Numerical Investigation of the Lateral Dynamic Behaviour of the *Anaconda*

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Abstract: This paper deals with the study of a particular single track vehicle, named *Anaconda*. Numerical simulations are performed to assess the vehicle's linear dynamic behavior. Indeed, multibody models of each component of the *Anaconda* and the one of the entire vehicle are developed and linearized around stationary states. The out-of-plane linearized sub-models are then used to have more insight in the lateral behaviour of the *Anaconda* and the influence of one of its component, the pedal module, on this behaviour is outlined. These tasks are carried out within the EasyDyn framework, an open source multibody library. Informative observations on the simulation results help to find out some features of the *Anaconda* concerning its linear dynamic behaviour; and some comment are made on the possibility of controlling its unstable eigenmodes.

1 INTRODUCTION

Anaconda is an in-line polycycle with reference to single track vehicles, like bicycle and motorcycle. It is composed of a head module which is a classical bicycle followed by some pedal modules as shown in Fig. 1.

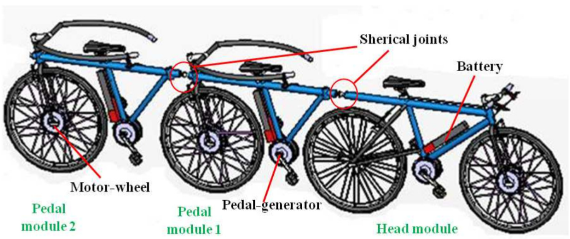

Figure 1: *Anaconda* with two pedal modules.

This vehicle can transport several people one on each module. Modules are connected each other by spherical joints, and each pedal module is equipped with a rear steered wheel so as its rider can contribute in the vehicle balance and help in the following the prescribed path; while the rider on the head module decide which path to follow.

In this conceptual model, electric generators provide energy when riders pedal. This energy is managed by a central unit in order to redistribute it in a proper manner to motor-wheels and store the exceeded energy in batteries (Verlinden and Kabeya, 2012; Kabeya and Verlinden, 2010).

Anaconda as bicycles are human-powered vehicles and nowadays the latter are used as healthy and less pollutant transportation means. Riding a bicycle can be learned intuitively and, when mastered, this activity becomes a second nature.

However, the dynamic behaviour behind the bicycle riding is more complex. The modularity of the *Anaconda* makes its dynamic behaviour much more complex than that of the bicycle and its investigation is a challenging task.

The main issues in studying single track vehicle are their stability characteristics and their dynamic behaviour; and literature contains papers outlining their dynamic studies (Sharp, 1985; Sharp, 1971; Limebeer and Sharp 1971; Cossalter, 2006, Meijaard et al., 2007).

This paper is concerned with the stability of an *Anaconda* composed of one pedal module. Thanks to the well established stability analysis of single track vehicle, the aim of this study is to describe the lateral stability of the *Anaconda* and to figure out the influence of the pedal module. Numerical simulations are carried out on multibody models of the vehicles; based on the EasyDyn framework. Models concerned in this study are those of the head module alone, the

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pedal module alone and an *Anaconda* with one pedal module. Simulation results allow us to get more insight in the lateral stability features of this vehicle and to figure out the pedal module influences.

2 EasyDyn FRAMEWORK AND VEHICLE'S MODELS

2.1 EasyDyn Framework

The studied mechanical models are developed according to a multibody approach influenced by EasyDyn (Verlinden et al., 2005; Verlinden et al., 2013).

EasyDyn is C++, open source and flexible, multibody library from the Department of the Theoretically Mechanics, Dynamics and Vibrations of the Faculty of Engineering of the University of Mons in Belgium.

