Fractional Order Tracking Control of Unmanned Aerial Vehicle in Presence of Model Uncertainties and Disturbances

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Abstract: An unmanned Aerial Vehicle (UAV) is a highly non-linear unstable system. In this work using fractional order calculus, a novel fractional order dynamics of UAV is proposed. The concept of fractional order depicts the more realistic behavior of UAVs. For proposed fractional order model, a fractional order sliding mode controller (SMC) is designed such that the desired path can be achieved by the UAV in finite-time. In addition to this model uncertainty and disturbance is considered in the system which is handled by the proposed robust SMC. Stability analysis is given for the fractional order SMC using fractional Lyapunov method. Simulations have been done for position and attitude tracking of UAV to demonstrate the efficacy of the proposed method.

1 INTRODUCTION

Recently, UAVs are being used for wide variety of applications some of them are transportation, survelliance (Aubry et al., 2014), forest trail destection (Giusti et al., 2015), agriculture purposes (Mogili and Deepak, 2018) etc. however to control a quadrotor is quite challenging due to its characteristics like high nonlinearity, underactuation propoerty and external disturbances. From the past few years it is a subject of interest for researchers to design a robust controller for quadrotor UAVs.

Although there are several controllers e.g. LQR controller (Cohen et al., 2020), Backstepping Controller (Yu et al., 2019), (Liu et al., 2016) developed and applied on the UAV, still a robust control scheme has been an interest of research. Sliding mode control (SMC) (Ríos et al., 2018) is one of the most popular and robust control technique which has the ability to rejects disturbances and uncertainties but at the cost of chattering (Boiko and Fridman, 2005). The chattering actuates the unwanted dynamics of the system which can deteriorate the system performance, hence disturbance rejection at a cost of deteriorated performance is not appreciable. Since quadrotor is a relative degree two type of system, a proper stable sliding surface is needed for the design of controller. Depending upon the type of surface the convergence of error can be asymptotic or finite-time. The asymptotic surface (Xiong and Zhang, 2016) shows slower convergence than the finite-time surface but finite-time surface or terminal sliding surface (Weidong et al., 2015), (Wang et al., 2016) cause singularity issue.

Most of the controllers discussed above are integer order control schemes. Recently, fractional order controllers (Chen et al., 2019)(Cajo et al., 2019)(Hua et al., 2019) have drawn much attention due to application of powerful processors. The fractional order terms provides an extra degree of freedom in terms of controller parameters which can be adjusted for better tracking performance. Some of the work on fractional order controller on UAV are as (Oliva-Palomo et al., 2019) presents a PI fractional order controller for quadrotor for only attitude control. A novel fractional controller has been proposed in (Izaguirre-Espinosa et al., 2018) for attitude control as well as position control of quadrotor. Conventinal SMC has been employed in (Shi et al., 2020) for position and attitude control of quadrotor where fractional order switching law is proposed to compensate the uncertainties on integer model of quadrotor. A fast terminal SMC (FTSMC) has been presented in (Labbadi et al., 2019) for faster convergence of tracking error however FTSMC is applied only on attitude control whereas conventional SMC is employed for position control and thus overall scheme doesn't provide faster convergence. All these schemes hasve been applied on applied on the integer order model of the

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quadrotor UAV. Moreover, less attention is given to the fractional-order based dynamics. Though we consider the dynamics of the quadrotor of integer order but practically it may not be of integer order because there may be some fractional order term exist which effects the dynamics of the quadrotor. Hence, a fractional order controller can increase the robustness and usability of controller if employed with the fractional order model of the quadrotor.

Motivated from the above discussion, a fractional order model of quadrotor has been considered instead of an integer order model In this work which is more practical and feasible with the real world model. Thereafter, A robust control law as fractional order sliding mode controller (SMC) has been presented for the quadrotor model while considering the uncertain dynamics. There is a trade-off has been done between the asymptotic surface and finite-time time surface using fractional order theory. Using fractional order theory, a novel fractional sliding surface is proposed for the quadrotor which improves the response of the surface as well as avoids the singularity issue of the finite-time sliding surface. Next for mitigating the chattering issue of SMC, a power rate reaching law along with a proportional term has been used in the control laws for position and attitude tracking. There are six fractional order control laws are designed for UAV where, three are position controllers which generates the thrust required and attitude reference for attitude controller while rest of the three are attitude controllers. The main contribution of the paper are summarised as follows:

1. A fractional order novel sliding surface is proposed so that the region of stability increased in left half plane.
2. A novel fractional order singularity free control law is proposed which rejects the model uncertainties present in the quadrotor.
3. Stability of the fractional sliding surface and the controller is given using Lyapunov stability theory.
4. Simulations are conducted for quadrotor position and attitude tracking and it is shown that the proposed technique is better than the existing second order twisting controller.

