




Identification of Critical Links within Complex Road Networks using Centrality Principles on Weighted Graphs

Nirupam Bidikar¹^a, Yunpeng Zhang^{*:2}^b and Fengxiang Qiao³^c

¹*Dept. of Computer Science, University of Houston, Houston, U.S.A.*

²*Dept. of Information and Logistics Technology, University of Houston, Houston, U.S.A.*

³*Innovative Transportation Research Institute, Texas Southern University, Houston, U.S.A.*

Keywords: Resilience, Graphs, Roadway Infrastructure, Geographic Information Systems, Centrality, Betweenness, Node Criticality, Link Criticality.


Abstract: Building resilient infrastructure has become a necessity in modern times. If a system can efficiently deal with failures, it is considered resilient. Roadways are some of the most vital infrastructures in the world. Their collapse due to unprecedented calamities would disrupt the normal functioning of society and cause significant financial loss. To minimize traffic jams and keep traffic flowing during such times, it is essential to identify important roads within a network and plan alternate routes to divert traffic. This study aims to find critical links in a road network and study their relationships with important nodes in the same network. It also highlights some traditional approaches and applies graph-theory concepts to measure node and edge importance within a network. An approach based on variable centrality is proposed. We have implemented our proposed system and evaluated its performance on multiple networks including a large scale statewide road network in Texas. Our preliminary experiments show promising results.


1 INTRODUCTION


Transportation networks are vital components in assessing a country or state's planning and infrastructure capabilities. They also play a crucial role in the region's economy and financial standing as they are one of the essential means of trade. Natural calamities and disasters are some of the events that massively affect (Mattsson and Jenelius, 2015) transportation infrastructure. Due to their sudden and unpredictable nature, no amount of preparation is sufficient to tackle them. (Karagyozev et al., 2012) showcased that around 44 million euros worth of damage had been generated on Bulgaria's transportation infrastructure as a result of natural disasters. In such situations, most people take alternate shortest routes to reach their destination. This may lead to users choosing roads which are unable to accommodate this new influx of vehicles causing blockages and traffic jams. This effect can cascade over through the network and can compromise multiple roads making them inop-

erable. To tackle such situations and be prepared to lessen their impacts, there is a need for effective methods to identify critical links or roads within a given network. Through proper planning, traffic has to be properly rerouted through these critical links which are able to function at a greater operating capacity until the primary roads are restored to their original state.

Road networks can be essentially represented as giant graphs, with intersections or cities being the nodes and roadways being the edges. Converting geospatial data like road networks into functional structures like graphs requires significant preprocessing to address topological errors resulting from, e.g., discretized digital coordinate systems. Once that hurdle is overcome, one can obtain a network where all concepts from graph theory like centrality can be directly applied. Finding alternate routes between a source and destination can be solved as k-shortest paths problem using routing algorithms like Dijkstra's Algorithm or Bellman-Ford Algorithm. These algorithms don't necessarily give routes which consider traffic volume and sudden change in flow of traffic. The critical links we seek should consider the overall connectivity and the operating capacity of the edge

^a  <https://orcid.org/0000-0002-3452-4094>

^b  <https://orcid.org/0000-0001-6208-9571>

^c  <https://orcid.org/0000-0001-9074-0288>

which can be calculated through graph properties such as centrality.

Traditional methods of finding centrality and node importance are not relevant in weighted and directed graphs (Opsahl et al., 2010). Recently, researchers have developed many metrics to represent these properties for weighted graphs accurately. On road networks, not all links have the same functional capacity and importance, and it is crucial to incorporate this distinction within analyses. This paper shows how essential links can be identified within complex road networks using centrality principles on weighted graphs. It also gives precedence to the structural properties of the network which has a significant impact on the proposed centrality metric. Road transport metrics like edge capacity and daily traffic value are considered as edge weights to analyze networks better on a higher level.