EasyDyn uses minimal coordinates to describe the kinematics of rigid bodies connected by joints, thanks

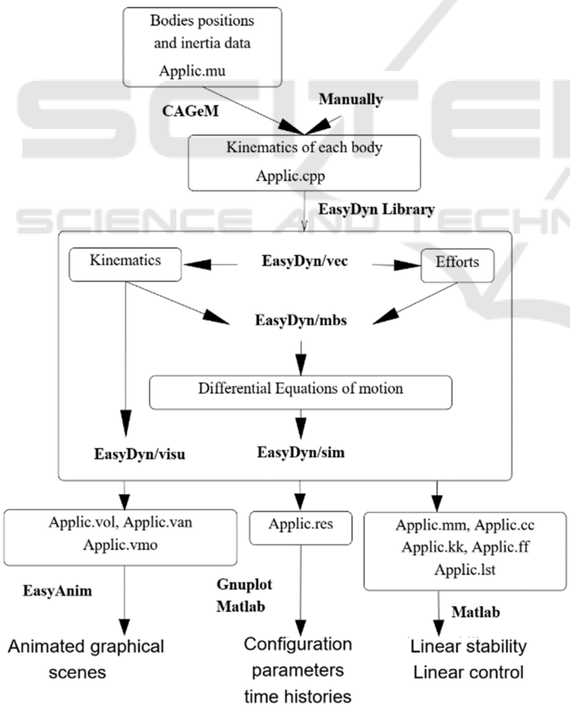

Figure 2: EasyDyn simulation data flow.

to homogeneous transformation matrices; and the principle of virtual power to derive the equations of motion which are then integrated according to the Newmark scheme (Newmark-$\frac{1}{4}$). Fig. 2 depicts the simulation data flow in the EasyDyn framework.

The process starts with a Mupad file (Applic.mu) which contains bodies' inertia data and their relative configurations expressed in term of the vehicle configuration parameters. A symbolic tool called CAGeM is used to generate symbolically the kinematics of the vehicle. The resulted C++ file (Applic.cpp) from CAGeM contains basic EasyDyn command lines; and the user can include in this file other EasyDyn command lines dedicated to his application. Among the output files of this process there are those required for the stability analysis of the multibody systems under this research.

2.2 Mechanical Model

The *Anaconda* mechanical model was presented in (Verlinden and Kabeya, 2012); where a multibody approach was used by taking into account the confirmed modelling assumptions made for single track vehicle: bodies are considered to be rigid, rider's lower body is firmly attached to the module frame, rider's upper body can rotate about the longitudinal axis of the module frame, tire-ground contact is modelled as force element.

Fig. 3 illustrates the mechanical models of an *Anaconda* composed of a head module with one pedal module and a pedal module alone.

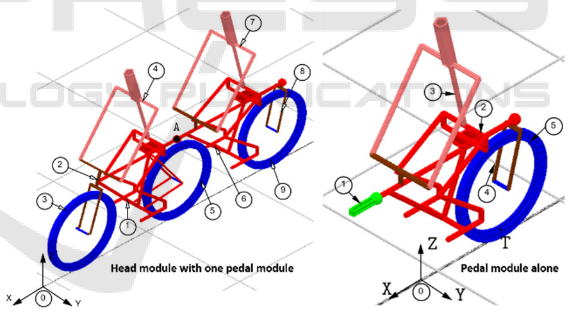

Figure 3: Riders-vehicle mechanical model.

The number of bodies (n_b) and the one of degrees of freedom (n_{cp}) of each system are summarized in Tabs. 1 and 2. Parameters defining these degrees of freedom are considered as configuration parameters.

Three vehicle's models are investigated in this work:

- The head module alone: $n_b = 6$, $n_{cp} = 10$;
- The pedal module alone: $n_b = 6$, $n_{cp} = 6$;
- The *Anaconda* with one pedal module: $n_b = 10$, $n_{cp} = 16$.

Table 1: Bodies and their relative degrees of freedom for the *Anaconda* with one pedal module.

