{i=1}^{7} \frac{1}{r_i} x_i(t)\, \dot{x}_i(t), \quad (41)$$

where,

$$\dot{x}_i(t) = r_i \left[\epsilon(t) - \frac{\omega_i}{r_i} x_i(t)\right]. \quad (42)$$

By replacing $\dot{x}_i(t)$ by its expression,

$$\dot{V}(x) = \sum_{i=1}^{7} x_i(t)\, \epsilon(t) - \sum_{i=1}^{7} \omega_i\, x_i(t)^2, \quad (43)$$

where,

$$\sum_{i=1}^{7} x_i(t)\, \epsilon(t) = U_{CRONE}(t)\, \epsilon(t) = \frac{1}{M} C_m(t)\, \epsilon(t), \quad (44)$$

which represents the input/output product associated with the power calculus.

For the CRONE to be passive, the following inequality must hold:

$$\dot{V} - \sum_{i=1}^{7} x_i(t)\, \epsilon(t) < 0. \quad (45)$$

However,

$$\dot{V} - \sum_{i=1}^{7} x_i(t)\, \epsilon(t) = -\sum_{i=1}^{7} \omega_i\, x_i(t)^2, \quad (46)$$

and $\omega_i\, x_i(t)^2 > 0$ thus inequality (45) holds and the CRONE controller is passive.

During Section 2, the passivity plant was shown as well as the passivity of both of the controllers through the analysis of analytical expressions.

The problem is the following: the fault detection has caused a switch, which takes place between two passive subsystems. It is, thus, necessary to prove the stable nature of the switching system in order to ensure the system's operating safety.

# 3 STABILITY OF THE SWITCHED SYSTEMS

The previous sections made it possible to demonstrate that the switch was done between two passive subsystems, namely on the one hand, the longitudinal model regulated by the CRONE and on the other hand, the longitudinal model regulated by the PI.

The objective is to prove the overall stability of the longitudinal speed controller, whatever the switching law.

For this, the continuous approach method (Nouillant et al., 2001) is developed. The principle is to build a state space representation of an augmented model, here the linearized model of the plant and the regulation, regardless the operating mode. In addition, only one Lyapunov candidate function denoted $V(x)$ is constructed.

The objective is to verify that the Lyapunov candidate function satisfies, for each operating mode, the Lyapunov criteria represented by equations (1) and (2) namely $V(x) > 0$ and $\dot{V}(x) \leq 0$.

If this is the case, the candidate function is a Lyapunov function common to both subsystems and the following theorem can be applied:

***Theorem 1:*** *(*Boyd *& al,* 1994*) (Sun and Ge, 2011). Let be the linear switched system $\dot{x}(t) = A_j\, x(t)$. If there exists a positive definite symmetric matrix $P \in R^{nxn}$ such that the following inequality is respected:*

$$A_j^T\, P + P\, A_j < 0, for\ j = 1{,}2\ , \tag{47}$$

*then the function $V(x) = x^T P\, x$ is a Common Quadratic Lyapunov Function (CQLF) for the system. The switching system is then stable whatever the switching law.*

***Remark 3.1:*** *This theorem is a sufficient condition but very conservative because it is difficult to obtain such a function.*

The matrices $A_j$ and $B_j$ of the augmented model are defined. In order to have a generic writing the matrices are defined in the most complex case, i.e. by taking into account the largest state and command vectors.

For this, the following state vector is considered: $x = [x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ v_x\ \omega]^T$ where $x_1$ ... $x_7$ represent the states of the regulation and $v_x, \omega$ respectively the longitudinal speed and the wheel's rotation speed, represent the states of the plant in the linearized model. In addition, the input vector is considered $u = V_{x_{ref}}$, which represent the reference longitudinal speed.