The paper is organized as follows. Section 2 highlights some of the research done in this field. Section 3 introduces several metrics used to quantify connectivity within a network. Section 4 describes data collection and processing methodology used to perform analyses. Section 5 shows the impact of destroying edges within a network using a synthesised graph and a real world graph. We also propose a solution to identify critical links in the network and demonstrate its performance on real world graphs. Finally we present conclusions of our work and future direction of our research.

2 RELATED WORK

(Gauthier et al., 2018) performed stress tests on links to find out critical connections in networks. The authors experimented with real-world scenarios, including traffic flows. They compared results with topological methods, which have a more significant computational overhead, showing that the criticality of links depends on the metric being evaluated. This method seems to be a viable option to determine critical links. While their research was limited to a small network, their results look promising. (Almotahari and Yazici, 2020) introduced link criticality index for ranking connections. They used network flows to evaluate the criticality of a link using readily available sensor and traffic data. They were able to find critical links in the network using only the top 20% of the origin-destination pairs. This approach might not fare well for all kinds of network structures. Network topology dramatically affects the performance of this algorithm. (Furno et al., 2018) proposed a framework to identify vulnerable nodes in large scale road net-

works. Road networks are modelled as graphs and big data techniques were used to improve performance. Betweenness centrality metric is used to evaluate the critical nodes. Resilience metrics - Vulnerability, efficient information exchange were used to evaluate their procedure. City scale networks were represented as undirected graphs independent of contextual traffic data. (Li et al., 2020) proposes a "Traffic Flow Betweenness index"(TFBI) to identify critical links in a network. The index is determined by shortest travel path, traffic flow, and origin-destination demand. Critical links determined using TFBI are selected and masked from the network to assess their impact. Compared to the traditional methods, their approach is computationally less intensive. However, their method does not consider non-linear effects due to degradation of connectivity in a sub-network affecting the rest of the network. (Herrera et al., 2016) analyzed the resilience of water distribution networks using graph concepts. Their approach involved calculating redundant paths between nodes and generating flow graphs to analyze edge operating capacity and maximum flow through the system. This approach is suitable for analyzing networks with a threshold for edge capacity and can scale for large networks.

(Bhatia et al., 2015) studied the Indian railway network, finding out critical links using percolation theory. They selected the giant component (largest connected part of the network) from the network to perform their analysis and established a metric "critical functionality" which is the ratio of nodes in giant component to that in the original network. Later they experimented by removing individual nodes and routes and observing the overall connectivity of the network. (Singh et al., 2015) developed a service built using PostGIS and pgRouting, which helps calculate alternate shortest paths in the event of a natural disaster or any similar incident that compromises an edge.

(Henning et al., 2017) developed a method to identify critical networks within small city networks using the centrality indices of edges. They propose a function that depends on the centrality indices, which classifies each edge as critical or non-critical. This paper serves as an essential basis for our solution as we are interested in analyzing the relation between node and edge importance in large-scale networks.

(Opsahl et al., 2010) published a study in which they have conducted extensive work to generalize centrality measures for weighted graphs networks and find the shortest paths among such networks. (Pasos and Cardoso, 2020) followed up on the previously mentioned paper and suggested improvements to the metrics. They suggested that using logarithmic ratios to calculate variable node centrality would minimize

errors and extend the range of varying parameter (α) as shown in the equations below.

$$C_D^{log,\alpha}(u) = \log\left(\frac{s_u}{k_u}\right) * \alpha + \log(k_u) \quad (1)$$

$$C_C^{log,\alpha}(u) = \log\left(\frac{C_C^w(u)}{C_C(u)}\right) * \alpha + \log(C_C(u)) \quad (2)$$

where, C_D represents degree centrality, C_C represents closeness centrality, s_u is strength of a node, and k_u is the degree of a node.

Most of the research done in this field is based on either topology alone or edge weights. As seen from the study by (Opsahl et al., 2010), we can see how both these factors are essential to determine centrality measures in a weighted graph. Our approach aims to solve this problem by extending the research done by (Passos and Cardoso, 2020) and apply it to large real world networks like road transportation systems. We also study the relation between node and edge importance in a complex network and how variable centrality is affected by changing the precedence given to weighted centrality.

