Estimating the Frequency of the Sinusoidal Signal using the Parameterization based on the Delay Operators

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Abstract: The article presents an algorithm for estimating the frequency of an offset sinusoidal signal. Delay operators are applied to the measured signal, and a linear regression model is constructed containing the measured signals and the constant vector depending on unknown frequency. For the vector regression model, the method cascade reduction is used. A reduction procedure is proposed that allows the original model to be reduced to a reduced one containing a smaller number of unknown parameters. Finally, using the classical gradient method was used to compare the efficiency of the proposed method.

1 INTRODUCTION

One of the main tasks in the design of automatic control systems is action alignment of parametrically indefinite disturbing influences on the control object. In the theory of linear systems, there is an internal model principle for solving such problems. It is necessary to build models of the reference and disturbing influences. In the case of harmonic disturbances, the model parameters will contain unknown frequencies. The initial conditions will be set by unknown displacement, amplitudes, and phases of the disturbing signal harmonics. In this case, it is necessary to apply adaptive internal models, which provide parametric identification possibility of the disturbing signal.

The task of estimating the parameters of sinusoidal signals is fundamental and, in addition to theoretical significance, has wide practical application (Stoica et al., 2000). Such problem can arise during the synthesis of a compensation system for a parametrically uncertain disturbance (Pyrkin et al., 2015), for example, in precision displacement systems (Aphale et al., 2008).

One of the fundamental problems of control theory is the problem of real-time frequency estimation for a signal consisting of several sinusoids. The problem is studied in many branches of science: signal processing, instrument making, adaptive control. The problem of frequency estimation is widely presented in practical applications, for example, in precision positioning systems in nanotechnology (Aphale et al., 2008), in dynamic positioning systems for vessels exposed to external disturbances such as waves, winds, and currents (Yohei Takahashi et al., 2007), in power systems for fault detection (Xia et al., 2012), (Phan et al., 2016), etc.

As a rule, identifying unknown parameters is posed from a set of measurements, estimating parameters in real-time using adaptive control, or compensating for disturbances. The problem of identifying harmonic signal constant frequency has been well studied over the last decade, and a large number of real-time algorithms have been developed. Many approaches solve these problems. The most famous is the least-squares method and its various modifications (Lijung.N, 1991). For real-time estimation, iterative forms of the least-squares method or gradient integral algorithms can be used. In (Pyrkin A.A. and S.A, 2015), an algorithm for continuous-time parametric estimation of all parameters of an indefinite disturbance with a deterministic polyharmonic structure is presented. Standard gradient estimate is used for identification. In (Vedyakova et al., 2020) algorithm for estimating an asymmetric exponentially decaying sinusoid is considered. This problem is a special case of the issue considered in this work in the case of one harmonic in the spectrum of the signal under study. The algorithm is based on the dynamic expansion of the regressor. In (Aranovskiy et al., 2016),

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algorithm of frequencies estimation of unbiased polyharmonic signal is presented. The algorithm is based on the use of dynamic expansion of the regressor and standard gradient estimation and provides exponential convergence to zero of the estimation error. In (Aranovskii, 2008), algorithm of unknown frequency estimation in continuous time of a displaced sinusoidal signal is considered. The algorithm has noise immunity to amplitude-limited measurement noises and provides asymptotic convergence to zero of the estimation error.

In this article the synthesis of devices for signal frequency estimation is considered. Parametrization is proposed to obtain a linear regression model, the vector of unknown parameters associated with the original signal parameters. The cascade reduction method is used to estimate the parameters (Bobtsov et al., 2010), (Iureva et al., 2020). Conditions are formulated under which the exponential convergence to zero of the estimation errors is ensured.

This paper is organized as follows: problem statement is described in Section 2; linear regression model is constructed in Section 3; in Section 4 the estimation algorithm is proposed, and exponential convergence of the estimation error to zero is proved; in Section 5 proposed algorithm computer simulation results are included confirming the efficiency of the approach and finally the conclusion.