Rest of the paper is organized as follows: preliminaries for fractional calculus is provided in section 2 and section 3 constitutes the problem formulation. Fractional order quadrotor model is presented in section 4 which is followed by the controller design in section 5 and stability analysis in section 6. Simulation results slong with the comparative analysis has been presented in section 7. Finally, conclusion and future scope is given in section 8.

2 PRELIMINARIES

The caputo fractional derivative (Odibat, 2006) of any function $f(\vartheta)$ is represented by:

$$D^{\beta}[f(\vartheta)] = \frac{1}{\Gamma(n-\beta)} \int_{t_0}^{t} \frac{f^{(n)}(\tau)}{(t-\tau)^{(\beta-n+1)}} d\tau \qquad (1)$$

where, $\Gamma(.)$ represents the Gamma function and $n-1 < \beta < n \in N$.
Fractional order integral (Odibat, 2006) of order $\alpha > 0$ can be expressed as:

$$J^{\alpha}[f(\vartheta)] = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} (t-\tau)^{(\alpha-1)} f(\vartheta) d\tau \qquad (2)$$

3 PROBLEM FORMULATION

The control architecture is shown in Fig .1 consists of two control loops where first one corresponds to inner or attitude control loop which runs at high frequency, another is position control loop which estimates the attitude reference to the attitude controller in terms of ϕ_d, θ_d and thrust to the quadrotor. The speed of the rotor is regulated by the pulse width modulated (PWM) signal.The generation of the desired PWM signal is controlled by the output of attitude controller i.e. torque and thrust from the position controller, which then actuates the motors of quadrotor.
The objective is to track the desired position and attitude in presence of uncertainties where the desired position x_d, y_d, z_d and desired yaw angle ψ_d are provided by the user. Here a novel fractional order sliding surface is proposed which increases the stability range of the error plane. After that a fractional order SMC is applied on the quadrotor which rejects the uncertainty present in the system.

So mathematically, the objective is

$$\lim_{t\to\infty} x \to x_d, \lim_{t\to\infty} y \to y_d, \lim_{t\to\infty} z \to z_d \text{ and } \lim_{t\to\infty} \psi \to \psi_d$$

4 QUADROTOR MODEL

Fig. 2 represents the pictorial view of quadrotors and direction of the forces acting on the four arms. The forces F_1, F_2, F_3 and F_4 works in the upwards direction to generate the desired thrust so that the quadrotor can fly in the qpwards direction. The sum of the total

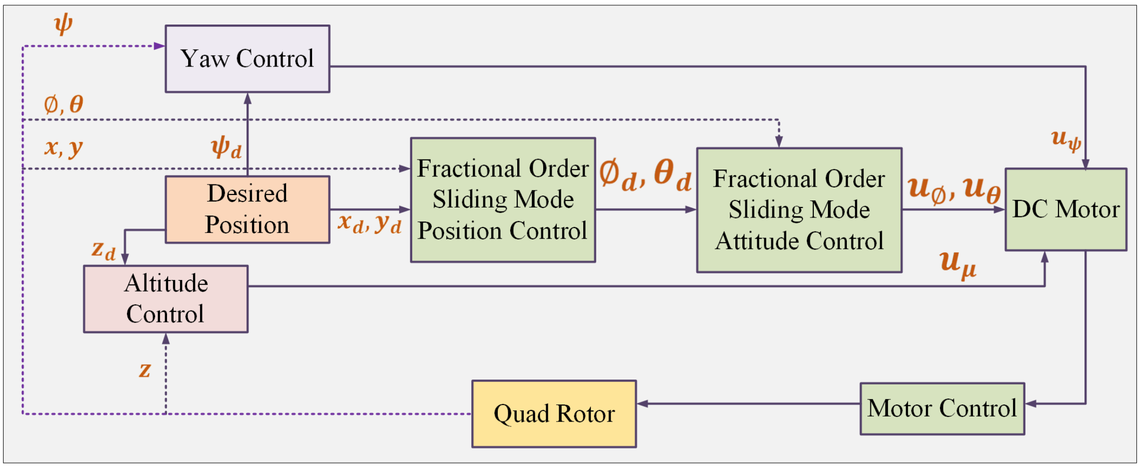

Figure 1: Control Architecture for UAV.