3 METRICS FOR CONNECTIVITY AND IMPORTANT COMPONENTS

In this section we introduce definitions of various metrics relevant to assessing the connectivity and resilience of a graph.

3.1 Average Shortest Path

It is the average distance between all possible pairs of points along the shortest possible paths. This metric gives an idea about how the network is connected overall and the effort it takes to transmit information among nodes.

3.2 Flow

Flow networks are graphs which showcase connectivity based on an edge's capacity. Every edge has a flow parameter which constitutes to the operating limits of the particular edge. The flow of an edge is fixed and cannot change. During operation, the information flow cannot exceed the set limit defined by the edge.

3.3 Closeness Centrality

Closeness Centrality depends on distance between a pair of nodes. Smaller the distance of a node with re-

maining nodes in a network, the greater it's centrality.

3.4 Betweenness Centrality

This metric measures the importance of a node in the network. It depends on the number of shortest paths passing through a vertex for a given pair of start and end points. Higher the centrality, greater is the influence of the node on the network meaning it's disruption can cause significant problems.

3.5 Service Area

A service area defines a region accessible from a starting point given a distance using the network. A test was performed on the highway network of Texas using the major cities as starting points and the output is shown in Figure 1.

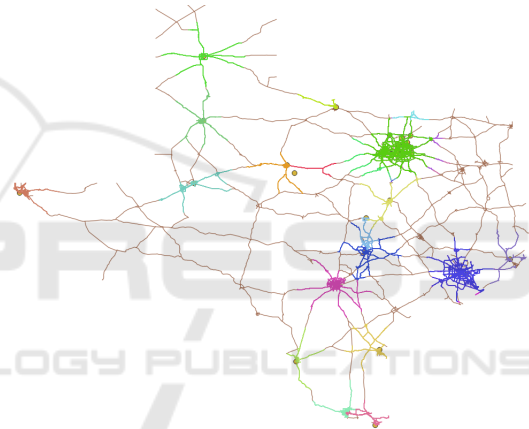


Figure 1: Service area for 100 KM.

3.6 Origin-destination Cost Matrix

OD cost matrix calculates shortest paths between pairs of nodes within a network. The distance calculated can be euclidean or along the polylines in the network. This metric is used in GIS for routing problems involving multiple origins and destinations.

4 DATA COLLECTION AND PREPROCESSING

The datasets used in this paper were collected for performing analysis in ESRI Shapefile format (ESRI and PaperdJuly, 1998) which consists of polylines and points. The Highway Performance Monitoring System (HPMS) data (TxDOT, 2019) contains highway performance metrics for the roadway network like average daily traffic volume and maximum

operating capacity. Texas roads were taken from TIGER/Line dataset (Bureau, 2020) which shows the major road system of the state. Texas cities data was obtained from Open Data Portal (Portal, 2020) hosted by TxDOT which is a repository for many geospatial datasets. We chose to add in another dataset for road network obtained from (Survey, 2014) to verify the effect of topography on our results. This graph is more generalized and less complex than the TIGER dataset.

The data was compiled together in a PostgreSQL database and a spatial index was created for fast retrieval and processing. Using the PostGIS and pgRouting extensions, the shapefiles were checked for topological errors like dangles, self loops, and parallel vectors, and shape length for each line segment was also calculated. The cities database was filtered based on population of city from year 2010. Top 30 cities were selected for analysis. Giant component (Bollobás, 2001) of the Texas road network was taken to exclude broken links as the network is too complex to manually repair. All analysis was done using QGIS (QGIS Development Team, 2021) toolbox and PyQGIS library for custom scripts.

5 IMPACT OF EDGE REMOVAL IN NETWORKS

In this section, we present details of our empirical work which show the effect of edge removal within a network.

