Real-time Robust Trajectory Control for Vehicle Platoons: A Linear Matrix Inequality-based Approach

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Keywords: Robust Control, Trajectory Planning, Linear Matrix Inequality, Vehicle Platooning.

Abstract: This paper proposes a solution to dynamically adjust vehicle platoon trajectories. The goal of the control algorithm is to keep the optimal interdistance between adjacent vehicles proceeding at cruising speed on a straight road. After a proposal of the interdistance required between neighboring vehicles, a robust decentralized controller based on a linear control law provides the speed profile for each component of the platoon. Its objective is to minimize the divergence in space in respect to the planned trajectories while assuring a safe span between adjacent members of the platoon. The results on a limited instance demonstrate the effectiveness of the proposed approach.

1 INTRODUCTION

Autonomous vehicles represent a topic of growing interest in systems engineering. The goal to design unmanned cars and to introduce them in the common traffic is one of the most ambitious projects in the current century, studied all over the world. Since the road is a complex environment, many parameters have to be taken into account for the car's autonomous motion. Firstly, smart sensors, located on the vehicles or on the infrastructures, have to acquire specific data such as videos and images from the surroundings in order to identify the elements useful to manage the vehicle tracking. Other devices have to be installed to transmit the information, in real time, to the control center which analyses and evaluates the data and sends the correct control commands to the vehicles. Some vehicles can send or receive automatically the information about their states from or toward a centralized decision maker in order to manage their own trajectory according to the reconstructed environment. When the surrounding is virtually rebuilt, the automated system can provide its control law by modifying the acceleration, the speed and/or the steering of the vehicle replacing the human intervention. Control techniques can also be applied to vehicle platooning, which refers to a set of cooperative automated vehicles which travel on the same trajectory in a string formation.

There are already several findings that show how autonomous driving could improve the throughput of the road, while reducing the fuel emissions, especially for platooning (Alam et al., 2014). While dealing with platoon, many factors have to be considered. In this case, in fact, each vehicle does not only follow its path, but also it interacts with the other members of the platoon. The involved vehicles retrieve information on the surroundings and have to proceed harmoniously with the rest of the system. The interdistance between adjacent vehicles has to be constantly monitored by means of safety zones, designed accordingly to road traffic rules and following principles introduced in (Chen and Wang, 2007) and (Bersani et al., 2015). The classification of safer or less safe areas leads the choice of the trajectory generator algorithm to be used.

In literature, many works tackle the trajectory planning problem for one vehicle. In (Alia et al., 2015), the authors used clothoid tentacles and occupancy grid to generate a feasible path for a vehicle and even allowing it to avoid obstacles. To achieve the same objective, in (Choi et al., 2008), the Bezier's curve method is adopted. However, the definition of the trajectory for a vehicle included in a platoon appears more complex.

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Bozzi, A., Zero, E., Sacile, R. and Bersani, C.
Real-time Robust Trajectory Control for Vehicle Platoons: A Linear Matrix Inequality-based Approach.
DOI: 10.5220/0010550704100415
In *Proceedings of the 18th International Conference on Informatics in Control, Automation and Robotics (ICINCO 2021)*, pages 410-415
ISBN: 978-989-758-522-7

In particular, unpredictable events during the transit due to external factors may cause vehicle approaching or detachment from the other members of the platoon. In this case, a control system which generates countermeasures, for each element of the system, to restore the correct position, speed and interdistance has to be introduced in order to guarantee passengers' safety. The approaches may be centralized or decentralized. In the first case, usually the leader regulates the vehicles' position by providing the optimal data for speed and acceleration to the followers (Graffione et al., 2020b), while in the second case, each member of the platoon uses as reference the distance and the speed of the preceding vehicle (Ghasemi et al., 2013).

In (Liu et al., 2019), the authors proposed an optimal platoon trajectory control method by optimizing accelerations of the vehicles in order to satisfy constraints related to travel delay and fuel consumption. In (Huang et al., 2019), a distributed control model is presented where each vehicle is independently controlled by itself and it shares only information about its motion state in order to perform vehicle merging or splitting. The same problem has also been tackled in (Graffione et al., 2020a) solved by a Model Predicted Control model (MPC). In this latter case, the leader coordinates the exchanged data with the vehicles in order to perform the planned manoeuvres.

This paper presents a new algorithm to ensure the correct maintenance of the distance between neighboring vehicles, by means of a robust controller that provide a linear control law. The offline procedure should be activated when vehicles seem to have trouble in keeping an optimal interdistance each other due to external factors.