(a) (b)

Figure 2: (a.) Quadrotor (b.) Direction of Forces acting on Four arms.

forces $u_\mu = F_1 + F_2 + F_3 + F_4$ is called the total thrust required to lift the quadrotor. The minimum thrust required to drag the UAV in the upward direction should be greater than the weight of the UAV. Therefore small UAVs required small thrust compared to big one to achieve the same height from the ground and hence takes less power.

4.1 Quadrotor Dynamics

Generally fractional order controllers are designed for integer order system. In (Hua et al., 2019) fractional order sliding mode controller is designed for integer order UAVs. It will be more realistic if one would take the dynamics of UAVs also fractional order. The fractional order dynamics of quadrotor in presence of model uncertainty and external disturbance, is presented here by taking the fraction order of the model of UAV given in (Singh et al., 2020). It is to be noted that in this paper a novel fractional model of the quadrotor is designed.

$$
\begin{aligned}
&D^\alpha \phi_1 = \phi_2\\
&D^\alpha \phi_2 = \delta f(\phi,t) + d_\phi(t) + \dot{\theta}\dot{\psi}\left(\frac{J_y - J_z}{J_x}\right) + \frac{u_\phi}{J_x}\\
&D^\alpha \theta_1 = \theta_2\\
&D^\alpha \theta_2 = \delta f(\theta,t) + d_\theta(t) + \dot{\phi}\dot{\psi}\left(\frac{J_z - J_x}{J_y}\right) + \frac{u_\theta}{J_y}\\
&D^\alpha \psi_1 = \psi_2\\
&D^\alpha \psi_2 = \delta f(\psi,t) + d_\psi(t) + \dot{\phi}\dot{\theta}\left(\frac{J_x - J_y}{J_z}\right) + \frac{u_\psi}{J_z}\\
&D^\alpha z_1 = z_2\\
&D^\alpha z_2 = \delta f(z,t) + d_z(t) + \frac{u_\mu}{m}(C\phi C\theta) - g\\
&D^\alpha x_1 = x_2\\
&D^\alpha x_2 = \delta f(x,t) + d_x(t) + \frac{u_\mu(C\phi S\theta C\psi + S\phi S\psi)}{m}\\
&D^\alpha y_1 = y_2\\
&D^\alpha y_2 = \delta f(y,t) + d_y(t) + \frac{u_\mu}{m}(C\phi S\theta S\psi - S\phi C\psi)
\end{aligned} \tag{3}
$$

where $C(.), S(.)$ corresponds to $cos(.)$ and $sin(.)$ respectively, ϕ_1, θ_1, ψ_1 are three attitude angles i.e. roll, pitch and yaw angles respectively whereas x_1, y_1, z_1 are positions of the quadrotors. There are total twelve states including angular velocities ϕ_2, θ_2, ψ_2 and translational velocities x_2, y_2, z_2. All these twelve states are controlled by four control inputs u_μ, u_ϕ, u_θ and u_ψ. $\delta f(\phi,t)$, $\delta f(\theta,t)$, $\delta f(\psi,t)$, $\delta f(x,t)$, $\delta f(y,t)$, $\delta f(z,t)$ are model uncertainty and $d_\phi(t)$, $d_\theta(t)$, $d_\psi(t)$ are external disturbances. The relation between rotor forces and four control inputs is given in (Singh et al., 2020). The dynamics of the quadrotor can be represented as second order fractional subsystems if the virtual control laws are selected as:

$$
\begin{aligned}
u_{x1} &= \frac{u_\mu}{m}(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi) \\
u_{y1} &= \frac{u_\mu}{m}(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi) \\
u_{z1} &= \frac{u_\mu}{m}(\cos\phi\cos\theta) - g
\end{aligned} \tag{4}
$$

Therefore,

$$u_\mu = m\sqrt{(u_{x1})^2 + (u_{y1})^2 + (u_{z1}+g)^2} \tag{5}$$

After considering all the virtual control inputs the dynamics of the quadrotor can be decoupled using six second order subsystems. Now objective is to design tracking controller such that desired positions and attitude angles are achieved.