5.1 Experiments with Dummy Graph Networks

Initial experiments proceed with generating graph structures with one of them being poorly connected and the other being well connected. We start remov-

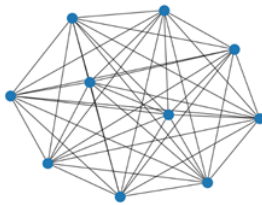


Figure 2: Well connected graph (WCG).

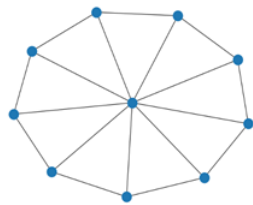


Figure 3: Poorly connected graph (PCG).

ing edges and calculate the metric “average shortest path length”. In Figure 4, the blue line represents the poorly connected graph shown in Figure 3 and the yellow line represents the well-connected graph

shown in Figure 2. The test was performed with both graphs having 30 nodes each. We chose to take the

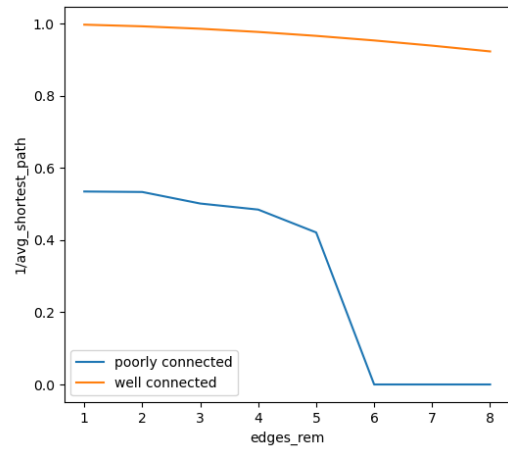


Figure 4: Edges removed vs $(avgshortestpathlength)^{-1}$.

inverse of the metric because the poorly connected graph gets split into multiple components as we keep removing nodes there by making the average shortest path length infinity. We can observe from Figure 4 that this value drastically drops for PCG whereas the decline is smoother for the WCG.

5.2 Experiments with Real Road Networks

A separate view was created by removing different highway systems. The shortest routes for an untouched network can be seen in Figure 5. These were plotted using Dijkstra’s algorithm.

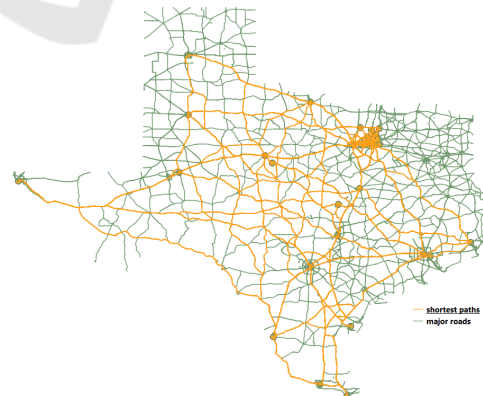


Figure 5: Regular Shortest Paths.

Removing interstate highways created measurable change in shortest paths between destinations as shown below in Figure 6. The extreme left of the state is cut off from the rest based on the our given network.

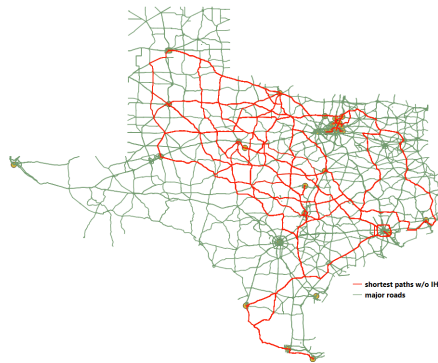


Figure 6: Shortest Paths without Interstate Highways.

Disabling state highways did not have as drastic of an effect as removing interstate highways as shown in Figure 7 hinting towards the apparent importance of Interstate highways.