Body		Joint			Relative conf. param.	
No.	name	name	w.r.t	No_{rDOF}	denoted	name
0	ground	fixed	ground	0	-	-
1	HM frame	free	ground	6	q_0 q_1 q_2 q_3 q_4 q_5	longit. disp. lateral disp. vertical disp. HM yaw angle HM pitch angle HM roll angle
2	HM fork	revolute	HM frame	1	q_6	HM steer angle
3	HM front wheel	revolute	HM fork	1	q_7	-
4	HM rider	revolute	HM frame	1	q_8	HM rider body lean angle
5	HM rear wheel	revolute	HM frame	1	q_9	-
6	PM frame	spherical	HM frame	3	q_{10} q_{11} q_{12}	PM rel. yaw angle PM rel. pitch angle PM rel. roll angle
7	PM rider	revolute	PM frame	1	q_{13}	PM rider body lean angle
8	PM fork	revolute	PM frame	1	q_{14}	PM steer angle
9	PM rear wheel	revolute	PM fork	1	q_{15}	-

Table 2: Bodies and their relative degrees of freedom for the pedal module alone.

Body		Joint			Relative conf. param.	
No.	name	name	w.r.t	n_{rDOF}	denoted	comment
0 (0)	ground	fixed	ground	0	-	-
1 (5)	Front moving body	prismatic	ground	0 0	- -	direction and velocity fixed
2 (6)	frame	spherical	Front moving body	3	q_0 (q_{10}) q_1 (q_{11}) q_2 (q_{12})	yaw angle pitch angle roll angle
3 (7)	rider	revolute	frame	1	q_3 (q_{13})	upper body lean angle
4 (8)	fork	revolute	frame	1	q_4 (q_{14})	steer angle
5 (9)	rear wheel	revolute	fork	1	q_5 (q_{15})	-

Taking advantages of the parametric and generic model of the *Anaconda* implemented in EasyDyn (Kabeya and Verlinden, 2011), the numerical model of the head module alone and the one of the *Anaconda* with one pedal module are derived from the same mechanical model.

The implementation of the pedal module alone is made apart from the mechanical model presented in Fig. 3. The pedal module is considered as a trailer towed by a front moving body, replacing the head module, whose motion is imposed. Physically, this corresponds to the hypothesis that the head module motion is not affected by the one of the pedal module.

2.3 Mathematical Model

For each vehicle's model, the n_{cp} second order equations of motion are derived and recasted in a matrix form as:

$$M(\underline{q}) \cdot \underline{\ddot{q}} + \underline{h}(\underline{q},\underline{\dot{q}}) = \underline{g}(\underline{q},\underline{\dot{q}},t) \qquad (1)$$

where:

- $\underline{q}$ is a $(n_{cp},1)$ vector gathering all the configuration parameters;
- M is a (n_{cp},n_{cp}) mass matrix;
- $\underline{h}$ is a $(n_{cp},1)$ vector gathering contributions of centrifugal and Coriolis forces;
- $\underline{g}$ is a $(n_{cp},1)$ vector gathering contributions of external forces.

The forces taken into account in these models are gravity and tyre-ground contact forces.

Furthermore, equations of motion are linearized around a stationary state, defined as the state in which the vehicle is let going straight ahead in a constant configuration position $\underline{q}^0$ and at a constant forward velocity. The linearized equations are given as:

$$M \cdot \Delta\underline{\ddot{q}} + CT \cdot \boldsymbol{\Delta}\underline{\dot{q}} + KT \cdot \boldsymbol{\Delta}\underline{q} = 0 \qquad (2)$$

where:

- $\Delta\underline{q} = \underline{q} - \underline{q}^0$ is the relative configuration parameter vector defined with respect the stationary state position $\underline{q}^0$;
- CT is a (n_{cp},n_{cp}) tangent damping matrix;
- KT is a (n_{cp},n_{cp}) tangent stiffness matrix.

For the linearization state, the lateral or out-of-plan dynamic is decoupled from the in-plane one (Koenen, 1983). Then sub matrices concerned with out-of-plane dynamic: the reduced mass matrix (M^r), the reduced tangent damping (C^r) and the reduced tangent stiffness matrix (K^r) are drawn from their respective counterparts of the entire linearized system by taking into account only the concerned configuration parameters ($\underline{q}^r$). The configuration parameters involved in the out-of-plane dynamic are:

- The lateral displacement, the yaw, roll and steer angles for the head module alone: $n_r = 4$;
- The yaw, roll and steer angles for the pedal module alone: $n_r = 3$;
- The combination of the above two configuration parameters for an *Anaconda* with one pedal module: $n_r = 7$.