4.2 Error Model

In this section error dynamics of x, y, z and ϕ, θ, ψ is given. Now, the second order fractional dynamics for x position is:

$$
\begin{aligned}
D^\alpha x_1 &= x_2 \\
D^\alpha x_2 &= \delta f(x,t) + d_x(t) + u_{x1}
\end{aligned} \tag{6}
$$

If the desired x position is x_d then the error will be:

$$
\begin{aligned}
e_{x1} &= x_1 - x_d \\
e_{x2} &= x_2 - D^\alpha x_d
\end{aligned} \tag{7}
$$

Hence, fractional order error dynamics for x position in presence of model uncertainty and disturbance is:

$$
\begin{aligned}
D^\alpha e_{x1} &= e_{x2} \\
D^\alpha e_{x2} &= \delta f(x,t) + d_x(t) + u_{x1} - D^{2\alpha} x_d
\end{aligned} \tag{8}
$$

Like wise error dynamics for rest of the five subsystems are:

$$
\begin{aligned}
D^\alpha e_{y1} &= e_{y2} \\
D^\alpha e_{y2} &= \delta f(y,t) + d_y(t) + u_{y1} - D^{2\alpha} y_d
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
D^\alpha e_{z1} &= e_{z2} \\
D^\alpha e_{z2} &= \delta f(z,t) + d_z(t) + u_{z1} - D^{2\alpha} z_d
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
D^\alpha e_{\phi1} &= e_{\phi2} \\
D^\alpha e_{\phi2} &= \dot\theta\dot\psi\left(\frac{J_y - J_z}{J_x}\right) + \delta f(\phi,t) + d_\phi(t) + u_\phi - D^{2\alpha}\phi_d
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
D^\alpha e_{\theta1} &= e_{\theta2} \\
D^\alpha e_{\theta2} &= \dot\phi\dot\psi\left(\frac{J_z - J_x}{J_y}\right) + \delta f(\theta,t) + d_\theta(t) + u_\theta - D^{2\alpha}\theta_d
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
D^\alpha e_{\psi1} &= e_{\psi2} \\
D^\alpha_{\psi2} &= \dot\phi\dot\theta\left(\frac{J_x - J_y}{J_z}\right) + \delta f(\psi,t) + d_\psi(t) + u_\psi - D^{2\alpha}\psi_d
\end{aligned} \tag{13}
$$

Now, in next section fractional order controller is designed for the quadrotor.

5 CONTROLLER DESIGN

In this section, a novel robust fractional order SMC has been proposed to counteract parametric uncertainty, external disturbances as well as unmatched uncertainty.

5.1 Fractional Order Sliding Surface Design

The proposed fractional order sliding surface is:

$$s_x(t) = D^{\alpha-1}e_{x2} + D^{\alpha-2}\Bigg[\Big[k_{x1}(|\, e_{x1}\,| + |\, e_{x1})\,|^\beta) + k_{x2} (|\, e_{x2}\,| + |\, e_{x2})\,|^\beta) + (sign(e_{x1})D^{1-\alpha}e_{x2})\Big] sign(e_{x2})\Bigg] \tag{14}$$

where, $\beta \in (0,1)$ is a positive constant. and k_{x1} and k_{x2} are positive tuning parameters. Taking the derivative of sliding surface eq.(14), we get

$$\dot s_x(t) = D^{\alpha}e_{x2} + D^{\alpha-1}\Bigg[\Big[k_{x1}(|\, e_{x1}\,| + |\, e_{x1})\,|^\beta) + k_{x2} (|\, e_{x2}\,| + |\, e_{x2})\,|^\beta) + (sign(e_{x1})D^{1-\alpha}e_{x2})\Big] sign(e_{x2})\Bigg] \tag{15}$$

After the reaching phase is achieved i.e. when $\dot{s}_x(t) = 0$, eq.(15) reduces to,

$$D^{\alpha}e_{x2} = -D^{\alpha-1}\Bigg[\Big[k_{x1}(\mid e_{x1}\mid + \mid e_{x1})\mid^{\beta}) + k_{x2}. \\ (\mid e_{x2})\mid + \mid e_{x2})\mid^{\beta}) + (sign(e_{x1})D^{1-\alpha}e_{x2})\Big] sign(e_{x2})\Bigg] \tag{16}$$

5.1.1 Error Dynamics in Sliding Mode

From Eq. (16) and (7), fractional dynamics for x position in sliding mode can be written as :

$$\begin{aligned} &D^{\alpha}e_{x1} = e_{x2} \\ &D^{\alpha}e_{x2} = -D^{\alpha-1}\Bigg[\Big[k_{x1}(\mid e_{x1}\mid + \mid e_{x1})\mid^{\beta}) + k_{x2}. \\ &(\mid e_{x2}\mid + \mid e_{x2}\mid^{\beta}) + (sign(e_{x1})D^{1-\alpha}e_{x2})\Big] sign(e_{x2})\Bigg] \end{aligned} \tag{17}$$

Likewise, error dynamics for rest of the quadrotor states can be obtained.

