A Novel Technique for Modeling Vehicle Crash using Lumped Parameter Models

Gulshan Noorsumar[a], Svitlana Rogovchenko[b], Kjell G. Robbersmyr[c], Dmitry Vysochinskiy[d] and Andreas Klausen[e]
Department of Engineering Sciences, University of Agder, Grimstad, Norway

Keywords: Mathematical Model, Collision Mitigation, Lumped Parameter Model, Piece-wise Linear Spring Stiffness, Vehicle Pitching, Maximum Dynamic Crush.

Abstract: This paper presents a novel technique for modeling a full frontal vehicle crash. The crash event is divided into two phases; the first until maximum crush and the second part when the vehicle starts pitching forward. This novel technique will help develop a three degrees of freedom (DOF) lumped parameter model (LPM) for crash and support in the vehicle development process. The paper also highlights the design process for reducing vehicle pitching in occupant protection load cases. The model has been validated against a finite element (FE) simulation of a full frontal crash of a Chevrolet Silverado developed by the National Highway Traffic Safety Administration (NHTSA), and the LPM shows good correlation with the FE test data.

1 INTRODUCTION

Vehicle crashes have been among the major causes of mortality in recent times (Du Bois et al., 2004). In October 2015, the European Commission had launched a study to identify the most common crash scenarios leading to serious injuries in a vehicle crash. The results of this study point to the fact that a frontal crash is the most common crash scenario, followed by a side impact, where occupants are severely injured (Noorsumar et al., 2020). Euro NCAP is a voluntary car safety assessment program introduced to ensure safer cars for occupants and vulnerable road users. This program has been instrumental in driving regulations across the globe and improving vehicle safety standards. During the past decades, several crash mitigation and avoidance techniques have been employed by vehicle design engineers to meet these stringent regulations. The vehicle front-end and side structures have been modified to improve energy absorption capability (Elkady and Elmarakbi, 2012). Vehicle design engineers have resorted to various methodologies to improve the vehicle structure to absorb energy in case of a crash and prevent intrusions in the occupant compartment. These methodologies have been partially successful in replacing full-time physical tests in the vehicle development process. The vehicle industry still conducts a lot of physical crash tests to validate the crash response generated from mathematical models.

One of the recent approaches is using finite element methods (FEM) to model the full vehicle impact scenario and conduct simulations to predict the vehicle and occupant injury values. (Benson et al., 1986) presented the calculations of crashworthiness design thereby, laying the foundations for application of FEM in the automotive industry. This technique has high accuracy in predicting injury values, however the process involves manual efforts and is computationally intensive. Lumped parameter models (LPM) were first used in modeling vehicle crash by (Kamal, 1970). In this paper the vehicle was represented by three lumped mass components and eight resistances representing the deformable parts in the vehicle. Mentzer et al. (Mentzer et al., 1992) employed real time crash data to determine parameters for LPM used to represent the crash scenario. The force deformation curves derived from these models helped determine predictive models aiding in vehicle

[a] https://orcid.org/0000-0002-6718-4508
[b] https://orcid.org/0000-0001-8002-4974
[c] https://orcid.org/0000-0001-9578-7325
[d] https://orcid.org/0000-0002-0453-0012
[e] https://orcid.org/0000-0002-5411-3655

Noorsumar, G., Rogovchenko, S., Robbersmyr, K., Vysochinskiy, D. and Klausen, A.
A Novel Technique for Modeling Vehicle Crash using Lumped Parameter Models.
DOI: 10.5220/0010529200620070
In *Proceedings of the 11th International Conference on Simulation and Modeling Methodologies, Technologies and Applications (SIMULTECH 2021)*, pages 62-70
ISBN: 978-989-758-528-9

development.

Recently, LPMs were used by Elkady et al. to develop a multi-DOF mathematical model to simulate a crash event with active vehicle dynamics control systems (VDCS) (Elkady and Elmarakbi, 2012; Elkady et al., 2012). The model replicated a full frontal and offset impact between two vehicles and compared the performance of a baseline vehicle with a vehicle equipped with VDCS features. It also includes a 3-DOF occupant impact model using Lagrangian formulation. Munyazikwiye et al. use a mass-spring-damper model with two lumped mass components representing a vehicle impacting a rigid barrier. After identifying the parameters, the model in this study shows good correlation with test data which demonstrates that a simple LPM can be used to represent the impact dynamics successfully (Munyazikwiye et al., 2013). Multi body modeling has also been used in the past for simulating vehicle dynamics model for realistic applications (Riegl and Gaull, 2018).