Moreover, it is to highlight that rider's upper body degree of freedom is frozen in these models. This configuration parameter together with the rider's legs are involved in the human control activities attempting to maintain the vehicle balance. Their influence where proved to be less significant with respect to the steer angle (Kooijman et al., 2009).

The out-of-plane matrices are recasted in an equivalent state space model:

$$\underline{\dot{x}} = A \cdot \underline{x} \qquad (3)$$

where:

- $\underline{x}$ is the reduced state vector of dimension $(n_s,1)$ with $n_s = 2{*}n_r$; and is equal to

$$\underline{x} = \begin{Bmatrix} \underline{q}^r \\ \underline{\dot{q}}^r \end{Bmatrix} \quad (4)$$

- A is the evolution matrix of dimension (n_s,n_s) defined from the reduced matrices as:

$$A = \begin{bmatrix} 0 & I \\ -(M^r)^{-1} K^r & -(M^r)^{-1} C^r \end{bmatrix} \quad (5)$$

with 0 and I the zero and identity matrices of appropriate dimensions.

The evolution matrix is used in the sequel for the computation of eigenvalues and eigenmodes of the vehicle's out-of-plane dynamic. Positions of eigenvalues in the complex plane will vary with the forward speed. The stability analysis of eigenmodes is rely on these positions.

3 SIMULATION RESULTS

3.1 Modes Determination Procedure

For each vehicle, simulations are made as follows:

- The vehicle is brought in a steady state condition letting it run straight ahead at a constant velocity. The velocity ranges from 0.2 to 10 m/s with a step of 0.1 m/s are selected;
- The linearization of the equations of motion is performed around this steady state configuration;
- The text files containing the matrices of the linearized equations of motion from EasyDyn are retrieved under Matlab where the subset of the lateral dynamics is extracted and recasted in an equivalent state space model.
- Eigenvalues are computed for each forward velocity and their evolution analysed.

The two first step are performed with EasyDyn. Combination of the information from eigenvalues evolution over the forward speed, mode shapes and their animations are used to distinguish them from each other.

3.2 Modes of the Head Module Alone

According to the considered mechanical model, six distinct head module's modes are observed from the eight eigenvalues computed. They are denoted from HMMode1 to HMMode6. Evolutions of their eigenvalues over the forward speed are given in Figs. 4 and 5.

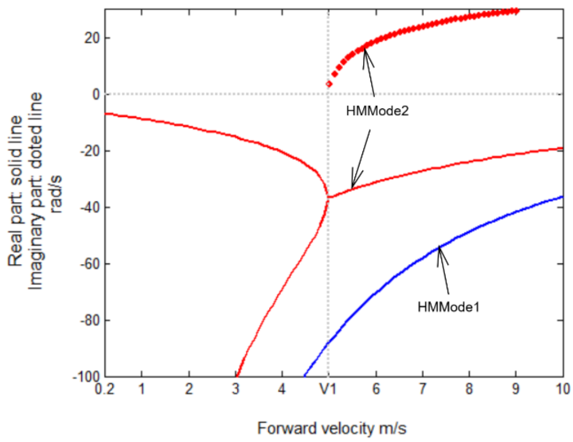

Figure 4: HM modes evolution over the forward speed range (HMMode1 and HMMode2).