5.2 Fractional Order Controller Design

Again revisiting eq.(15)

$$\dot{s}_x(t) = D^{\alpha}e_{x2} + D^{\alpha-1}\Bigg[\Big[k_{x1}(\mid e_{x1}\mid + \mid e_{x1}\mid^{\beta}) + k_{x2}. \\ (\mid e_{x2}\mid + \mid e_{x2}\mid^{\beta}) + (sign(e_{x1})D^{1-\alpha}e_{x2})\Big] sign(e_{x2})\Bigg] \tag{18}$$

Substituting value of $D^{\alpha}e_{x2}$ from eq.(8) in eq.(18) and substituting $\dot{s}_x(t) = -k_{x3}s_x - k_{x4}\mid s_x\mid^{\gamma} sign(s_x)$ the control law u_{x1} will be:

$$\begin{aligned} &-k_{x3}s_x - k_{x4}\mid s_x\mid^{\gamma} sign(s_x) = \delta f(x,t) + d_x(t) + u_{x1} \\ &-D^{2\alpha}x_d + D^{\alpha-1}\Bigg[\Big[k_{x1}(\mid e_{x1}\mid + \mid e_{x1}\mid^{\beta}) + k_{x2}. \\ &(\mid e_{x2}\mid + \mid e_{x2}\mid^{\beta}) + (sign(e_{x1})D^{1-\alpha}e_{x2})\Big] sign(e_{x2})\Bigg] \end{aligned} \tag{19}$$

where, $\gamma \in (0,1)$ is a positive constant. and k_{x3} and k_{x4} are positive tuning parameters. Further simplifying

$$\begin{aligned} &u_{x1} = -k_{x3}s_x - k_{x4}\mid s_x\mid^{\gamma} sign(s_x) - \delta f(x,t) - d_x(t) \\ &+D^{2\alpha}x_d - D^{\alpha-1}\Bigg[\Big[k_{x1}(\mid e_{x1}\mid + \mid e_{x1}\mid^{\beta}) + k_{x2}. \\ &(\mid e_{x2}\mid + \mid e_{x2}\mid^{\beta}) + (sign(e_{x1})D^{1-\alpha}e_{x2})\Big] sign(e_{x2})\Bigg] \end{aligned} \tag{20}$$

For rejecting the model uncertainties and disturbances the control law is modified to,

$$\begin{aligned} &u_{x1} = -k_{x3}s_x - k_{x4}\mid s_x\mid^{\gamma} sign(s_x) - (\delta_{x1} + \delta_{x2})sign(s_x) \\ &+D^{2\alpha}x_d - D^{\alpha-1}\Bigg[\Big[k_{x1}(\mid e_{x1}\mid + \mid e_{x1}\mid^{\beta}) + k_{x2}. \\ &(\mid e_{x2}\mid + \mid e_{x2}\mid^{\beta}) + (sign(e_{x1})D^{1-\alpha}e_{x2})\Big] sign(e_{x2})\Bigg] \end{aligned} \tag{21}$$

where δ_{x1} and δ_{x2} are positive tuning parameters. Like wise control laws for rest of the position and Euler angels can be calculated. Control law for y position tracking is:

$$\begin{aligned} &u_{y1} = -k_{y3}s_y - k_{y4}\mid s_y\mid^{\gamma} sign(s_y) - (\delta_{y1} + \delta_{y2})sign(s_y) \\ &+D^{2\alpha}y_d - D^{\alpha-1}\Bigg[\Big[k_{y1}(\mid e_{y1}\mid + \mid e_{y1}\mid^{\beta}) + k_{x2} \\ &k_{y2}(\mid e_{y2}\mid + \mid e_{y2}\mid^{\beta}) + (sign(e_{y1})D^{1-\alpha}e_{y2})\Big] sign(e_{y2})\Bigg] \end{aligned}$$