This work is organized as follows. Section 2 provides a deeper description of the safety zones classification and presents the proposed robust control model. Section 3 shows a case study with simulation and results, and finally Section 4 includes conclusions and future developments.

2 MODEL AND METHOD

2.1 System Definition

The control of the platoon trajectory and the related behaviour of the leader and the followers requires an accurate model of the vehicles motion in order to guarantee robustness and string stability.

To generate dynamically the set of trajectories, a kinematic model of the vehicle has been used:

$$x_i(t+1) = x_i(t) + v_i(t)\Delta t + w_i(t) \qquad t = 0,...,T-1 \tag{1}$$

where x_i is the position of the i^{th} vehicle, v_i its velocity, w_i the possible disturbance and Δt the sampling time. This simple approach ensures a fast modification of the conditions and paired with limiters it can provide the bounding of the input respecting physical constraints.

In matrix form, the aforementioned system of M vehicles may be formalized as follows:

$$\underline{x}(t+1) = A\underline{x}(t) + \Delta t B\underline{v}(t) + \underline{w}(t) \quad t = 0,...,T-1 \tag{2}$$

with A and B identical matrices $\in \mathbb{R}^{M\times M}$.

In addition, it is assumed that each vehicle can access to the information about its own position and on both the preceding and the following vehicle positions. So:

$$y_i = C_i x_i \tag{3}$$

and for the generic i^{th} vehicle:

$$C_i = \begin{bmatrix} 0 & ... & 0 & 1 & 0 & 0 & 0 & ... & 0 \\ 0 & ... & 0 & 0 & 1 & 0 & 0 & ... & 0 \\ 0 & ... & 0 & 0 & 0 & 1 & 0 & ... & 0 \end{bmatrix} \tag{4}$$

where $C_i \in \mathbb{R}^{3\times M}$.

The first and last vehicles of the platoon have slightly different $C_i \in \mathbb{R}^{2\times M}$ matrix:

$$C_1 = \begin{bmatrix} 1 & 0 & ... & 0 \\ 0 & 1 & ... & 0 \end{bmatrix} \quad C_M = \begin{bmatrix} 0 & ... & 1 & 0 \\ 0 & ... & 0 & 1 \end{bmatrix} \tag{5}$$

The specific structure of those matrices assures that each vehicle monitors its own position and the position of the vehicle which precedes and follows in the string formation, as an example, by sensors such as lidar and radar.

However, (Pates et al., 2017) and (Gao et al., 2017) demonstrate that, to obtain better performances of the control models regarding the scalability of the system or in case of noisy measurements, it is recommended to provide each vehicle also with information coming from the leader's position.

2.2 Safety Zones

In the management of the platoon trajectory, it is significant to monitor the interdistance between each couple of consecutive vehicles, in order to apply a proper controller able to handle specific goals according to the different situations during the driving sessions. When two vehicles get closer, a safety aspect must be taken into account. In the proposed approach, to classify critical situations, the interdistance between two adjacent vehicles may be represented as three colored states, green (optimal), yellow

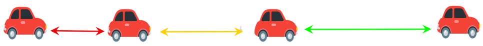

Figure 1: Safety zones division.

(sub-optimal) and red (critical). When a vehicle is too close to the adjacent one, its position may be represented by a red zone. The state of the vehicle turns in yellow, whenever a safer-distance is re-established according to a predefined threshold in space. The green state represents a safe condition in respect to correct interdistance among vehicles.

A schematic representation of safety zones is shown in Fig.1 According to the state color, the objective of the control changes. The green zone is a collision-free zone, in this case, the vehicle has to keep a given speed, while maintaining the interdistance. On the other hand, the red zone represents collision-risk, as the distance with the preceding car is not wide enough to guarantee safety. In this occurrence, the vehicle should perform a braking maneuver and even stops itself in order to avoid a possible collision, if for some external reason, the preceding vehicle starts braking. The yellow intermediate zone is by far the most interesting, since a vehicle must pay attention to the preceding car because the interdistance is not small enough to induce an accident, but not wide enough to allow a careless drive.

Hereinafter, the bounds of each zone are defined as follows (distance in m, velocity in km/h):

$$d_{red} \leq \frac{3 \times V}{10} \quad d_{green} \geq \left(\frac{V}{10}\right)^2 \quad d_{red} < d_{yellow} < d_{green} \tag{6}$$

given accordingly to the reaction and the braking time of a human driver.