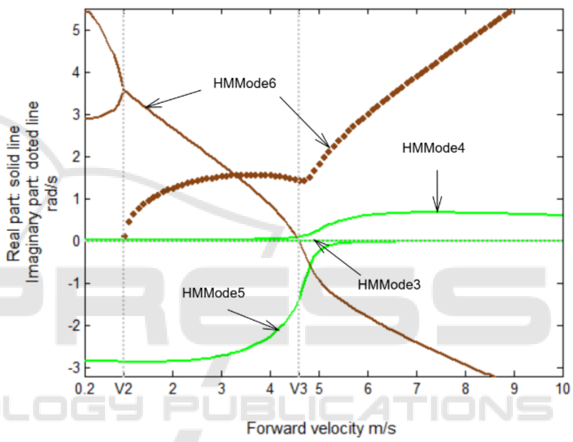

Figure 5: HM modes evolution over the forward speed range (HMMode3, HMMode4, HMMode5 and HMMode6).

HMMode1, 2, 3 and 5 are stable modes whereas HMMod4 and 6 are unstable ones. HMMode6 evolves to the stable region as the forward speed increases with a crossing speed V3 equal to 4.6 m/s. All modes are non-oscillatory except HMMode2 and 6 that start in this form with two branches that merge (at V1 = 5 m/s and V2 = 1 m/s) and became oscillatory.

Some of these modes are common with the ones of single track vehicles: HMMode1, upper branch of HMMode2, HMMode5 and HMMode6. They correspond to the classical wobble, caster, capsize and weave modes.

Let us mention that HMMode1 is characterized by a dominant steer angle motion in opposite phase with the one of the roll angle as it can be seen in Fig. 4. In this figure, aside the mode shape (left) there is a screenshot (right) that illustrate its vibrational behaviour.

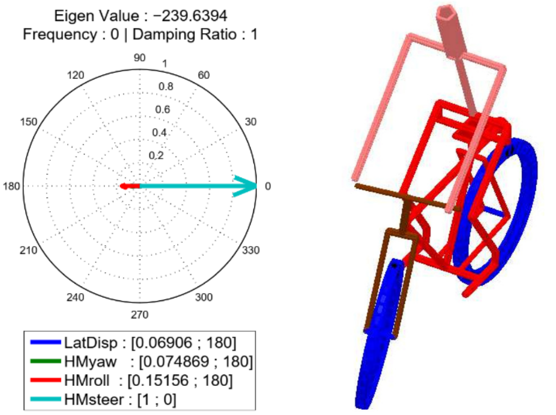

Figure 6: HMMode1 shape (right) and screenshot of its vibrational motion (left) at 2 m/s.

This faster mode is responsible of the counter steering phenomenon. In this investigated mechanical model, HMMode1 is stable and non-oscillatory as the wobble mode of a simplified motorcycle (Cossalter and Roberto, 2015). The unstable and oscillatory wobble encounter in bicycle model is due to the implementation of the front frame flexibility and the tire dynamics (Sharp, 2008; Dressel and Rahman, 2010).

Moreover, HMMode3 has an eigenvalue equal to zero and characterized by a large lateral displacement as HMMmode4. This behaviour is observed also with HMMode4 and 5 when they are close the HMMode3 (below 4.2 m/s and above 5.5 m/s, respectively).

3.3 Modes of the Pedal Module Alone

The six eigenvalues computed exhibit four distinct pedal module modes denoted from PMMode1 to 4. Their evolutions over the forward speed are given in Fig. 7.

Furthermore, PMMode1 to 3 are stable modes whereas PMMode4 is the only unstable mode.

PMMode1 is also non-oscillatory like HMMode1 but slower with time constant value varying from 6E-4 to 5E-2 second. It is characterized by a roll motion in antiphase with those of the yaw and steer angles. This can be seen on the mode shape in Fig. 8 (left). Below 8.6 m/s, the steer motion is the dominant one; and above this speed the roll motion become dominant. This antiphase configuration feature between the roll and the steer angles characterizes the steer into the lean manoeuvre required to keep the pedal module in equilibrium. Which means that the rear steered handlebar play its designed role. The steer into the lean manoeuvre is illustrated in Fig. 8 (right).