Occupant injury prediction is an area of research where the vehicle-occupant interaction in a vehicle impact scenario is studied and the injury patterns of occupants in the car are determined with mathematical models. Large vehicle deceleration has been identified as one of the main causes of head and chest injuries, and vehicle rotational motions in different axis also lead to occupant injuries (Chang et al., 2006). In a full frontal impact, vehicle pitch and drop are significantly greater compared to rolling and yawing motions. In the recent past, increasing focus on unbelted occupants to meet FMVSS 208 (Federal Motor Vehicle Safety Standards) requirements has led researchers to observe that vehicle pitch and drop contributed to higher head and neck injury values. The objective of a vehicle structure is not just to absorb energy and optimize crash pulses, but also to minimize vehicle pitch and drop (Chang et al., 2006; Woitsch and Sinz, 2013). Chang et al. have developed an FE model to study vehicle pitch and drop in body-on-frame vehicles. The model is correlated to barrier tests and also tries to predict factors affecting vehicle pitch and drop in a crash event (Chang et al., 2005). The research from Chang et al. points to the fact that design of vehicle rails plays an important role in the load distribution during an impact scenario for body-on-frame vehicles. The out-of-plane bending of the vehicle rails increases the role of a vertical component of the barrier force, causing an imbalance in the vehicle, leading to forward pitching on the vehicle. Wei et al. have estimated the relationship between energy absorbing components and the crash pulse, establishing the fact that the bumper and the front rails both significantly contributing to the energy absorption in a full frontal crash event (Wei et al., 2016).

In this paper, we simplify the system by splitting the vehicle motion into two phases corresponding to

- the horizontal linear motion, and
- the rotation of the vehicle body.

We have decided to replicate a full frontal vehicle crash event at 56 kilometers per hour (kmph) employing an LPM with multiple DOFs to predict

- the maximum deformation in the vehicle to absorb energy, and
- the pitch angle of the vehicle due to the crash response.

2 METHODOLOGY

Literature documents that a crash event leads to pitching, rolling and yawing of the vehicle along with the deceleration of the vehicle and movement in horizontal and vertical directions. It is difficult to model the impact scenario in different axes and generating the governing equations. It was also observed that the time for the vehicle to attain minimum velocity after impact also coincides with the maximum deformation on the vehicle.

In this study, we separate the horizontal translational motion from the vertical motion during the impact event. In a full frontal crash event the vehicle is observed to be experiencing forward pitching; whereas the effect of rolling and yawing can be neglected. Taking into account these assumptions we split the crash event into two phases:

- time till maximum deformation and minimum vehicle velocity after start of crash event t_1, and
- time after maximum deformation to the end of the crash event t_2.

2.0.1 FEM Simulations

In this study, finite element simulation for a 2014 Chevrolet Silverado (Administration et al., 2016) running at 56 kmph and hitting a frontal barrier at 0% offset was conducted. The FE model was developed by National Crash Analysis Center (NCAC) in collaboration with NHTSA (National Highway Traffic Safety Administration) through the reverse engineering process (Administration et al., 2016). The FE model consisting of 1476 parts, 2,741,848 nodes and 2,870,507 elements has been correlated to NHTSA Oblique Test and Insurance Institute for Highway Safety (IIHS) Small Overlap Front Test. The FE model weighs 2582 kg which is close to the physical test vehicle weighing 2624 kg. It replicates the material and geometrical properties of the physical vehicle (Singh et al., 2018).

The FE model was run on LS-DYNA with 32 CPUs in an HPC environment and the corresponding curves generated were used for the parameter estimation and validation of the LPM in MATLAB Simulink. In the FE simulation, the acceleration of some nodes on the vehicle body are recorded by the solver LS-DYNA. These nodes are selected by the user at the preprocessing stage. This process was employed to determine the acceleration of the vehicle CG as well as the barrier forces, to be used for validation in this study. Figure 1 shows the FE model used in the simulations.

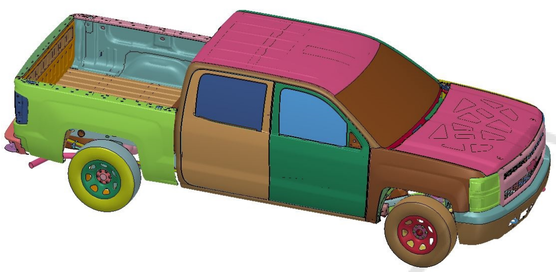

Figure 1: FE Model of a 2014 Chevrolet Silverado developed by NCAC.

This FE model generated the piecewise linear curve data for spring and damper coefficients. The algorithm uses Newton-Euler numerical integration to achieve the computed values and predict the time for maximum dynamic crush of the vehicle. The algorithm developed is explained in the next section.

2.1 Lumped Parameter Model

The LPM developed is a single-mass system with a spring and damper unit in the front, known as Kelvin model, representing the bumper system and the deformable system. The front springs allow translational motion only in the direction of x-axis (Huang, 2002). The suspension of the vehicle is represented by a pair of springs and dampers which allow translation in the vertical z-axis and rotation around the y-axis. The center of mass of the vehicle has 3 DOFs making this system fairly complex to solve in a single system. The lumped mass body can move in the direction of horizontal (x) and vertical (z) axes along with rotation around one (y) axis. The center of gravity (CG) of the vehicle is located at a distance l_f from the front end and l_r from the rear end suspension points. The distance l_0 represents the distance of the CG from the front occupant compartment zone.