More in details, these bounds have been assigned accordingly to the rules of the Italian road traffic, which asserts as a recommended distance the sum of the space traveled during the human reaction and the braking time, while the minimum threshold is the reaction space only. Nevertheless, these distances might even be reduced, considering that autonomous vehicles perform faster computation and input prompt with respect to human being. In other words, the unmanned vehicles' reaction time is considerably lower. Coherently with this reasoning, (Alam et al., 2014) has shown that especially for heavy duty vehicles (HDV) platooning the minimum distance to avoid collision can be reduced.

However, in this paper it has been preferred to maintain the canonical range. The proposed control model will be applied considering the yellow intermediate state of criticality.

2.3 Robust Control for Yellow Zone

The proposed robust control model deals with a situation in which vehicles are not optimally spaced (i.e. they are in yellow zone) and while they are proceeding with a given velocity. The behaviour in green and red zone is assumed to be correctly performed by assigned controllers thus they are not considered in this paper.

The robust controller's purpose in the yellow zone is to restore the desired interdistance minimizing the speed variation, while keeping the planned trajectory, despite disturbances that affect the system.

Thus, the trajectory of the *i-th* vehicle has to be modified with respect to the desired reference path a priori defined $x_i^d(t)$, leading to apply the following change of variable:

$$\begin{cases} \tilde{x}_i(t) = x_i(t) - x_i^d(t) \\ \tilde{v}_i(t) = v_i(t) - v_i^d(t) \end{cases} \tag{7}$$

The problem can be defined as minimax problem:

$$\inf_{v} \sup_{w \neq 0} \frac{J(v,w)}{||w||^2} \tag{8}$$

subject to (2) where the cost function J is designed as:

$$J(v,w) = \sum_{i=1}^{M} \sum_{t=0}^{T-1} \alpha_i \tilde{x}_i^2(t) + \gamma_i \tilde{v}_i^2(t) + \sum_{i=2}^{M} \sum_{t=0}^{T-1} \beta_i (\tilde{x}_i(t) - \tilde{x}_{i-1}(t))^2 \tag{9}$$

In (9) α_i,β_i and γ_i represent gains which give primary importance respectively to tracking the trajectory, restoring the optimal interdistance and minimizing the difference from the desired velocity. The leader vehicle is considered as an element of the platoon (i.e. it is unmanned as the others) with the specific feature that it does not have to care about keeping some distance to a preceding vehicle.

Theorem 1. *Let's consider a time horizon of two intervals, i.e.* $t = 0, \dots T$ *and* $T = 1$*. The trajectory of the platoon described by* $\underline{x}(t)$ *can be modified in real-time, according to the problem defined by (8) and (2) by an optimal control law* $v(t) = Kx(t)$*, which is linear, where* K *is the solution of the following linear matrix inequality (LMI):*

$$\min_{\theta, K} \theta \tag{10}$$

s.t.

$$K = \begin{bmatrix} k_{1,1} & k_{1,2} & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & k_{2,1} & k_{2,2} & k_{2,3} & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots & k_{M,M-1} & k_{M,M} \end{bmatrix} \tag{11}$$

$$\begin{pmatrix} \hat{\mathbf{Q}}_{xx} - \theta\mathbf{I} + \hat{\mathbf{Q}}_{xv}\mathbf{KC} + \mathbf{C}^T\mathbf{K}^T\hat{\mathbf{Q}}_{vx} & \mathbf{C}^T\mathbf{K}^T \\ \mathbf{KC} & -\hat{\mathbf{Q}}_{vv}^{-1} \end{pmatrix} \leqslant 0 \tag{12}$$

and $\theta > \theta^{\star}$*:*

$$\inf_{\mu\in S} \sup_{0\neq x\in\mathbb{R}^n} \left(\begin{pmatrix} x \\ \mu(Cx) \end{pmatrix}^T Q \begin{pmatrix} x \\ \mu(Cx) \end{pmatrix} \right) / \left(\|x\|^2\right) = \theta^{\star} \tag{13}$$

Proof. From the cost function of (9) we can deduce the matrix Q_J for the minimax problem, so partitioned:

$$Q_J = \begin{bmatrix} Q_{xx} & Q_{xv} \\ Q_{vx} & Q_{vv} \end{bmatrix} \tag{14}$$

$$\begin{cases} Q_{xx} = \begin{bmatrix} \alpha_1+\beta_2 & -\beta_2 & ... & 0 & 0 \\ -\beta_2 & \alpha_2+\beta_2+\beta_3 & ... & 0 & 0 \\ ... & ... & ... & ... & ... \\ 0 & 0 & ... & \alpha_{M-1}+\beta_{M-1}+\beta_M & -\beta_{M-1} \\ 0 & 0 & ... & -\beta_{M-1} & \alpha_M+\beta_M \end{bmatrix} \\ Q_{vv} = diag(\gamma_1, \gamma_2, ...\gamma_M) \\ Q_{xv} = Q_{vx} = 0 \end{cases} \tag{15}$$