The matrices $A_j$ and $B_j$ are as follows:

$$A_j = diag(-r_i\ \omega_i(j), -r_7, A), \tag{48}$$

where, $i = 1, \dots, 6$ and $A$ represents the matrix of the linearized model of the plant (see equation (21)).

$$B_j = \begin{bmatrix} r_1(j) \\ r_2(j) \\ r_3(j) \\ r_4(j) \\ r_5(j) \\ r_6(j) \\ r_7 + k(j) \\ 0 \\ 0 \end{bmatrix}. \tag{49}$$

Index *j* represents the operating mode in which the system is, i.e.:

- If $j = 1$, the CRONE is in operation and regulates the system, thus $r_1 \rightarrow r_7$ and $\omega_1 \rightarrow \omega_6$ have the numerical values associated with the PFD of the CRONE controller and $k = 0$.
- If $j = 2$, the PI is in operation and regulates the systems, thus $r_1 = r_2 = \cdots = r_6 = 0, \omega_1 = \cdots = \omega_6 = 0, r_7 = r_{PI}$ and $k = k_{PI}$.

In order to study the stability of the switched system, the energy storage function is defined and denoted $V(x)$. This function is equal to the sum of the energy storage function of the regulation system and the energy storage function of the plant.

$$V(x)(j) = \frac{1}{2}\sum_{i=1}^{7} \frac{1}{r_i(j)}\ x_i^2(j) + \frac{1}{2} M_t\ v_x(t)^2 + 2 * \frac{1}{2} J_r\ \omega(t)^2. \tag{50}$$

This function can be rewritten under a matricial form such as:

$$V(x) = x^T P_j x, \tag{51}$$

where, $P_j = diag\left(\frac{1}{2}\frac{1}{r_i}(j), \frac{1}{2}M_t, J_r\right) > 0$ with $i = 1, \dots, 7$.

$V(x)$ is considered as the candidate Lyapunov function.

The objective is to calculate the derivative of $V(x)$ and to check if it meets the Lyapunov criteria for both operating mode.

$$\dot{V}(x) = \sum_{i=1}^{7} \frac{1}{r_i(j)}\ x_i(j)\ \dot{x}_i(j) + M_t\ \dot{v}_x v_x + 2J_r\ \dot{\omega}\ \omega, \tag{52}$$

where, $\dot{x}_i(j) = -r_i\omega_i(j)\ x_i$ for $i = 1, \dots, 6$ and $\dot{x}_7(j) = -r_7(j)\ x_7$.

By replacing $\dot{x}_i(j)$, $\dot{v}_x$ and $\dot{\omega}$ with their respective expressions, equation (52) can be rewritten as :

$$\dot{V}(x) = \sum_{i=1}^{6} -\omega_i x_i(j)^2 - x_7^2 + a v_x^2 + b v_x \omega + c\omega^2 + 2\omega c_m. \tag{53}$$

Equation (53) can be decomposed in two functions: $\dot{V}_{REG}(x)$, which represents the derivative of the storage function for the regulation and $\dot{V}_{PRO}(x)$, which represents the derivative of the storage function for the plant.

$$\dot{V}_{REG}(x) = \sum_{i=1}^{6} -\omega_i\, x_i(j)^2 - x_7^2. \tag{54}$$

$$\dot{V}_{PRO}(x) = a\, v_x^2 + b\, v_x\omega + c\, \omega^2 + 2\, \omega\, c_m, \tag{55}$$

In order to check if the candidate function $V(x)$ is a Lyapunov function, the sign of (50) is first studied for both case.

When $j = 1$, the CRONE is regulating the system. Moreover, $v_x(t)^2$ , $\omega(t)^2 > 0$ and according to section 2.3, $r_i > 0$.

As a result, $V(x)(j = 1)$ is positive definite.

When $j = 2$, the PI is regulating the system. Thus, equation (50) can be rewritten such as:

$$V(x)(j = 2) = \frac{1}{r_{PI}}\, x_{PI}^2(j) + \frac{1}{2} M_t\, v_x(t)^2 + 2 * \frac{1}{2} J_r\, \omega(t)^2. \tag{56}$$

As $r_{PI} > 0$, $V(x)(j = 2)$ is positive definite.

At this stage, in order to conclude on the nature of the candidate function $V(x)$, the sign of (53) has to be studied. For this purpose, the sign of (54) and (55) are studied for both functioning cases.

For the first case, when $j = 1$ , $\dot{V}_{REG}(x) = \sum_{i=1}^{6} -\omega_i\, x_i(j)^2 - x_7^2$. As $\omega_i > 0$, the function $\dot{V}_{REG}$ is negative definite.