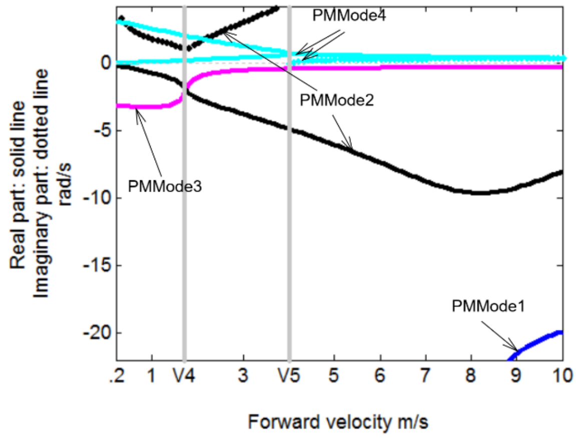

Figure 7: PM modes evolution over the forward the forward speed range.

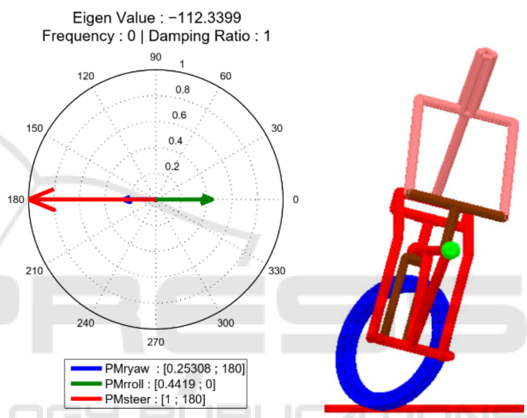

Figure 8: PMMode1 shape (right) and screenshot of its vibrational motion (left) at 3 m/s.

In addition, PMMode2 behaves the same way as PMMode1 but in an oscillatory manner with frequencies varying from 0.64 Hz at the beginning of the simulation process (at 0.2 m/s) to 3 Hz at 10 m/s. A the forward speed of 1.7 m/s a frequency minimum value of 0.17 Hz is reached together with a maximum damping ration of 84%.

The yaw and the steer angle motions are the only ones involved in PMMode3 and PMode4. PMMode3 is a non-oscillatory mode whereas PMMode4 is a quasi-oscillatory one above V5 = 4.1 m/s. This unstable begins with two non-oscillatory branches that merge at V5 in an oscillatory form with a maximum frequency equal to 5.7E-2 Hz reached at 8.8 m/s. The yaw and steer motions evolves in (quasi) antiphase configuration for these pedal module modes (see Fig. 9 (left)). The screenshot of PMMode4 shown in Fig. 9 (right) suggests that as time increases, higher yaw angle will be reached (indeed slowly) due to the unstable nature of this mode. This will lead the pedal module to hit the front one.

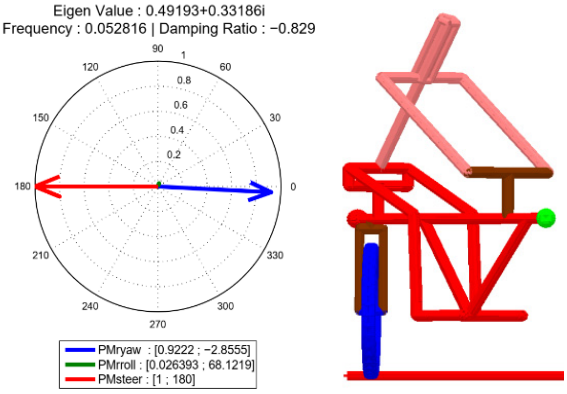

Figure 9: PMMode1 shape (right) and screenshot of its vibrational motion (left) at 3 m/s.

3.4 Modes of the *Anaconda* with One Pedal Module

From the fourteen eigenvalues computed, only ten distinct modes are found out to be distinct for the *Anaconda* with one pedal module. They are denoted from AMode1 to 10. Figs. 10, 11 and 12 depict the evolutions of their eigenvalues with the forward speed.

The first figure (Fig. 10) is concerned with stables modes having lower time constant in the speed range.

