The Estimation of Sustainable Development Trajectories of the Regional Socio-economic System

Karolina Ketova[a] and Daiana Vavilova[b]
Department of Applied Mathematics and Information Technologies, Kalashnikov Izhevsk State Technical University, Studencheskaya Street, Izhevsk, Russia

Keywords: Regional Socio-Economic System, Economic-Mathematical Modeling, Investments, Sustainable Development Trajectories, Effective Volume of Labor Resources.

Abstract: The article is solved the problem of constructing sustainable development trajectories of the socio-economic system on the example one of the regions of the Russian Federation. The problem is formulated on the basis of a dynamic macro-model of the regional economy, where the development factors are productive capital and effective volume of labor resources. The effective volume of labor resources is a synergy of the quantity of the factor (the number of labor resources) and the quality of the factor (the employee's labor efficiency). The produced at the macro level product is allocated to investments in maintaining and expanding productive capital, labor resources, and consumption in the macroeconomic regional system. The inclusion of the factor of the effective volume of labor resources in the macro model is a distinctive feature of this management problem statement. The solving the problem algorithm is to build trajectories of sustainable economic development: the trajectories of balanced sustainable growth and the optimal trajectory of the socio-economic system, which put this system on the trajectory of balanced sustainable growth. The statistical base of the calculations is the demography data, the volumes of investment in the production and social-educational sphere of the Udmurt Republic. To solve the problem of identifying unknown parameters of the model, the period 2000-2019 is used. The optimal investment rates are calculated to allow the economic system to reach the trajectory of balanced sustainable growth by 2025. The proposed methodology can be used to build trajectories of sustainable development of socio-economic systems, as well as to conduct parametric model calculations to identify factors of sustainable economic growth.

1 INTRODUCTION

The stable growth of socio-economic system indicators in the region is laid down in the construction of the development strategy, which determines the amount of financing for the production and social spheres of activity. The construction of a development strategy should be carried out using formalized methods of economic and mathematical modeling. This approach to problem solving provides scientific results and allows its practical application.

In this article, search of sustainable development trajectories of the socio-economic system is carried out by the model presented in (Belenky and Ketova, 2006; Ketova, 2013), where it is considered as the initial regional model. A distinctive feature of this problem statement is the inclusion of the factor of the effective volume of labor resources as the leading factor involved in the creation of the final product.

Like many factors influencing the behavior of socio-economic systems, which are explicitly involved in the construction of development trajectories and do not have an initially formalized mathematical form and statistical content, the factor of effective volume of labor resources consists of quantitative and qualitative components. The quantitative component is the number of labor resources. The qualitative component is formed from the calculation of the efficiency of worker's labor.

Generalized gross produced product is distributed for consumption, investments in maintaining and expanding productive capital and investments aimed

[a] https://orcid.org/0000-0001-7143-1930
[b] https://orcid.org/0000-0002-2161-4402

Ketova, K. and Vavilova, D.
The Estimation of Sustainable Development Trajectories of the Regional Socio-economic System.
DOI: 10.5220/0010586901070115
In *Proceedings of the International Scientific and Practical Conference on Sustainable Development of Regional Infrastructure (ISSDRI 2021)*, pages 107-115
ISBN: 978-989-758-519-7

at developing the labor capital of the region (investments in the social and educational sphere).

Labor efficiency is an significant indicator for development of the socio-economic system. The Decree of Russian Federation's President of the "On the Strategy of Economic Security of the Russian Federation for the period up to 2030" was signed, according to which, among other things, it should be ensured to counteract modern challenges to the country's economic security, and prevent crisis phenomena in the resource and raw materials, production, scientific and technological and financial spheres. Among the statistical indicators, analyzing the degree of implementation of the Strategy, there is a category of "labor efficiency", which is reflected by the labor productivity index (Caron, Fally and Markusen, 2020).