For the second case, when $= 2$ , $\dot{V}_{REG}(x) = -x_7^2$, then $\dot{V}_{REG}$ is negative definite.

Despite the operating mode, the derivative of the storage function for the controller is always defined negative. Therefore the sign of (53) depends on the sign of (55), which is independent of the functioning mode.

***Lemma 3.1***: *(Khalil, 2002). If a system is passive with a positive storage function $V(x)$, then the origin is stable is the sense of Lyapunov by considering $V(x)$ as a Lyapunov function candidate. Then, $\dot{V}(x) \leq 0$.*

The approach presented in section 2.2.2 and 2.2.3 shows that firstly the plant is passive and secondly that both $v_x(t)^2$ and $\omega(t)^2$ are positive. As a result, the storage function of the plant, which is $V_{pro}(x) = \frac{1}{2} M_t v_x(t)^2 + 2\frac{1}{2} J_r \omega(\mathrm{t})^2$ is positive definite. By using Lemma 3.1, equation (55) is then negative definite.

Equations (54) and (55), which respectively represents the derivative of the storage function for the controller and the plant are both negative definite. In this case, equation (53), which represents the sum of these two functions is as well negative definite.

Since the candidate function $V(x)$ is positive definite and its derivative is negative definite for both operating modes, $V(x)$ is therefore a common quadratic Lyapunov function. As the result, by application of Theorem 1, the regulated longitudinal model is stable.

# 4 CONCLUSIONS

Fault management systems are employed to ensure the operational safety of the Advanced Driver Assistance Systems. These include a diagnostic part that detects and locales the fault, and a reconfiguration part that follows the detection, allowing you to switch to a functional mode or a degraded mode. The reconfiguration can take the form of a switch. As the latter can be a source of instabilities, it is therefore necessary to ensure the stability of the overall system despite the presence of switching.

Thus, the purpose of this work was to study the stability of switched regulated systems following reconfiguration due to the detection of a fault on the calculator of the Automated Cruise Control system.

In order to answer this problem, the passivity of the undisturbed plant modelled by a longitudinal non-linear bicycle model, as well as the Partial Fraction Decomposition of the PI and CRONE controllers, was studied in section 2.

For the plant, the passivity was demonstrated through the sign study of the analytical expression of the storage function as well as its derivative. The same approach was applied to de Partial Fraction Decomposition of both controllers.

The study of passivity concludes that the switch occurs between two passive subsystems, namely the plant controlled by the CRONE and the plant controlled by the PI.

Finally, in order to show the passive and therefore stable nature of the switched regulated systems, the continuous approach has been developed. The latter consists in building an augmented model through a state space representation whose structure is independent of the operating mode. This state space representation contains the controllers and the plant.

Then, a single candidate function of Lyapunov is defined and represents the sum between the storage function of the regulation and the storage function of the plant. A sign study of this function and its derivative leads to the conclusion that the candidate function meets Lyapunov's stability criteria and therefore that the regulated longitudinal bicycle model is passive and thus stable, despite the switch between CRONE and PI controllers.

The safety and security aspect have been proven through these works. The perspectives, however, related to an aspect of comfort.

Indeed, when the CRONE controller is operational, the PI operated in open loop. In some cases, the switch can generate a significant discontinuity in the control signal. These abrupt variations can engender sources of discomfort for the passengers, particularly in terms of sudden variation of acceleration.

Regarding the application area, this work was developed around a longitudinal model without disturbances and on a dry, straight and plane road. It would be interesting in terms of perspectives, to expand the model used, to make it more realistic and generic with regard to real driving scenarios.

On the one hand, disturbances such as gusts of wind, slopes of the road, poor road adherence or non-uniform loading can be considered and the other hand, other vehicle-specific dynamics such as lateral and yaw dynamics can be taken into account.

Then, a study of the stability associated with reconfiguration, regardless of driving scenarios, would allow verifying the genericity of the reconfiguration.

In the longer term, the idea is to study reconfiguration and stability on the architecture of the Automated Driving that is more complex with an application on driving-aid functions such as artificial intelligence-based decision-making or planning algorithms whose mathematical model is more difficult even impossible to obtain.

These perspectives will enhance the operating safety of the generic architecture of an Automated Driving vehicle in both highway and urban environments.

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