2 PROBLEM FORMULATION

Consider the measured offset sinusoidal signal:

$$y(t) = \sigma + \nu sin(\omega t + \varphi), \tag{1}$$

where $\omega \in R_+$ is signal frequency, $\nu \in R_+$ – stationary amplitude, $\sigma \in R$ – is the bias, $\varphi \in$ – is rare phase and the number of signal harmonics $y(t)$. Parameters: $\omega, \sigma, \nu, \varphi$ are considered unknown.

It is required to form estimations $\hat{\omega}(t)$ of the frequencies that ensure the convergence of the estimation error of $\tilde{\omega}(t) = \omega - \hat{\omega}(t)$ to zero under the following assumptions:
Assumption 1: Signal consists of one harmonic offset.
Assumption 2: Minimum frequency $\underline{\omega}$ and maximum frequency $\bar{\omega}$ are known.

3 PARAMETRIZATION

Consider the measurable harmonic signal (1) with exponentially damped amplitude and bias. On the first step the goal is to find linear regression model with measurable variables and constant parameter associated with an unknown frequency ω.

Consider two signals:

$$y_1(t) = y(t-\lambda), t \geq \lambda, \tag{2}$$
$$y_2(t) = y(t-2\lambda), t \geq 2\lambda. \tag{3}$$

where $\lambda \in R_+$ is chosen delay value.
Remark 1: The delay value λ from (2) and (3) should be chosen such that $\lambda < \frac{\pi}{\bar{\omega}}$.

The output signals (2) and (3) can be rewritten explicitly:

$$y_1(t) = \sigma + a_1 \nu sin(\omega t + \varphi) - b_1 \nu cos(\omega t + \varphi), \tag{4}$$
$$y_2(t) = \sigma + a_2 \nu sin(\omega t + \varphi) - b_2 \nu cos(\omega t + \varphi), \tag{5}$$

where $a_1 = cos\omega\lambda, b_1 = sin\omega\lambda, a_2 = cos2\omega\lambda, b_2 = sin2\omega\lambda$ and remark that $a_2 = 2a^2{}_1 - 1, b_2 = 2a_1 b_1$.

Subtract from (1) multiplied by a_1 equation (4) and subtract from (1) multiplied by $2a^2{}_1 - 1$ equation (5). Then is obtain:

$$a_1 y(t) - y_1(t) = (a_1 - 1)\sigma + b_1 \nu cos(\omega t + \varphi) \tag{6}$$
$$y(t)(2a_1{}^2 - 1) - y_2(t) = = (2a_1{}^2 - 2)\sigma + 2a_1 b_1 \nu cos(\omega t + \varphi), \tag{7}$$

Then subtract from (6) multiplied by $2a_1$ (7) and obtain:

$$2a_1 (a_1 y(t) - y_1(t)) - (2a_1{}^2 - 1)y(t) + y_2(t) = = \sigma(-2a_1 + 2), \tag{8}$$
$$y_2(t) + y(t) = -2(a_1 - 1)\sigma + 2a_1 y_1(t). \tag{9}$$

Equation (9) can be written in the form of a linear regressor with respect to two parameters a_1, σ:

$$\psi(t) = \xi^T(t)\Theta, \tag{10}$$

where

$$\psi(t) = y(t) + y_2(t), \tag{11}$$
$$\xi^T = \begin{bmatrix} y_1(t) & 1 \end{bmatrix}, \tag{12}$$
$$\Theta = \begin{bmatrix} 2a_1 \\ -2(a_1 - 1)\sigma \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}. \tag{13}$$

4 PARAMETER ESTIMATION

In the previous section linear regression model (10) was gained. In this section the method for parameter estimation is proposed. Estimation algorithm is presented based on the cascade reduction method (Bobtsov et al., 2010).

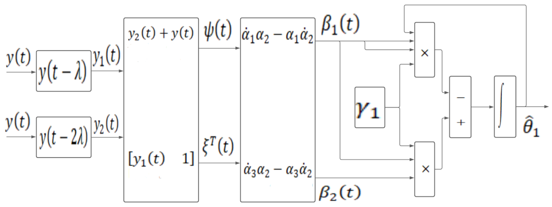

Figure 1: Block diagram of the algorithm (18).