The labor productivity is an indicator of the economic efficiency of labor potential and technologies (Shumilina and Tsvil, 2019; Jaume, 2021). Labor efficiency is also characterized the ability to create goods and services and reflects the level of well-being of the population. At the same time, high labor efficiency affects the potential of labor resources, formed for the future (Ketova, 2007; Ketova, Rusyak and Derendyaeva, 2013).

At present, there is no single coherent theory of assessment of the labor efficiency. A detailed analysis of this problem is presented in (Galiullin, Ermakov and Simonova, 2017; Tavani and Zamparelli, 2021), where several generalized points of view are accepted.

In many educational institutions on the economics of enterprise and labor, in academic dictionaries, the concept of "labor efficiency" is synonymous of the "labor productivity".

According to the International Labor Organization's methodology (ILO), labor productivity is the ratio of gross domestic product (GDP) to the total number of employees. According to the Organization for Economic Cooperation and Development's methodology (OECD), labor productivity is defined as the ratio of GDP to hours worked. The Russian Federal State Statistics Service calculates the labor productivity at the macro level as a result of dividing the GDP volume index by the change in total labor costs.

Another criterion for assessing labor efficiency is wages. Usually, it is the main source of income for most citizens. Wages reflect the level of well-being of the population (Aranzhin, 2019; Heil, 2020).

The issues of reasearching the relationship between labor productivity and its payment are considered in many scientific works (Smirnov and Sannikov, 2008; Jung and Lim, 2020). The growth in labor productivity must be accompanied by a proportional increase in wages. It improves economic efficiency indicators (Park and Rieu, 2020; Da Silveira and Lima, 2021).

The relationship between labor productivity and wages differs from region to region and from country to country. Also, the degree of interconnection of these indicators differs by the territorial regional aspect (Drobot, Makarov and Yarikova, 2019; Varlamova and Larionova, 2020; Laskiene, Pekarskiene and Kontautiene, 2021). So, generalized assessment of the level of wages is an objective characteristic of labor efficiency and allows using it in the model of socio-economic development of the regional socio-economic system of the Udmurt Republic (Ketova, 2013).

The quantitative component of the effective volume of labor resources is calculated as a result of solving the problem of modeling and forecasting demographic dynamics, which is presented in detail in (Belenky and Ketova, 2006; Rusyak and Ketova, 2008). The constructed forecast functions of the labor force and the total population of the region are explicitly introduced into the model of optimal management of the socio-economic system.

Generalized gross produced product is allocated for consumption, investments in maintaining and expanding productive capital and investments i effective volume of labor resources in the region.

The algorithm for solving the problem is to estimate trajectories of sustainable economic development.

The trajectory, along which the socio-economic system should move to achieve high results of economic development, is called the trajectory of balanced sustainable growth. Since the studied socio-economic system, as a rule, is not initially located on it, then it must reach this trajectory of sustainable development. To do this, we construct the trajectory, called the trajectory of the optimal trajectory of movement of a specific socio-economic system, which put this system onto the trajectory of balanced sustainable growth.

Solving the problem of estimating sustainable development trajectories of socio-economic systems, taking into account such an indicator as labor efficiency, is great practical importance. The solution of this problem is made on the example of the regional socio-economic system – the Udmurt Republic (UR).

2 RESEARCH MATERIALS AND METHODS

2.1 Problem Definition

The following main provisions are adopted.

1. At the macro level, when modelling the economic dynamics of the region, we consider generalized indicators: gross regional product Y (GRP), consumption C, productive capital K (PC) investment in the PC I, the effective size of the labour resources Z, investment in workforce development in the region E.