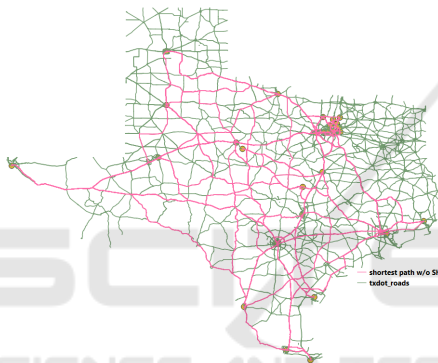


Figure 7: Shortest Paths without State Highways.

Removal of US highways did not affect the end to end connectivity much but it did considerably increase the cost of travel between destinations as seen from Figure 8.

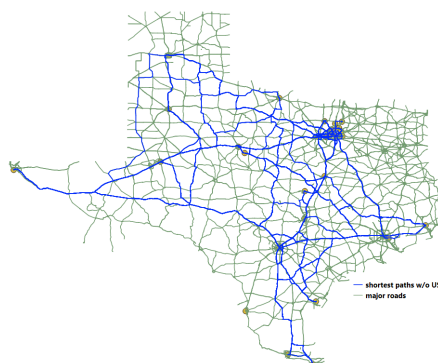


Figure 8: Shortest Paths without US Highways.

Based on the empirical observations from the experiments so far, we considered OD cost matrix as a basis to study road network deterioration. We generated tables for all 4 views and selected Houston, Texas

as the origin and 6 other cities as destinations which are spread across the state to highlight end to end connectivity which is shown in Figure 9 below.

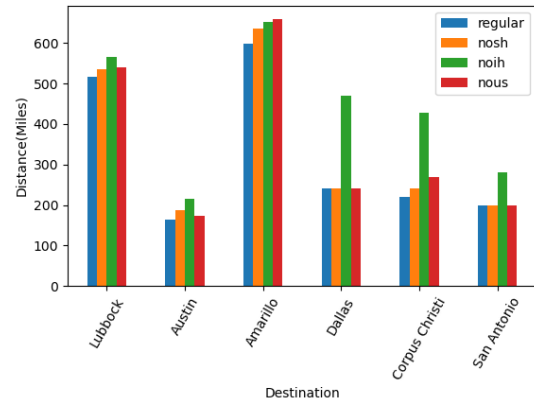


Figure 9: Shortest Path Variation on edge removal.

We can see that for major cities like Corpus Cristi, San Antonio, and Dallas, Interstate Highways play a crucial role in fast connectivity. For Austin, alternate routes almost have the same distance as the original shortest path which indicates disrupting the direct link to Austin would not affect overall travel time to that city.

6 PROPOSED SOLUTION

We propose a procedure to assess criticality of an edge based on the importance of the nodes it connects using local characteristics of the node and its connectivity with the remaining network. The nodes are evaluated using the variable centrality metric and is also indirectly used to find out critical links. Initial exploratory analysis with variable closeness centrality did not show favorable results, which led us to try Betweenness, as defined in Equation 3.

$$C_B^{log,\alpha}(u) = \log\left(\frac{C_B^w(u)}{C_B(u)}\right) * \alpha + \log(C_B(u)) \quad (3)$$

Variable Betweenness centrality $C_B^{log,\alpha}(u)$ depends on weighted betweenness centrality $C_B^w(u)$ which takes into account the impact of an edge and regular betweenness centrality $C_B(u)$ which considers connectivity of a given node with rest of the network.

We begin with calculating centrality metrics for the weighted and unweighted networks. Then we compute the variable centralities using Equation 3 by varying α for different views on the network based on the node's individual and global impact.

The nodes are ranked based on $C_B^{log,\alpha}(u)$, and the

top 900 are selected. We combine the different shortest path views as shown in section 5.2 and a spatial join is performed to check the percentage of nodes that lie on these edges in shortest paths between the cities. (Bröhl and Lehnertz, 2019) showcases the relationship between node-edge importance based on the centrality metrics. Their study shows that around 60% of the critical nodes ranked using betweenness centrality lie on the network’s shortest paths. Inspired by this approach, we choose the shortest paths to validate our critical nodes and also extract critical edges based on this approach. Attributes like path length, average daily traffic volume, maximum operating capacity are used as edge weights to calculate centrality metrics.