In this case we sequentially integrate equation (9), i.e.:

$$\int_0^t (y_2(\tau)+y(\tau))\,d\tau = \\ = \theta_1 \int_0^t y_1(\tau)d\tau + \theta_2 \int_0^t d\tau, \qquad (14)$$

Introduce the notation:

$$\alpha_1(t) = \int_0^t (y_2(\tau)+y(\tau))d\tau,$$
$$\alpha_2(t) = \int_0^t d\tau,$$
$$\alpha_3(t) = \int_0^t y_1(\tau)d\tau.$$

and sequentially first divide by $\alpha_2(t)$, and then differentiate the last relation. Then get:

$$\dot{\alpha}_1\alpha^{-1}{}_2 - \alpha_1\dot{\alpha}_2\alpha^{-2}{}_2 = \\ = \theta_1(\dot{\alpha}_3\alpha^{-1}{}_2 - \alpha_3\dot{\alpha}_2\alpha^{-2}{}_2). \qquad (15)$$

Divide (15) into two parts by α_2^{-2}, and obtain:

$$\dot{\alpha}_1\alpha_2 - \alpha_1\dot{\alpha}_2 = \theta_1(\dot{\alpha}_3\alpha_2 - \alpha_3\dot{\alpha}_2). \qquad (16)$$

Introduce the following notation: $\beta_1 = \dot{\alpha}_1\alpha_2 - \alpha_1\dot{\alpha}_2, \beta_2 = \dot{\alpha}_3\alpha_2 - \alpha_3\dot{\alpha}_2$.

Then equation (16) takes the next form:

$$\beta_1(t) = \theta_1\beta_2(t). \qquad (17)$$

whence follows an identification algorithm in the form:

$$\dot{\hat{\theta}}_1(t) = -\gamma_1\theta_1(t)\beta^2{}_2(t) + \gamma_1\beta_2(t)\beta_1(t). \qquad (18)$$

where $\gamma_1 \in R_+$ is the chosen constant that provides exponential convergence of the estimation error to zero.

From (17) and (18) can be obtained the differential equations for errors checking: $\tilde{\theta}_1(t) = \theta_1 - \hat{\theta}_1(t)$.

Frequency Estimation

It follows from (18) that:

$$\hat{\omega}(t) = \frac{1}{\lambda} arccos\left(\frac{\hat{\theta}_1}{2}\right). \qquad (19)$$

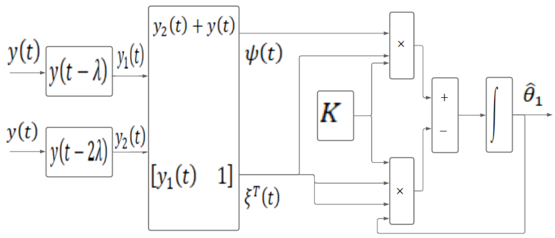

Figure 2: Block diagram of the algorithm (22).

Since the function domain (19) is the subset of R, it is necessary to put some restrictions on it $\hat{\theta}_1(t)$. Under Assumption possible values of θ_1 satisfying the inequality:

$$2\cos\bar{\omega}\lambda \le \theta_1 \le 2\cos\underline{\omega}\lambda. \qquad (20)$$

Rewrite estimation for $\hat{\theta}_1$, which would satisfy the next equation:

$$\dot{\hat{\theta}}_1(t) = Pr(-\gamma_1\theta_1(t)\beta^2{}_2(t) + \gamma_1\beta_2(t)\beta_1(t)). \qquad (21)$$

The projection $Pr(*)$ allows condition (20) to be satisfied so that the estimation remains qualitatively the same (P.A.Loannou, 2012).

5 NUMERICAL EXAMPLES

In this section the simulation results are presented. These results illustrate the efficiency of proposed estimation algorithm. All simulations have been performed in MATLAB Simulink.