Matrices thus obtained refer to a dynamic system. In order to perform an offline computation of the trajectory we need to transform the system from dynamic to static. According to the available data on the state and on the disturbance at initial time (i.e. $\underline{x}(0)$ and $\underline{w}(0)$, hereinafter respectively x_0 and w_0 for sake of notation) the following problem has to be solved:

$$\inf_{v_0} \sup_{x_0,w_0} \left(\frac{x_0^T Q_{xx} x_0 + v_0^T Q_{vv} v_0 + x^T(1) Q_{xx} x(1)}{||x_0, w_0||^2} \right) \tag{16}$$

By writing $x(1)$ as a function of $x(0)$ from (2) and developing the computation, (16) may be rewritten as:

$$\inf_{v_0} \sup_{x_0,w_0} \left(\frac{x_0^T(Q_{xx}+A^TQ_{xx}A)x_0 + v_0^T(Q_{vv}+B^TQ_{vv}B) + x_0^TA^TQ_{xx}Bv_0 + v_0^TB^TQ_{xx}Ax_0}{||x(0), w(0)||^2} \right) \tag{17}$$

The numerator in (17) can be represented in the matrix form as:

$$\begin{bmatrix} x_0 \\ u_0 \end{bmatrix}^T \hat{Q} \begin{bmatrix} x_0 \\ u_0 \end{bmatrix} \tag{18}$$

More in detail, the matrix $\hat{Q}$ is composed as follows:

$$\hat{Q} = \begin{bmatrix} \hat{Q}_{xx} & \hat{Q}_{xv} \\ \hat{Q}_{vx} & \hat{Q}_{vv} \end{bmatrix} = \begin{bmatrix} Q_{xx}+A^TQ_{xx}A & A^TQ_{xx}B \\ B^TQ_{xx}A & R+B^TQ_{vv}B \end{bmatrix} \tag{19}$$

and it holds all the data needed to solve the LMI problem of Theorem 1.

Furthermore, Theorem 1 of (Gattami et al., 2012) demonstrates that there exist linear decision $\mu_i(C_ix) = K_iC_ix$, for $i = 1,...,N$ where the finite value θ^* of the game represented by equation (13) is achieved. □

Corollary 1.1. *The control can be applied on wider time horizon, i.e.* $T > 1$*, applying results in (Gattami and Bernhardsson, 2007), Section VII.*

Table 1: Initial speeds.

Vehicle	speed [m/s]
1	18.79
2	17.55
3	17.67
4	15.59
5	14.66

3 CASE STUDY

The case study refers to a five-vehicle platoon (M=5) which is moving along a rectilinear path. A decreasing in the speed profile of the vehicles which the platoon consists of is considered.

The main goal of the control law is indeed to restore the correct distance, which is assumed to be at least 29 meters according to (6) and provided that platoon is willing to proceed at the cruising speed of 15m/s, thus slightly reducing the velocity of the first vehicles. In other words, the robust control aims at spacing the vehicles with the correct interdistance with smaller as possible variation of their speed.

Initial speeds used for our simulation are listed in Table 1. This happens because they are not optimally spaced as between two neighboring vehicles intervenes a distance of 20 meters (caused by external factors prior to the start of the simulation), and thus platoon is trying to actuate some techniques to increase the span.

In this scenario the noise is assumed to be additive white Gaussian, with zero mean and known covariance.

Gains are maintained unitary for demonstration purposes, but in a real implementation they need to be tuned empirically according to their individual importance. In particular, it is reasonable to prioritize the restoring of the optimal interdistance when vehicles seem to face difficulties in exiting the yellow zone due to external factors. This will cause them to significantly reduce their speed to obtain better stability while resuming initial platoon shape.

Fig.2 shows the deviation of the vehicle position with respect to the planned one. While in the very first instants, a huge difference appears between desired and actual trajectories, at the end of the simulation which lasts 50 seconds, the convergence is achieved. The results demonstrate the effectiveness of the robust controller which restores the initial and optimal state for the vehicles in the string formation.

Fig.3 displays the interdistance between vehicles during the simulation. Also in this case, the predefined values (29 meters) are recovered carrying out the main objective of the control law.