7 RESULTS

Not all edges within a network have the same level of importance. The statistics such as daily traffic volume and maximum operating capacity suggest that some roads are more important than others. We observed how ranking using different centrality metrics would change edge identification.

7.1 Complex Graph

Out of 12,118 nodes in our network, around 4,614 lie on the shortest paths between the selected 30 cities. The top 900 nodes in intervals of 100, are selected based on the centrality metrics. In each value range, we check the percentage of nodes lying on the shortest paths, which are shown in Figure 10. Between-

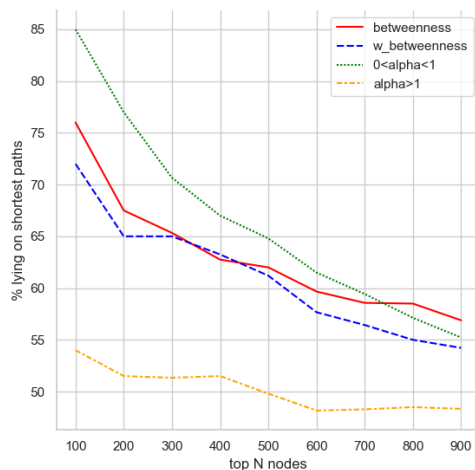


Figure 10: Plot showing percentage of nodes lying on Shortest Paths with respect to top 900 Nodes.

ness gives a higher rank to nodes with better local connectivity and the maximum number of shortest

paths passing through them irrespective of edge importance. Variable Centralities ($\alpha > 1, \alpha < 1$) show a more realistic output considering the entire network while evaluating the node.

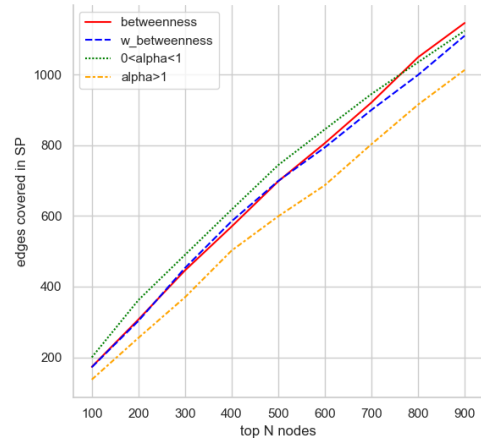


Figure 11: Edges covered when certain number of nodes are selected.

A plot to show edge coverage as ranking range (top 100 - 900 nodes) was increased is shown in Figure 11. From this plot, we can observe that the metric with a steeper line covers greater edges within the shortest paths. Increasing α makes variable centrality very selective about the nodes it chooses. This leads to the solution missing out on some edges with reasonable properties (as seen from Figure 12).

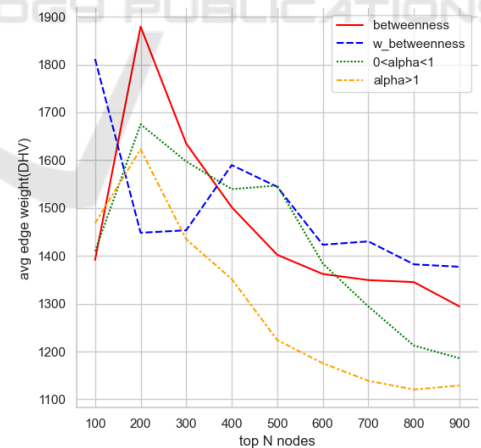


Figure 12: Average accumulated edge weight with respect to top 900 Nodes.