Control law for altitude z tracking is:

$$\begin{aligned} &u_{z1} = -k_{z3}s_z - k_{z4}\mid s_z\mid^{\gamma} sign(s_z) - (\delta_{z1} + \delta_{z2})sign(s_z) \\ &+D^{2\alpha}z_d - D^{\alpha-1}\Bigg[\Big[k_{z1}(\mid e_{z1}\mid + \mid e_{z1}\mid^{\beta}) + k_{z2}. \\ &(\mid e_{z2}\mid + \mid e_{z2}\mid^{\beta}) + (sign(e_{z1})D^{1-\alpha}e_{z2})\Big] sign(e_{z2})\Bigg] \end{aligned}$$

Control law for roll ϕ tracking is:

$$\begin{aligned} &u_{\phi} = J_x\Bigg(-k_{\phi3}s_{\phi} - k_{\phi4}\mid s_{\phi}\mid^{\gamma} sign(s_{\phi}) - \dot{\theta}\dot{\psi}(\frac{J_y - J_z}{J_x}) \\ &-(\delta_{\phi1} + \delta_{\phi2})sign(s_{\phi}) + D^{2\alpha}\phi_d - D^{\alpha-1}\Bigg[\Big[k_{\phi1}(\mid e_{\phi1})\mid \\ &+\mid e_{\phi1})\mid^{\beta}) + k_{\phi2}(\mid e_{\phi2})\mid + \mid e_{\phi2})\mid^{\beta}) + (sign(e_{\phi1})\times \\ &D^{1-\alpha}e_{\phi2})\Big] sign(e_{\phi2})\Bigg]\Bigg) \end{aligned}$$

Control law for pitch θ is:

$$u_\theta = J_y\Bigg(-k_{\theta 3}s_\theta - k_{\theta 4} \mid s_\theta \mid^\gamma sign(s_\theta) - \dot{\phi}\psi(\frac{J_z - J_x}{J_y}) - (\delta_{\theta 1}+\delta_{\theta 2})sign(s_\theta) + D^{2\alpha}\theta_d - D^{\alpha-1}\Bigg[\Big[k_{\theta 1}(\mid e_{\theta 1} \mid + \mid e_{\theta 1} \mid^\beta) + k_{\theta 2}(\mid e_{\theta 2} \mid + \mid e_{\theta 2} \mid^\beta) + (sign(e_{\theta 1})\times D^{1-\alpha}e_{\theta 2})\Big] sign(e_{\theta 2})\Bigg]\Bigg)$$

Control law for yaw θ is:

$$u_\psi = J_z\Bigg(-k_{\psi 3}s_\phi - k_{\psi 4} \mid s_\psi \mid^\gamma sign(s_\psi) - \dot{\phi}\dot{\theta}(\frac{J_x - J_y}{J_z}) - (\delta_{\psi 1}+\delta_{\psi 2})sign(s_\psi) + D^{2\alpha}\psi_d - D^{\alpha-1}\Bigg[\Big[k_{\psi 1}(\mid e_{\psi 1} \mid + \mid e_{\psi 1} \mid^\beta) + k_{\psi 2}(\mid e_{\psi 2}) \mid + \mid e_{\psi 2}) \mid^\beta) + (sign(e_{\psi 1})\times D^{1-\alpha}e_{\psi 2}\Big] sign(e_{\psi 2}\Bigg]\Bigg)$$

6 STABILITY ANALYSIS OF CONTROLLER

For the stability analysis, two different Lyapunov function have been taken where one is for reaching phase stability analysis and the other is for sliding phase.

6.1 Reaching Phase Stability

For reaching phase we have to show that reachability law $\dot{s}_x(t) = -k_{x3}s_x - k_{x4} \mid s_x \mid^\gamma sign(s_x)$ converges to zero. Let us take Lyapunov candidate as:

$$V_r = \mid s_x \mid \tag{22}$$

The derivative of V_r is

$$\dot{V}_r = sign(s_x)\dot{s}_x \tag{23}$$

Substituting derivative of sliding surface using eq.(15) in eq.(23)

$$\dot{V}_r = \Bigg(D^\alpha e_{x2} + D^{\alpha-1}\Bigg[\Big[k_{x1}(\mid e_{x1}) \mid + \mid e_{x1}) \mid^\beta) + k_{x2}(\mid e_{x2} \mid + \mid e_{x2}) \mid^\beta) + (sign(e_{x1})D^{1-\alpha}e_{x2})\Big] sign(e_{x2})\Bigg]\Bigg) \times sign(s_x) \tag{24}$$