2.2 Vehicle Crash Model: Phase I

First we model only the translational movement of the vehicle along the horizontal axis and hitting the barrier at 0% offset. The mathematical model is developed in Simulink which replicates the maximum vehicle deformation till the time of maximum crush t_m. This value also corresponds to the time when the vehicle attains zero or minimum velocity. It should be noted that the vehicle may not achieve zero velocity by the time of maximum deformation if the vehicle front end is not able to absorb energy to undergo plastic deformation. The mathematical model uses a single DOF equation with a front spring-damper unit. The stiffness of the spring is tuned to represent the maximum deformation of the vehicle at a particular speed. For this problem we have assumed a speed of 56 kmph (NHTSA regulations for frontal crash). The motion of suspension system in the model has been neglected during this phase of the event scenario. Figure 2 represents the vehicle in a deformed state. The Simulink model predicts the time till maximum deformation of the vehicle and the maximum displacement of the vehicle CG.

The prediction of the values of spring deformation coefficient k and damper coefficient c used in the general equation of motion have been a challenge for researchers in the past (Marzbanrad and Pahlavani, 2011), (Pawlus et al., 2011). There have been several parameter estimation studies conducted in the past to determine the stiffness of the vehicle front in a crash event. The behaviour of the front end system is highly nonlinear but it has been approximated by a piecewise linear relationship (Munyazikwiye et al., 2017; B Munyazikwiye et al., 2018).

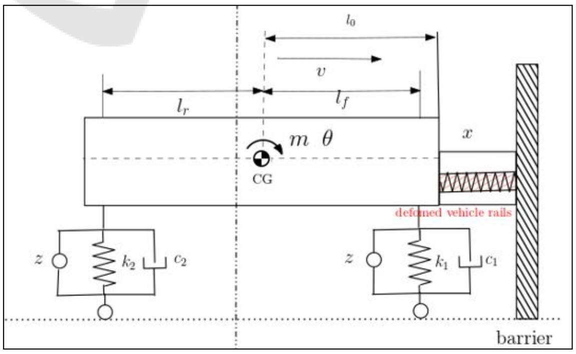

Figure 2: Vehicle representation in Phase 1 of the event: Deformed front end.

Equation of Motion:

$$m\ddot{x} + c\dot{x} + kx = Q_i \tag{1}$$

where, $Q_i = 0$ (i.e. no force component is added here); k is the spring coefficient; c is the damper coefficient for the bumper model.

In the model developed for the first phase, the spring and damper coefficients are parameterized using a gradient-descent optimization algorithm developed in (Klausen et al., 2014) for a single mass-spring-damper system. The code searches for a global minima by performing 100 re-runs of gradient descent optimization, each with randomly generated initial parameter values. The algorithm was modified to improve the correlation between the test and computed values. The non-linear force-deformation curve for spring-damper system has been assumed to be piecewise-linear with six breakpoints in the curve. The forces on the spring are calculated using the general relationship between the force and deformation for a spring-damper system (Elkady and Elmarakbi, 2012), see Figure 3. The stiffness of the spring k and the spring force component F_k vary according to the deflection values in the spring and are defined as follows.

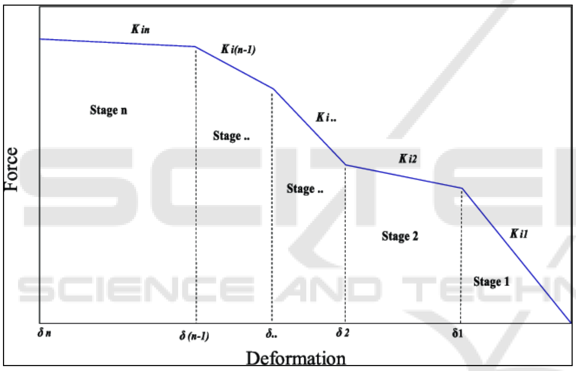

Figure 3: General piecewise force-deformation characteristics (Elkady and Elmarakbi, 2012).

The spring stiffness and damping coefficients in the model, are defined as the piecewise-linear functions of x and $\dot{x}$, respectively. These functions are

$$k(x) = \begin{cases} \frac{(k_2-k_1)\cdot|\hat{x}|}{x_1} + k_1 & \text{for, } |\hat{x}| \leq x_1, \\ \frac{(k_3-k_2)\cdot(|\hat{x}|-x_1)}{(x_2-x_1)} + k_2 & \text{for, } x_1 \leq |\hat{x}| \leq x_2 \\ \frac{(k_4-k_3)\cdot(|\hat{x}|-x_2)}{(x_3-x_2)} + k_3 & \text{for, } x_2 \leq |\hat{x}| \leq x_3 \\ \frac{(k_5-k_4)\cdot(|\hat{x}|-x_3)}{(x_4-x_3)} + k_4 & \text{for, } x_3 \leq |\hat{x}| \leq x_4 \\ \frac{(k_6-k_5)\cdot(|\hat{x}|-x_4)}{(x_5-x_4)} + k_5 & \text{for, } x_4 \leq |\hat{x}| \leq x_5 \\ \frac{(k_7-k_6)\cdot(|\hat{x}|-x_5)}{(C-x_5)} + k_6 & \text{for, } x_5 \leq |\hat{x}| \leq C \end{cases}$$