When no preference is given to weighted centrality, we fall back to our original Betweenness metric. This has less node and edge coverage due to main ranking factor being local connectivity. Overall, when $0 < \alpha < 1$, our metric performs the best, having respectable node, edge coverage, and reasonable average edge weights.

7.2 Simplified Graph

The simplified graph has 15,309 nodes and 16,662 edges. There are no multiple edges between any two pairs of nodes in this graph. We could not join the weights from the HPMS dataset to this graph for a similar metric analysis. Hence, we focus only on the topology and how it affected our results.

6,762 nodes lie on the shortest paths, and there exist 7,281 edges that belong to the shortest path among the 30 cities. From Figure 13, we can see that it is consistent with the definition of betweenness. All of

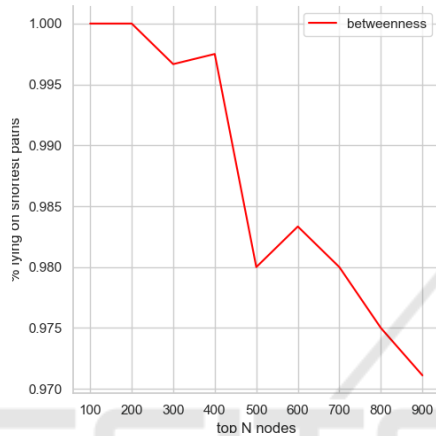


Figure 13: Percentage of nodes lying on Shortest Paths with respect to top 900 Nodes.

the top 200 nodes ranked by betweenness lie on the shortest paths. Similarly, if we look at Figure 14, the number of edges covered increases almost linearly as the range of nodes increases.

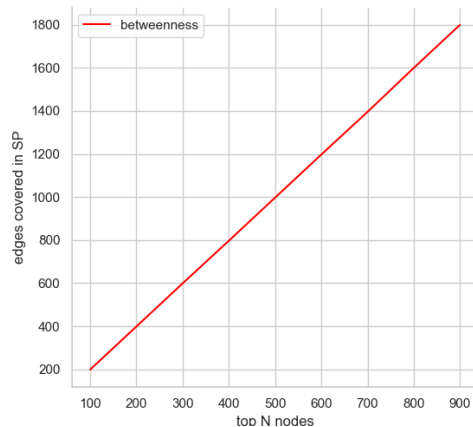


Figure 14: Edges covered in Shortest Paths with respect to top 900 Nodes.

Comparing the results of this graph with those in Section 7.1, we can see the effect that the topology has on the centrality and its subsequent effect on edge importance.

8 CONCLUSIONS

Identifying critical links within road networks is extremely important for transportation and planning. Our research has shown the impact of disabling major highway links on inter-city travels within a State. We also showcased a method to rank critical nodes based on variable centrality and extract respective edges originating or terminating on them under the assumption that edge criticality is directly dependent on node criticality. Below we list other insights from our research and the future direction.

Topology has had a considerable influence on the experiment results. The Complex graph had many illegal values for the variable centrality metric due to topological errors and disconnected components in several places. This made our approach very sensitive towards even the smallest error. In future research, we aim to experiment using more real-world road networks and develop ways to handle these errors. Our approach currently works by considering the network as a single mode network (Opsahl et al., 2010). We would like to extend our research by viewing it as a two mode network and testing variable centrality on it.

Inferring edge centrality in a complex network from node centrality is a challenging problem. The prevalent edge ranking algorithms need significant improvement to predict the shortest path links when the network is complex. Variable centrality ranking often (incorrectly) better ranks nodes not lying on the shortest paths. We can also infer that there exist other shortest paths with similar weighted distances but a different route within the network. Our future research would improve the variable centrality metric by considering additional graph properties covering the entire network. Our procedure used a metric similar to recall (Van Rijsbergen, 2004) for evaluating edge ranking algorithms that assess the number of correctly guessed links on the shortest path. Another assessment of interest would be precision (Van Rijsbergen, 2004) like metric, which would quantify the number of irrelevant links ranked higher than correct links. We are also interested in theoretically investigating the limitations of the existing ranking algorithms since they rely on very myopic information about the graph. We also would like to explore flow analysis within traffic networks and combine it with variable centrality to validate the results. Measuring traffic flow through extracted critical edges can be a good metric to determine criticality of those links.