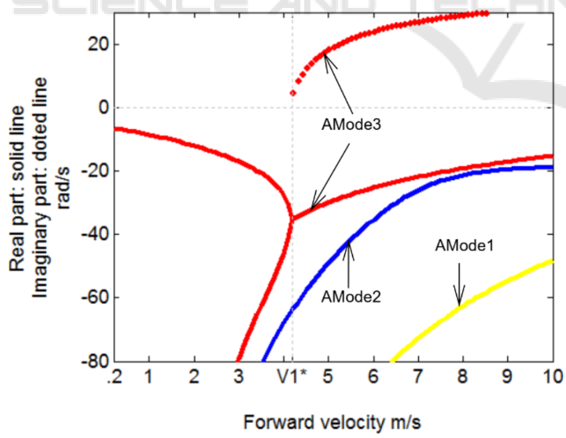

Figure 10: First group of stable modes of the *Anaconda* (AMode1, AMode2 and AMode3).

Moreover, each mode of the *Anaconda* with one pedal module is found out to be a combination of one mode of the head module and another one of the pedal module; the head and the pedal modules being considered alone as mentioned above.

Fig. 11 and 12 illustrate the remainder stable modes and the unstable ones respectively.

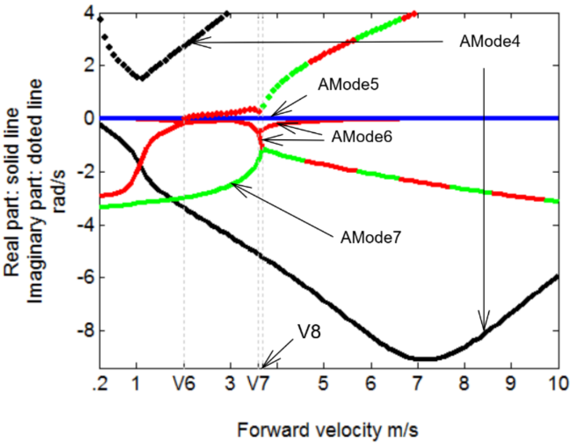

Figure 11: Second group of stable modes of the *Anaconda*: AMode4, AMode5, AMode6 and AMode7.

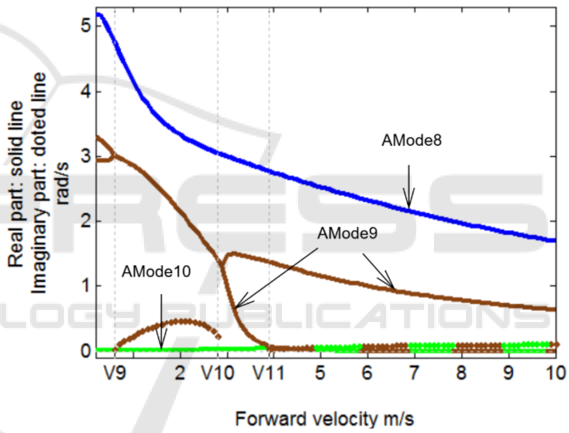

Figure 12: Unstable modes of the *Anaconda*: AMode8, AMode9 and AMode10.

In summary, Tab. 3 relates each *Anaconda* mode with its combination.

In addition, AMode1 and AMode2 are non-oscillatory evolving over the speed range with an increasing time constants like HMMode1 and PMMode1.

AMode3 is oscillatory and a replication of HMMode2. The weak influence of the pedal module on this mode can be observed on shapes of both modes (see Fig. 13). In these modes, the dominant motion is the one of head module yaw angle. AMode4, AMode5 and AMode10 are other cases of complete replication of a mode over the speed range. Particularly, Fig. 14 shows the replication of PMMode2 in AMode4.

Table 3: *Anaconda* modes combinations.