The damper characteristics are defined similar to the spring characteristics in the model

$$c(\dot{x}) = \begin{cases} \frac{(c_2-c_1)\cdot|\hat{\dot{x}}|}{\dot{x}_1} + c_1 & \text{for, } |\hat{\dot{x}}| \leq \dot{x}_1, \\ \frac{(c_3-c_2)\cdot(|\hat{\dot{x}}|-\dot{x}_1)}{(\dot{x}_2-\dot{x}_1)} + c_2 & \text{for, } \dot{x}_1 \leq |\hat{\dot{x}}| \leq \dot{x}_2 \\ \frac{(c_4-c_3)\cdot(|\hat{\dot{x}}|-\dot{x}_2)}{(\dot{x}_3-\dot{x}_2)} + c_3 & \text{for, } \dot{x}_2 \leq |\hat{\dot{x}}| \leq \dot{x}_3 \\ \frac{(c_5-c_4)\cdot(|\hat{\dot{x}}|-\dot{x}_3)}{(\dot{x}_4-\dot{x}_3)} + c_4 & \text{for, } \dot{x}_3 \leq |\hat{\dot{x}}| \leq \dot{x}_4 \\ \frac{(c_6-c_5)\cdot(|\hat{\dot{x}}|-\dot{x}_4)}{(\dot{x}_5-\dot{x}_4)} + c_5 & \text{for, } \dot{x}_4 \leq |\hat{\dot{x}}| \leq \dot{x}_5 \\ \frac{(c_7-c_6)\cdot(|\hat{\dot{x}}|-\dot{x}_5)}{(v_0-\dot{x}_5)} + c_6 & \text{for, } \dot{x}_5 \leq |\hat{\dot{x}}| \leq v_0 \end{cases}$$

where, k is the spring coefficient; c is the damper coefficient; $\hat{x}$ is the computed vehicle deformation; $\dot{x}$ is the vehicle velocity; $\hat{\dot{x}}$ is the computed vehicle velocity; v_0 is the velocity at the time of maximum dynamic crush; C is the maximum dynamic crush;

F_k and F_c are the built-up spring and damping forces defined by the following equations

$$F_k = k(x) \cdot x, \tag{2}$$

$$F_c = c(\dot{x}) \cdot \dot{x} \tag{3}$$

The proposed algorithm uses an optimization approach to minimize an objective function. The objective function to be minimized is the error function $E(\Theta,t)$ where Θ denotes the unknown variables in the mode. The error function is defined as follows:

$$E(\Theta,t) = E_1(\Theta,t) + E_2(\Theta,t) + E_3(\Theta,t), \text{where} \tag{4a}$$

$$E_1(\Theta,t) = |(a_{FE} - a_{LPM})| \tag{4b}$$

$$E_2(\Theta,t) = |(v_{FE} - v_{LPM})| \tag{4c}$$

$$E_3(\Theta,t) = |(x_{FE} - x_{LPM})| \tag{4d}$$

where, a is the acceleration; v is the vehicle velocity; and x is the displacement.

The error function $E(\Theta,t)$ determines the difference between the FE and computed values at every point, and the optimization algorithm tries to minimize these error values by altering $\Theta = [k_i, c_i] \forall i \in [1,7]$. The corresponding spring and damper coefficient values developed from this minimization algorithm have been discussed in the results section of the paper.

2.3 Vehicle Crash Model: Phase II

The second phase for the model describes what happens after the instant the vehicle achieves maximum dynamic crush and minimum velocity. The vehicle starts to pitch forward at this instant. Several studies were conducted to understand the reason behind the vehicle pitching forward (Chang et al., 2005; Chang et al., 2006), suggesting that for body-on-frame vehicles one of the reasons is the out of plane bending in vehicle rails leading to a vertical force component in the moment balance equation. The vertical force component is added to gravity force acting downwards and creates an imbalance of loading which leads to the vehicle pitching. The prediction of this pitching angle is important for determining the injury to occupants and a low pitching angle influences occupant protection design in a vehicle. In this phase of the event as shown in Figure 4 and Figure 5, we consider only vertical motion of the suspension springs and a rotation about the y-axis with angle θ.