ACKNOWLEDGMENT

This work is funded by Acquisition of an Advanced Traffic Management Systems for Cyber and Physical Security Research, UH High Priority Area Research Equipment Award, University of Houston, U.S.A.

REFERENCES

- Almotahari, A. and Yazici, A. (2020). Practice friendly metric for identification of critical links in road networks. *Transportation Research Record*, page 0361198120925475.
- Bhatia, U., Kumar, D., Kodra, E., and Ganguly, A. R. (2015). Network science based quantification of resilience demonstrated on the indian railways network. *PLoS one*, 10(11):e0141890.
- Bollobás, B. (2001). The evolution of random graphs—the giant component. In *Random graphs*, volume 184, pages 130–59. Cambridge university press Cambridge.
- Bröhl, T. and Lehnertz, K. (2019). Centrality-based identification of important edges in complex networks. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 29(3):033115.
- Bureau, U. C. (2020). Tiger/line shapefiles.
- ESRI, U. and PaperJuly, W. (1998). Esri shapefile technical description. *Comput. Stat*, 16:370–371.
- Furno, A., El Faouzi, N.-E., Sharma, R., Cammarota, V., and Zimeo, E. (2018). A graph-based framework for real-time vulnerability assessment of road networks. In *2018 IEEE International Conference on Smart Computing (SMARTCOMP)*, pages 234–241. IEEE.
- Gauthier, P., Furno, A., and El Faouzi, N.-E. (2018). Road network resilience: how to identify critical links subject to day-to-day disruptions. *Transportation research record*, 2672(1):54–65.
- Henning, S., Biemelt, P., Abdelgawad, K., Gausemeier, S., Evers, H. H., and Trächtler, A. (2017). Methodology for determining critical locations in road networks based on graph theory. *IFAC-PapersOnLine*, 50(1):7487–7492.
- Herrera, M., Abraham, E., and Stoianov, I. (2016). A graph-theoretic framework for assessing the resilience of sectorised water distribution networks. *Water Resources Management*, 30(5):1685–1699.
- Karagyozov, K., Razmov, T., Todorova, M., Varadinova, J., and Dzhaleva-Chonkova, A. (2012). Impact of natural disasters on transport systems—case studies from bulgaria. *Report from the International Panel of the WEATHER project funded by the European Commission's 7th framework programme. Sofia*.
- Li, F., Jia, H., Luo, Q., Li, Y., and Yang, L. (2020). Identification of critical links in a large-scale road network considering the traffic flow betweenness index. *PLoS one*, 15(4):e0227474.
- Mattsson, L.-G. and Jenelius, E. (2015). Vulnerability and resilience of transport systems—a discussion of recent research. *Transportation Research Part A: Policy and Practice*, 81:16–34.
- Opsahl, T., Agneessens, F., and Skvoretz, J. (2010). Node centrality in weighted networks: Generalizing degree and shortest paths. *Social networks*, 32(3):245–251.
- Passos, R. S. and Cardoso, D. O. (2020). Extrema analysis of node centrality in weighted networks. *arXiv preprint arXiv:2011.00429*.
- Portal, T. O. D. (2020). Texas cities.
- QGIS Development Team (2021). *QGIS Geographic Information System*. QGIS Association.
- Singh, P. S., Lyngdoh, R. B., Chutia, D., Saikhom, V., Kashyap, B., and Sudhakar, S. (2015). Dynamic shortest route finder using pgrouting for emergency management. *Applied Geomatics*, 7(4):255–262.
- Survey, U. G. (2014). National atlas of the united states.
- TxDOT (2019).
- Van Rijsbergen, C. J. (2004). *The geometry of information retrieval*. Cambridge University Press.