Let us compare the proposed algorithm with other identification. The gradient descent method was taken as an example.

The device for estimating parameters based on gradient descent has the form:

$$\dot{\hat{\Theta}} = K\xi\left(\psi - \xi^T\hat{\Theta}\right), \qquad (22)$$

where $K \in R_+$ is the chosen constant that provides exponential convergence of the estimation error to zero. Different signal was taken to check the algorithm operation. This signal belongs to earlier considered algorithm for two different harmonics: $y(t) = 4 + 2sin(2t+2)$ and $y(t) = 5 + 2sin(4t+1)$. Delay statements are used with the following delay values:

Method Cascade Reduction: $\gamma_1 = 1, \gamma_1 = 20$ and $\lambda = 0.1, 0.3, 0.5$.

The simulation results are shown in figures 3, 4, 5, 6.

Method Gradient Descent: $K = 0.1, K = 0.5$ and $\lambda = 0.1, 0.3, 0.5$.

The simulation results are shown in figures 7, 8, 9, 10.

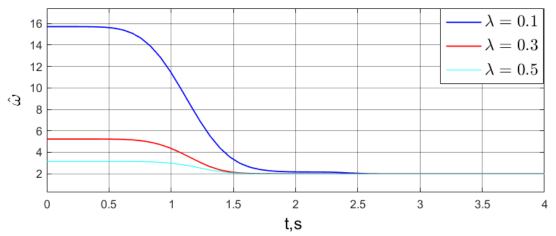

Figure 3: Parameter estimation transients for signal $y(t) = 4 + 2sin(2t + 2)$ at $\gamma_1 = 1$ (Method Cascade Reduction).

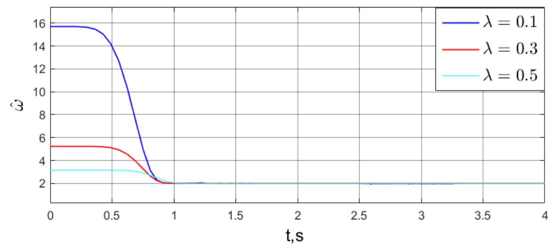

Figure 4: Parameter estimation transients for signal $y(t) = 4 + 2sin(2t + 2)$ at $\gamma_1 = 20$ (Method Cascade Reduction).

Remark 2: There is an optimal value at which the speed is maximum in the gradient method, and for the cascade reduction method, show that with an increase in the value, the convergence time is much faster, the speed can be increased infinitely. At the same time, looking at the diagrams, we can see that when changing the delay operator, the method of reduction is almost unchanged, but for the gradient method, the convergence time increases quite a lot and the overshoot increases.

For the case of unknown parameters, numerical modeling was carried out, which illustrated that when using the cascade reduction method, the oscillations in the parameter estimates were significantly lower, and the response time was much faster than when using the gradient descent method. For the slope reduction method in both cases, the temporary time to estimate the signal parameters is 450 seconds, compared with 2 second for the cascading method.

The simulation results show that when using the cascade reduction algorithm, the parameter estimates are significantly lower and the response time is much faster than when using the gradient method, and there is almost no overshoot when using the cascade reduction method. Thus, the cascade reduction method may be preferable for use in practical problems.

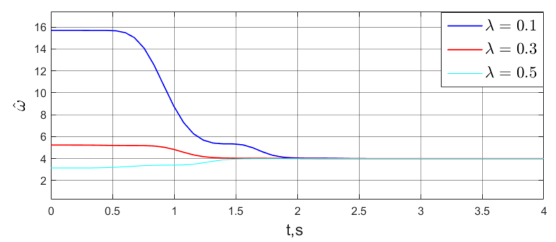

Figure 5: Parameter estimation transients for signal $y(t) = 5 + 2sin(4t + 1)$ at $\gamma_1 = 1$ (Method Cascade Reduction).