After substituting the value of $D^\alpha e_{x2}$ from error dynamics and inserting the control law u_{x1} eq.(24) will become:

$$\dot{V}_r \leq -sign(s_x)(k_{x3}s_x + k_{x4} \mid s_x \mid^\gamma sign(s_x)) \tag{25}$$

Using $sign(s_x)s_x = \mid s_x \mid$ and $sign^2(s_x)s_x = 1$. The Lyapunov derivative will become-

$$\dot{V}_r \leq -k_x(\mid s_x \mid + \mid s_x \mid^\gamma) \leq -k_x \mid s_x \mid \tag{26}$$

where, $k_x = min(k_{x3}, k_{x4})$. Eq.(26) is negative definite hence it is stable.

6.2 Sliding Phase Stability

for sliding phase stability we have to show that both the error states of x position tracking converges to zero. Let us take Lyapunov candidate as:

$$V_s = \mid e_{x1} \mid + \mid e_{x2} \mid \tag{27}$$

The derivative of V_s is

$$\dot{V}_s = sign(e_{x1})\dot{e}_{x1} + sign(e_{x2})\dot{e}_{x2} \tag{28}$$

Eq.(23) can also be written as using property of fractional order theory

$$\dot{V}_s = sign(e_{x1})[D^{1-\alpha}D^\alpha e_{x1}] + sign(e_{x2})[D^{1-\alpha}D^\alpha e_{x2}]$$

From eq.(8) and eq.(16) substituting the values of $D^\alpha e_{x1}$ and $D^\alpha e_{x2}$ in the above Eq.

$$\dot{V}_s = sign(e_{x1})[D^{1-\alpha}e_{x2}] - sign(e_{x2})D^{1-\alpha}D^{\alpha-1}\times \Bigg[\Big[k_{x1}(\mid e_{x1}) \mid + \mid e_{x1}) \mid^\beta) + k_{x2}(\mid e_{x2}) \mid + \mid e_{x2}) \mid^\beta) + (sign(e_{x1})D^{1-\alpha}e_{x2})\Big] sign(e_{x2})\Bigg] \tag{29}$$

After simplifying eq.(29), we get

$$\dot{V}_s = -\Big[k_{x1}(\mid e_{x1}) \mid + \mid e_{x1} \mid^\beta) + k_{x2}(\mid e_{x2}) \mid + \mid e_{x2}) \mid^\beta\Big]$$

which is negative definite. Hence both the reaching phase and sliding phase of designed fractional order controller is stable.

7 SIMULATION RESULTS

Proposed approach has been validated in MATLAB where following set of parameters specification has been taken as:

Mass (m)	1.0 Kg
Inetria (I_{xx})	1.676×10^{-2}
Inetria (I_{yy})	1.676×10^{-2}
Inetria (I_{zz})	2.314×10^{-2}

The gain parameters value has been chosen by trial and error as. $k_{x1} = k_{x2} = 2, k_{y1} = k_{y2} = 2, k_{z1} = k_{z2} = 2$ and $k_{\phi 1} = k_{\phi} = 1.2, k_{\theta} = k_{\theta} = 1.2, k_{\psi} = k_{\psi} = 1.4$. The fractional derivatives are selected as $\alpha = 0.6$ and $\beta = 0.7$. The proposed approach has been validated for two cases i.e. quadrotor hovering at 1.25m and spiral trajectory tracking. A comparison has also been provided against twisting controller (Shtessel et al., 2017) and the superiority of presented approach is validated.

7.1 Quadrotor Hovering

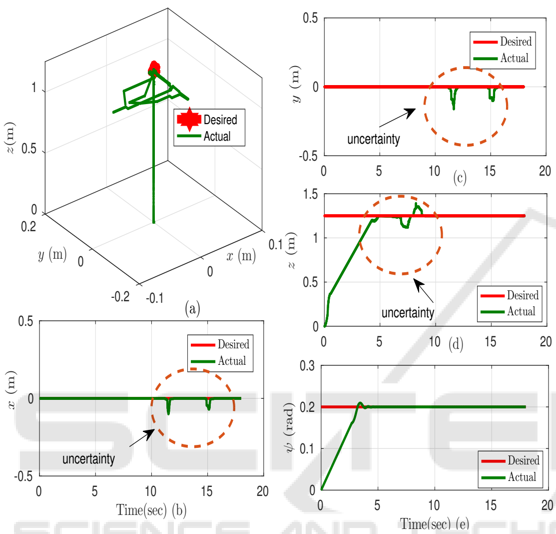

Figure 3: Hovering of UAV using proposed method.