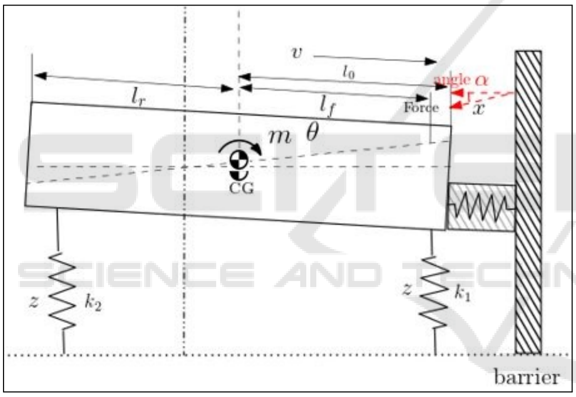

Figure 4: Vehicle representation in Phase II of the event: Vehicle Pitching forward.

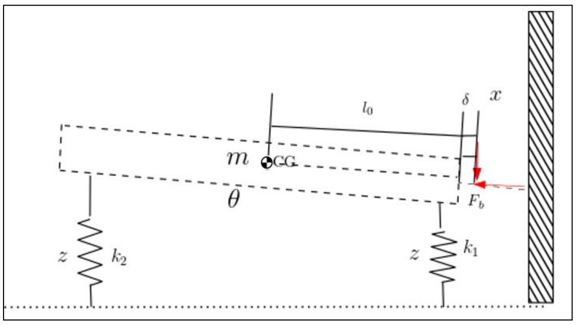

Figure 5: Vehicle representation in Phase II with forces acting on the vehicle and suspension springs in play.

We use the Lagrangian Formulation (Goldstein et al., 2002):

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} + \frac{\partial D}{\partial q_i} = Q_i \tag{5}$$

where, in the general case, $L = T - V$, T is the total kinetic energy of the system equal to the sum of the kinetic energies of the particles, $q_i\, i = 1..n$ are generalized coordinates and V is the potential energy of the system. For dissipation forces a special function D must be introduced alongside L, Q_i is the external force acting on the system, which in this case is the vertical force component experienced by the vehicle at the time of maximum dynamic crush.

Kinetic Energy:

$$T = \frac{1}{2}J\dot{\theta}^2 + \frac{1}{2}m\dot{x}^2 \tag{6}$$

Potential Energy :

$$V = \frac{1}{2}k_1(x - l_f\theta)^2 + \frac{1}{2}k_2(x + l_r\theta)^2 \tag{7}$$

Dissipation Energy :

$$D = \frac{1}{2}c_1(\dot{x} - l_f\dot{\theta})^2 + \frac{1}{2}c_2(\dot{x} + l_r\dot{\theta})^2 \tag{8}$$

The values of k_1, k_2, c_1, c_2, l_f and l_r are taken from standard automotive parameters from literature data (Savaresi et al., 2010). Table 1 shows the parameter values adopted from this study.

Table 1: Automotive Parameters set (Savaresi et al., 2010).

Symbol	Value	Unit	Meaning
M	400	kg	Sprung mass
m_{ij}	50	kg	Unsprung masses (i = front, rear and j = left, right)
I_x	250	kg.m^2	Roll inertia
I_y	1400	kg.m^2	Pitch inertia
t	1.4	m	Front and rear axle
l_f	1.4	m	COG-front distance
l_r	1	m	COG-rear distance
r	0.3	m	Nominal wheel radius
h	0.7	m	Chassis COG height
k_f	30,000	N/m	Front suspension linearized stiffness (left, right)
k_r	20,000	N/m	Rear suspension linearized stiffness (left, right)
c_f	1500	N/m/s	Front suspension linearized damping (left, right)
c_r	3000	N/m/s	Rear suspension linearized damping (left, right)
k_t	200,000	N/m	Tire stiffness (front, rear and left, right)
β	50	rad/s	Suspension actuator bandwidth

The value of the vehicle mass m and the moment of inertia J for the lumped mass system has been calculated from the FE model of the vehicle. The governing equations of motion are

$$\begin{aligned} Q_i =& J\ddot{\theta} + (k_1 l_f^2 + k_2 l_r^2)\theta + (c_1 l_f^2 + c_2 l_r^2)\dot{\theta} \\ &+ (k_2 l_r - k_1 l_f)x + (c_2 l_r - c_1 l_f)\dot{x} \end{aligned} \tag{9}$$

$$\begin{aligned} -Q_i =& m\ddot{x} + (k_1 + k_2)x + (c_1 l_f + c_2 l_r)\dot{\theta} \\ &+ (k_2 l_r - k_1 l_f)\theta + (c_1 + c_2)\dot{x} \end{aligned} \tag{10}$$

3 RESULTS AND DISCUSSION

In this section we compare the results of the LPM model with FE data generated from LS-DYNA simulations for a Chevrolet Silverado vehicle at 56kmph with a full frontal impact loadcase.

3.1 Phase I

As mentioned in the previous section, Part I of the event simulates the time till maximum deformation of the vehicle; the spring and damper coefficients are determined using the Gradient Descent Optimization with an error function explained in the previous section. The computed and test (FE) values are plotted in Figure 6 and shows good correlation of results. The algorithm predicts the stiffness and damping coefficient values as shown in Figures 7 and 8.