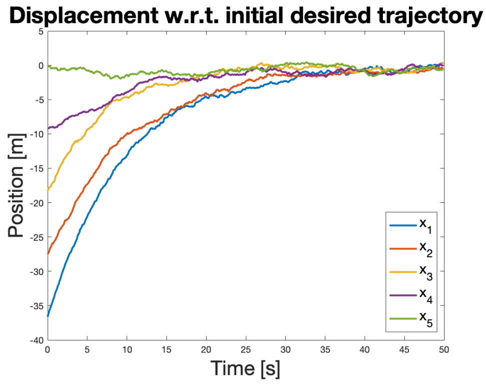

Figure 2: Divergence in vehicles' position with respect to the planned one.

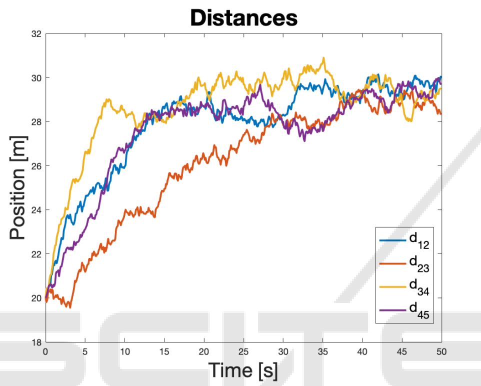

Figure 3: Distance between neighboring vehicles.

Finally, Fig.4 displays the velocity trends, which are coherent with the expectations: first vehicles need to slow down to ensure a correct repositioning of the other members of the platoon reaching out the stability after about 50 seconds. After that, they can proceed at cruising speed and, in cooperative way, increase both the speed and interdistance with other control techniques.

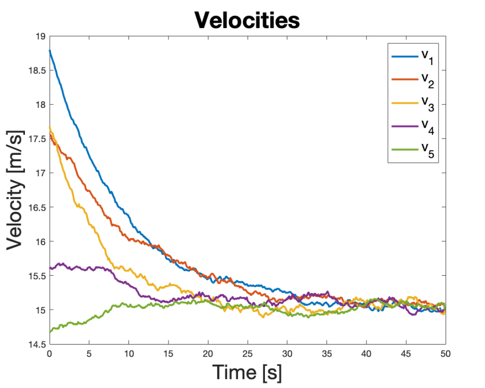

Figure 4: Trend of the velocities.

Table 2: Vehicles' state at the end of the simulation.

Vehicle	Velocity [m/s]	Distance from preceding car [m]
1	14.92	/
2	15.00	29.78
3	15.08	29.41
4	15.01	28.98
5	14.99	29.02

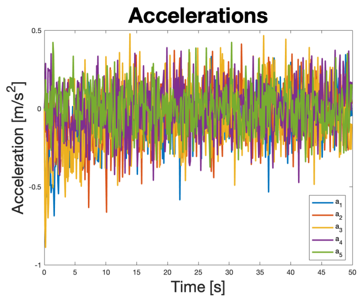

Figure 5: Vehicles' accelerations.

Table 2 sums up vehicles' velocity and distance with the preceding element of the platoon, validating our control algorithm.

4 CONCLUSION

In this paper, a robust control approach has been used to restore the optimal interdistance between neighboring vehicles in a platoon trajectory planning.

The interdistance that can occur between two adjacent vehicles has been classified in three different levels according to growing safe conditions. The proposed control model is dedicated to manage the vehicle's position and speed in the sub-optimal yellow condition which is very sensitive in the design of vehicle's trajectory. Here, in fact, vehicles may assume many different behavior, from the most aggressive to the most conservative one.

The proposed control low restores the optimal interdistance between each vehicle of the platoon by reducing the variation of the speed profile. In other words, the objective is to settle and maintain the platoon cruising speed with vehicles optimally spaced.

Results show (see Fig.5) that the acceleration required to perform the maneuver is feasible (i.e. it oscillates between values in -1 and 1), allowing us to have some confidence when testing the trajectory on a more realistic model.

In addition, the resistance to disturbances and the linearity of the control law allow this algorithm to be used when fast modification of the planned trajectory is required due to unforeseen approaching among vehicles. For future improvements we aim to test out the control algorithm on a large set of unmanned car or on curvilinear trajectories. This involves the analysis of the curve parameters and the computation of a control law both for the acceleration and the yaw angle separately.

Moreover, we are interested in verifying if the periodic computation of gains could improve the overall performance of the platoon. In our simulation, in fact, gains are estimated before the activation of the robust controller and maintained until the end of the experiment. By periodically checking the gain optimality, it may be possible to track better the trajectories profile during the evolution of the system.

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