Here, the objective is to takeoff the quadrotor to $z_d = 1.25m$ and hover thereafter at this height. To check the robustness of the proposed controller, a disturbance has been added at hovering in all of the three directions x, y and z at different time instants, the maximum bound on the magnitude of disturbances in all the three directions are $0.2 * sint$. Disturbances are applied in all the three directions at different instants of time to check the robustness of the controller. The simulation results are shown in Fig 3 where we see that the quadrotor successfully reaches at 1.25 m and hover as shown in 3(a) in spite of the disturbance. Thus we can conclude from Fig 3 that the quadrotor effectively counteract the disturbance and hover continuously at 1.25 m.

7.2 Spiral Trajectory Tracking

Now, the quadrotor is required to track the spiral shaped trajectory which is generated as follows by desired positions as: $x_d = 01 * sin(0.15 * t)$, $y_d = 2 * cos(0.2 * t)$ and $z_d = 1.5 * t$. The proposed approach is compared with (Shtessel et al., 2017) as well and obtained results are shown in Fig 4.

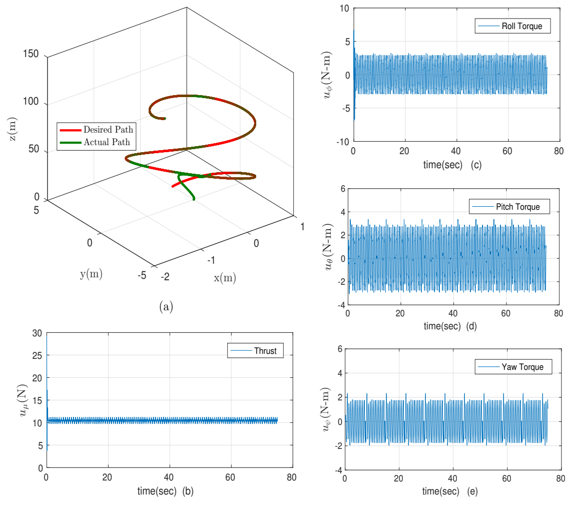

Figure 4: UAV tracking spiral shaped trajectory using Twisting controller.

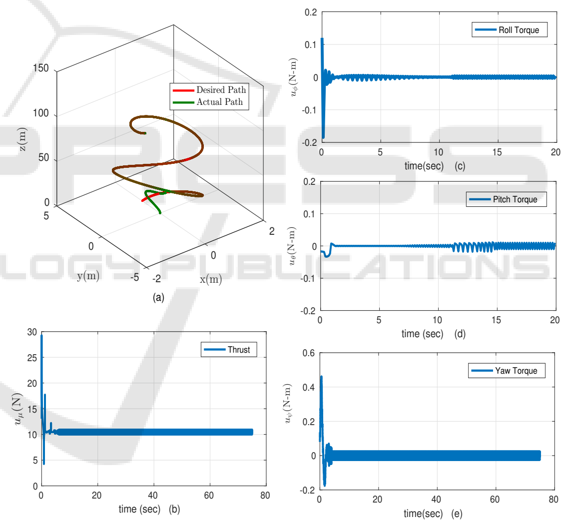

Figure 5: UAV tracking spiral shaped trajectory using proposed fractional order controller.

The tracking results obtained by proposed approach are shown in Fig 5. Form Fig 4(a) and 5(a), we see that both of the approaches show good performance in term of disturbance rejection. However, the control effort required for tracking is having more chattering as shown in Fig 4(b),(c),(d) and (e) as compared to the proposed approach in 5(b),(c),(d) and (e). Due to the large chattering, it may actuate the unmodelled dynamics which affects the actuator's performance and thus the approach (Shtessel et al., 2017) is not feasible in real-time scenario whereas proposed approach show less chattering and can be implemented in real-time.

8 CONCLUSION AND FUTURE SCOPE

In the proposed work novel fractional order model is presented for the quadrotor which is more real for practical applications. The fractional order parameters achieves the tracking accuracy of the controller. The comparative study has been done with the proposed method with the existing second twisting controller in terms of chattering attenuation and controller effort. The future scope of the present work is to design controller for multiagent quadrotor systems in presence of communication bandwidth limitation using fractional order theory. The present work is just a proof of the concept to be validated on more complex systems.

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