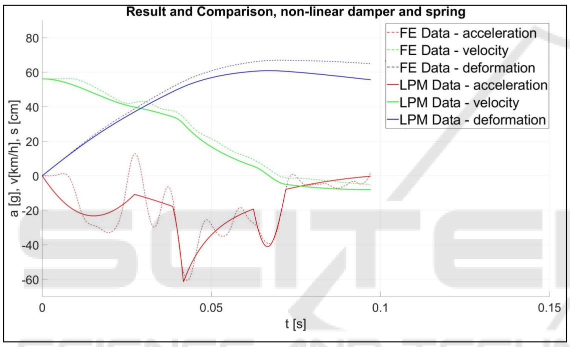

Figure 6: Plot of computed and test values for parameter model.

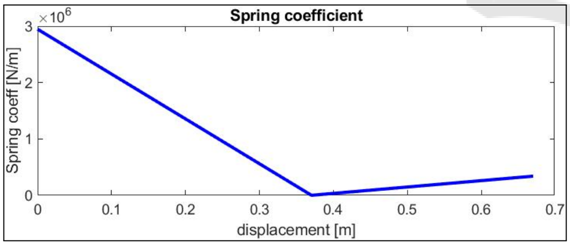

Figure 7: Spring coefficient obtained from the algorithm.

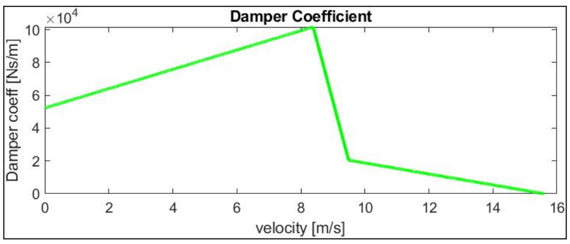

Figure 8: Damper Coefficient obtained from the algorithm.

The output from the Gradient Descent Optimization algorithm is used to predict the deformation and vehicle velocity in a MATLAB Simulink model;

Figure 9 shows a plot of maximum vehicle deformation vs test deformation and the plot shows good correlation. A similar plot (Figure 10) was generated to compare the velocity of the vehicle at the CG (in the case of LPM at the CG of the lumped mass). The time the vehicle attains zero velocity is similar in the plots but there is a small difference after 0.04s. The reason for this deviation can be attributed to the spring and damper characteristics which are approximated for this study using a piece-wise linear function. The model can be improved using a non-linear function for the spring stiffness and damping characteristics. If the model is simulated beyond the time the vehicle attains zero velocity, a rebound is observed in the velocity. This velocity rebound could be due to the internal strain energy store in the springs, and it would be interesting to investigate this further in future research.

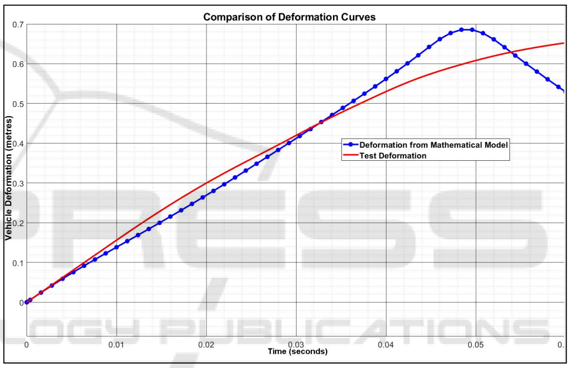

Figure 9: Displacement of the vehicle CG curves comparison for LPM vs FE model.

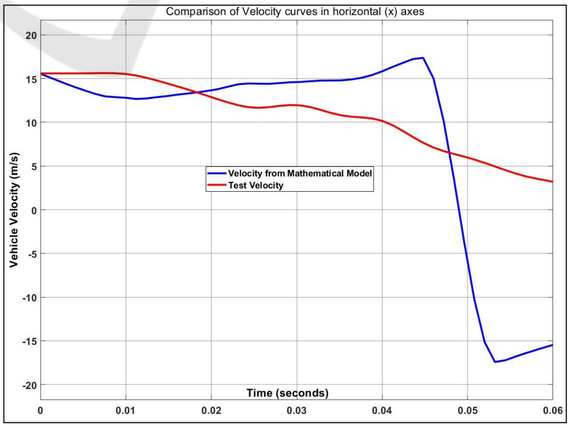

Figure 10: X-Velocity curve comparison for LPM vs FE model.

3.2 Phase II

The prediction for the second part of the model using Simulink was conducted and plotted against the data from FE model. The force Q_i in the governing equations is the vertical component of the barrier force experienced by the vehicle in the crash. The force curve is derived from the FE model and inputted into the Simulink model to improve prediction. However, it will be of interest to mathematically explain this force component in terms of residual impact energy after absorption. The Simulink model is run with numerical integration (variable timestep- ode 45) and the velocity of the lumped mass in z-direction along with the pitching angle is compared to data from FE model.The comparison with other numerical integration methods was kept out of scope of this study.