AMode	Combination
AMode1	HMMode1 + PMMode1
AMode2	HMMode2 lower branch + PMMode1
AMode3	HMMode2 + fixed pedal module
AMode4	fixed head module + PMMode2
AMode5	HMMode3 + fixed pedal module
AMode6	fixed head module + PMMode3 (v < V6) Deformed shape of AMode6 (V6 < v < V7)
AMode7	HMMode5 + fixed pedal module (v < V7) HMMode5 + PMMode3 (v > V7)
AMode8	HMMode5 + PMMode4
AMode9	HMMode6 + PMMode4
AMode10	fixed head module + PMMode4

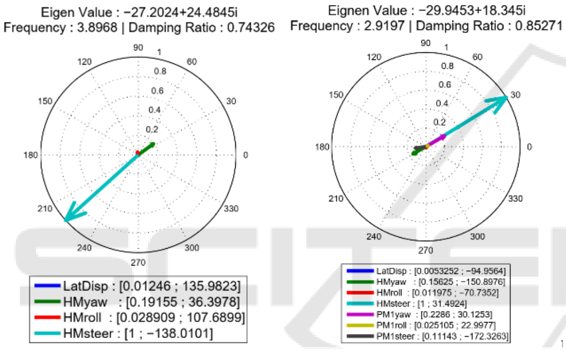

Figure 13: Shapes of HMMode2 and AMode3.

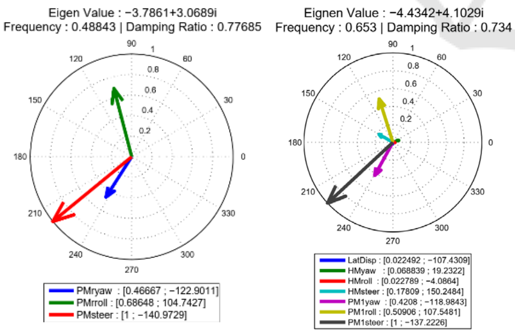

Figure 14: Shapes of PMMode2 and AMode4.

Indeed, AMode6 begins with two non-oscillatory branches below V6 = 2 m/s (replication of PMMode3 for the lower branches). These branches merge at this speed in a quasi-oscillatory form up to V7 = 3.6 m/s. In this speed range, the replication of PMMode3 is slightly deformed by the presence of the lateral displacement motion in anti-phase with the one of the pedal module steer angle and in quasi-phase with the one of the pedal module yaw angle. Beyond V8, AMode7 is non-oscillatory and a replication of HMMode5. From this speed, it merges with the lower branch of AMode6 in an oscillatory form. In this latter form, modes HMMode5 and PMMode3 are combined.

It is emphasized that when the real part of an eigenvalue is close to zero, the corresponding mode shape is characterized by a dominant lateral displacement motion.

The capsize (HMMode5) and weave HMMode6) are the bicycle modes involved in the combination of the unstable modes of the *Anaconda*: AMode8, AMode9 and AMode10. Except AMode8 which evolves over the speed range in a non-oscillatory manner, AMMode9 and AMMode10 switch from one form to another over the speed range (V9 = 0.6 m/s, V10 = 2.9 m/s and V11 = 3.9 m/s).

They are known to be controllable at any forward speed. Their combination with PMModes4 yields unstable modes with real parts below *5 rad/s* (but still decreasing with the forward speed); so as the unstable modes of the *Anaconda can be controlled in the human capabilities*.

4 CONCLUSIONS

This study is concerned with the out-of-plane dynamic of an *Anaconda* with one pedal module. The linearization of nonlinear equations of motion of the vehicle around a stationary state is required and numerical simulations are carried out for some forward speed in a speed range to get more insight in the lateral behaviour of the *Anaconda*. All these tasks are accomplished thanks to a co-simulation process between EasyDyn and Matlab. Taking advantage of the dynamic behaviours of the *Anaconda*'s components it was found out that component modes are combined each other or replicated in order to form the one of the *Anaconda*. Particularly, the only one unstable mode of the pedal module considered alone is involved in the *Anaconda*'s unstable modes. The analysis of the evolution of these unstable modes of the *Anaconda* help to draw conclusions that tackle a control issue for driving the *Anaconda* by human drivers.

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