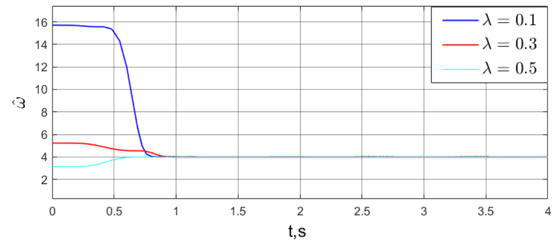

Figure 6: Parameter estimation transients for signal $y(t) = 5 + 2sin(4t + 1)$ at $\gamma_1 = 20$ (Method Cascade Reduction).

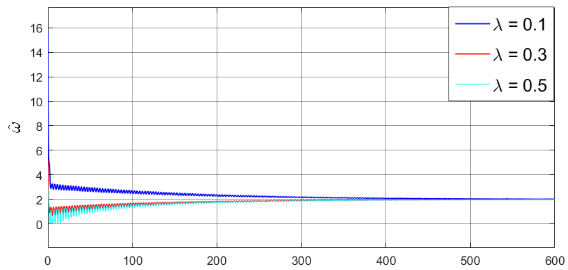

Figure 7: Parameter estimation transients for signal $y(t) = 4 + 2sin(2t + 2)$ at $K = 0.1$ (Method Gradient Descent).

6 CONCLUSIONS

In the article the problem harmonic signal parameters definition is considered. New parameterization method based on operator delay application to measurable signal is applied to construct linear regression model. Methods for producing estimates of the frequency of a harmonic signal are presented, making it possible to obtain estimates of the parameters at a predetermined time. Computer simulation has been carried out to illustrate the performance, demonstrating the parametric convergence of the algorithm variable (19), (22) to the correct value. Obtained algorithms

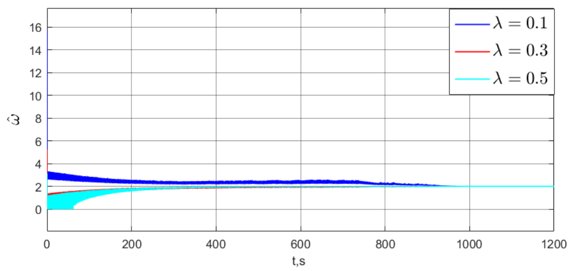

Figure 8: Parameter estimation transients for signal $y(t) = 4 + 2sin(2t + 2)$ at $K = 0.5$ (Method Gradient Descent).

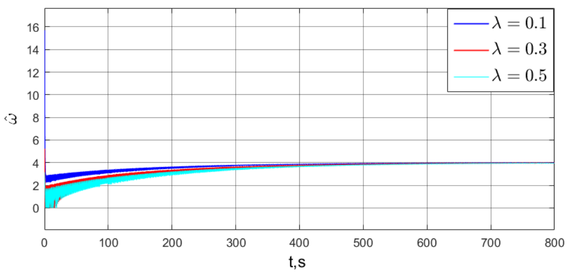

Figure 9: Parameter estimation transients for signal $y(t) = 5 + 2sin(4t + 1)$ at $K = 0.1$ (Method Gradient Descent).

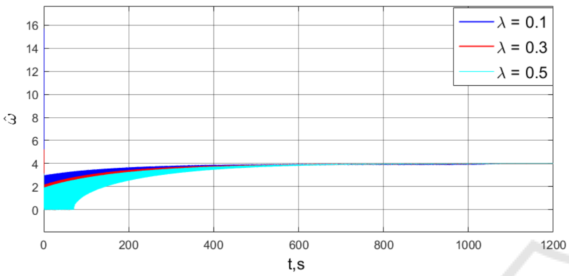

Figure 10: Parameter estimation transients for signal $y(t) = 5 + 2sin(4t + 1)$ at $K = 0.5$ (Method Gradient Descent).

are supposed to compensate vertical inertial accelerations in estimating gravity anomalies on moving object. Future investigations will be devoted to extending the methodology to the case of multisinusoidal signal estimation.

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