Figure 11 compares the forward pitching angle for the FE model and the LPM developed in this study. The pitch angle comparison shows a similar trend observed in both the curves; the vehicle starts to pitch around the same time during the crash event which is crucial to designers planning airbag deployment in vehicles and other active features on cars. The pitch angle curve for the simulation LPM peaks higher than the FE data at the start of the vehicle rotation but slowly follows the FE data curve showing comparable maximum pitch angle values which is also an important observation for a vehicle safety design team. The linear approximation for the spring and damper coefficients can be a contributing factor to the difference in the values between the curves along with the barrier force definition in the model. There might be energy losses in the model which have not been accounted for in this study.

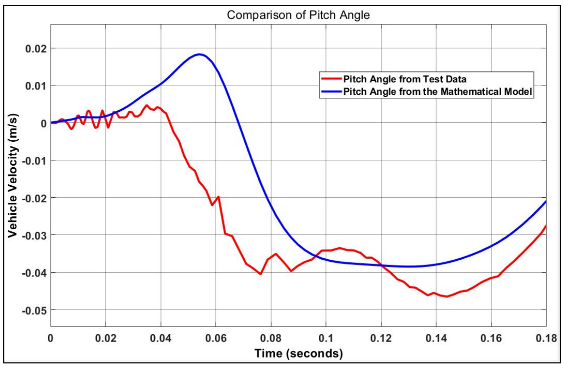

Figure 11: Forward Pitch Angle curve overlay for LPM vs FE model.

Figure 12 compares the z-velocity (vertical velocity) in the body with the curves generated from FE data. The trend in the curve is similar but the peak values are not matching in this simulation model. One of the contributing factors to this deviation is the use of standard linear spring and damper coefficient values for the model (used from literature data). The linear value for the spring and damper coefficients can lead to the difference in the values for this parameter as well. The values of l_f and l_r can also be tuned further to represent the Chevrolet Silverado (2015) model used in this study. However, the authors have intentionally avoided fine tuning these values assuming that this data may not be available to vehicle development team at the start of the design process. This makes it inevitable to use standard values for automotive parameters.

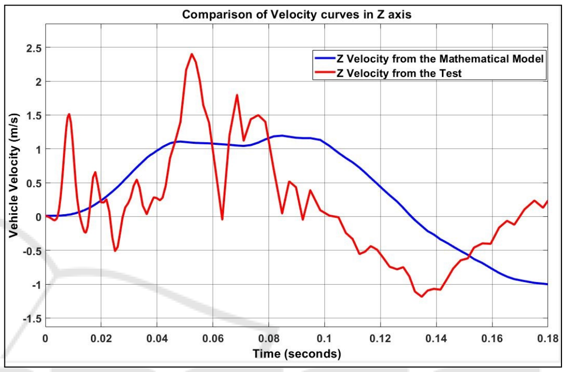

Figure 12: Z-Velocity curve comparison for LPM vs FE model.

4 CONCLUSION AND NEXT STEPS

The novel technique developed in this paper for modeling a full frontal vehicle crash event successfully predicts the event kinematics. The study demonstrates that the two phase simulation model can describe a highly complex dynamical multiple DOF system with few equations and parameters, making the process of using LPMs very simple and reliable for safety design engineers. The study also highlights that parameter identification is an important part of accident reconstruction process and its correlation has an influence on the deformation and velocity during vehicle crash. One of the major implications from the model developed in this study is the design of vehicle rails, as a contributing factor to vehicle pitching forward.

This assumption used to arrive at a simpler LPM model providing reliable results include the following:

- The spring and damper characteristics are assumed to be piecewise-linear with six breakpoints although they are non-linear in physical systems.

- The vehicle acceleration is assumed to be zero at the time pitching starts in the crash event.
- Energy losses like friction and heat losses in the vehicle during the crash event are neglected to simplify the problem.
- Only vehicle rotations about the y-axis (pitching) are considered for modeling in the full frontal impact scenario; rotations about other axes are considered negligible and not impacting the occupant injuries.

The next steps in this study include creating a mathematical expression for the force components in the mathematical model for the second phase which includes pitching of the vehicle. The mathematical expression for the force would help vehicle design teams to reduce pitching on the vehicle by changing the force components acting on the vehicle during a collision. The prediction for maximum deformation can be improved by using a non-linear force deformation curve for the spring stiffness curve with larger breakpoints along with including energy losses in the model. The model currently uses standard spring and damper coefficients for the suspension model which can be tuned to match a particular vehicle being studied; also including the mass and the distance of the CG from the vehicle suspension connections.

ACKNOWLEDGMENTS

The authors would like to thank University of Agder for the support to conduct this research.

REFERENCES

Administration, N. H. T. S. et al. (2016). Crash simulation vehicle models. *URL: https://www. nhtsa. gov/research-data/databasesand-software*.

B Munyazikwiye, B., Vysochinskiy, D., Khadyko, M., and G Robbersmyr, K. (2018). Prediction of vehicle crashworthiness parameters using piecewise lumped parameters and finite element models. *Designs*, 2(4):43.

Benson, D., Hallquist, J., Igarashi, M., Shimomaki, K., and Mizuno, M. (1986). Application of dyna3d in large scale crashworthiness calculations. Technical report, Lawrence Livermore National Lab.

Chang, J. M., Ali, M., Craig, R., Tyan, T., El-Bkaily, M., and Cheng, J. (2006). Important modeling practices in cae simulation for vehicle pitch and drop. *SAE Transactions*, pages 62–72.

Chang, J. M., Rahman, M., Ali, M., Tyan, T., El-Bkaily, M., and Cheng, J. (2005). Modeling and design for vehicle pitch and drop of body-on-frame vehicles. *SAE transactions*, pages 329–338.

Du Bois, P., Chou, C. C., Fileta, B. B., Khalil, T. B., King, A. I., Mahmood, H. F., Mertz, H. J., Wismans, J., Prasad, P., and Belwafa, J. E. (2004). Vehicle crashworthiness and occupant protection.

Elkady, M. and Elmarakbi, A. (2012). Modelling and analysis of vehicle crash system integrated with different vdcs under high speed impacts. *Open Engineering*, 2(4):585–602.

Elkady, M., Elmarakbi, A., and MacIntyre, J. (2012). Enhancement of vehicle safety and improving vehicle yaw behaviour due to offset collisions using vehicle dynamics. *International journal of vehicle safety*, 6(2):110–133.

Goldstein, H., Poole, C., and Safko, J. (2002). Classical mechanics.

Huang, M. (2002). *Vehicle crash mechanics*. CRC press.

Kamal, M. M. (1970). Analysis and simulation of vehicle to barrier impact. *SAE Transactions*, pages 1498–1503.

Klausen, A., Tørdal, S. S., Karimi, H. R., Robbersmyr, K. G., Ječmenica, M., and Melteig, O. (2014). Mathematical modeling and optimization of a vehicle crash test based on a single-mass. In *Proceeding of the 11th World Congress on Intelligent Control and Automation*, pages 3588–3593. IEEE.

Marzbanrad, J. and Pahlavani, M. (2011). A system identification algorithm for vehicle lumped parameter model in crash analysis. *International Journal of Modeling and Optimization*, 1(2):163.

Mentzer, S. G., Radwan, R. A., and Hollowell, W. T. (1992). The sisame methodology for extraction of optimal lumped parameter structural crash models. Technical report, SAE Technical Paper.

Munyazikwiye, B. B., Karimi, H. R., and Robbersmyr, K. G. (2013). Mathematical modeling of vehicle frontal crash by a double spring-mass-damper model. In *2013 XXIV International Conference on Information, Communication and Automation Technologies (ICAT)*, pages 1–6. IEEE.

Munyazikwiye, B. B., Karimi, H. R., and Robbersmyr, K. G. (2017). Application of genetic algorithm on parameter optimization of three vehicle crash scenarios. *IFAC-PapersOnLine*, 50(1):3697–3701.

Noorsumar, G., Robbersmyr, K., Rogovchenko, S., and Vysochinskiy, D. (2020). Crash response of a repaired vehicle-influence of welding uhss members. Technical report, SAE Technical Paper.

Pawlus, W., Karimi, H. R., and Robbersmyr, K. G. (2011). Development of lumped-parameter mathematical models for a vehicle localized impact. *Journal of mechanical science and technology*, 25(7):1737–1747.

Riegl, P. and Gaull, A. (2018). Modeling and validation of a complex vehicle dynamics model for real-time applications. In *Proceedings of 8th International Conference on Simulation and Modeling Methodologies, Technologies and Applications - SIMULTECH,*, pages 403–413. INSTICC, SciTePress.

Savaresi, S. M., Poussot-Vassal, C., Spelta, C., Sename, O., and Dugard, L. (2010). *Semi-active suspension control design for vehicles*. Elsevier.

Singh, H., Ganesan, V., Davies, J., Paramasuwom, M., Gradischnig, L., Wood, P., and Mogal, V. (2018). Structural countermeasure/research program mass and cost increase due to oblique offset moving deformable barrier impact test. Technical report.

Wei, Z., Karimi, H. R., and Robbersmyr, K. G. (2016). Analysis of the relationship between energy absorbing components and vehicle crash response. Technical report, SAE Technical Paper.

Woitsch, G. and Sinz, W. (2013). Influence of pitching and yawing during frontal passenger vehicle crash tests on driver occupant's kinematics and injury. *International journal of crashworthiness*, 18(4):356–370.