2. We distinguish two population groups: the total population of the region $P(t)$ and the working population $L(t)$ (produce GRP). The ratio of these two population groups $\lambda = L/P \in (0,1)$, because $0 < L(t) < P(t)$. The curves $P(t)$ and $L(t)$ are obtained as a result of solving the problem of demographic dynamics and are introduced into the model in an exogenous way. The ratio is calculated using the formula:

$$\lambda(t) = \frac{L(t)}{P(t)} = \frac{\left(\int_0^{\tau_l} \varepsilon_m(t,\tau)\rho_m(t,\tau)d\tau + \int_0^{\tau_l} \varepsilon_w(t,\tau)\rho_w(t,\tau)d\tau\right)}{\int_0^{\tau_l} \rho(t,\tau)d\tau} \quad (1)$$

В формуле (1) $\rho(t,\tau)-$ age distribution function of the population τ per year t (density), $\varepsilon(t,\tau)-$ percentages of people age τ, which participate in the production of GRP per year t, $\tau_l -$ lifetime.

The subscripts of $\varepsilon(t,\tau)$ and $\rho(t,\tau)$ make it possible to divide into women and men to clarify the accuracy of the calculations, since the time for leaving the economically active age is still different here.

3. The effective volume of labor resources $Z(t)$ depends on the average efficiency of one employee $z(t)$ and the number of labor resources: $Z(t) = L(t)z(t)$.

4. The management problem is considered in continuous time with a finite planning interval $[t_0,\ t_T]$; δ – discount coefficient.

5. The phase variables of the model are the productive capital K and the efficiency of one employee $z(t)$, change in time is described by the equations: $\dot{K} = I - \eta_k K$ and $\dot{z} = \lambda \frac{E}{L} - \eta_z Z$; η_k and η_z are the coefficients of retirement of the relevant factors of production.

6. The volume of output is determined by the production function $Y = F(K,Z)$. It is an upward convex function, monotonically increasing in each variable, as well as a linearly homogeneous function: $F(K,Z) = LF(K/L,\ Z/L) = LF(k,z)$. Here $k = K/L$ and $z = Z/L$ are unit (per worker) values of productive capital and the effective volume of labor resources, respectively.

7. There is a distribution of the produced product $Y = C + I + E$ into 3 parts for every year $\forall t \in [t_0, t_T]$: investment I, E in production factors K, Z, respectively, and consumption C (Figure 1).

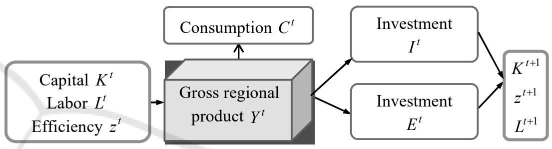

Figure 1: Economy reproduction cycle diagram.

8. In the socio-economic system $\forall t \in [t_0, t_T]$ a minimum level of consumption $C_{\min}$ must be maintained, which means that $C = \overline{C} + C_{\min}$. Then $\overline{Y} := Y - C_{\min}$, $\overline{F}(K,Z) := F(K,Z) - C_{\min}$, and there are natural restrictions of the problem $0 < C_{\min} < F(K,Z);\ \overline{Y} = \overline{C} + I + E \geq 0,\ \overline{C}, I, E \geq 0$.

9. Management in the socio-economic system is carried out according to the vector $s = (s_c, s_k, s_z)$, where are $s_c = C/\overline{Y} -$ the consumption rate, $s_k = I/\overline{Y} -$ the investment rate in K, $s_z = E/\overline{Y} -$ the investment rate in E, and $s_c + s_k + s_z \leq 1$.

10. Differential equations for phase variables taking into account $s_c = C/\overline{Y}$, $s_k = I/\overline{Y}$, $s_z = E/\overline{Y}$ and due to the transition from absolute values to unit ones $k = K/L$, $\overline{y} = \overline{Y}/L$, $c_{\min} = C_{\min}/P$, $\overline{y} = \overline{Y}/L = \overline{F}(K,Z)/L = \bar{f}(k,z) = f(k,z) - c_{\min}/\lambda$, take the form: $\dot{k} = s_k f(k,z) - \tilde{\eta}_k k,\ \tilde{\eta}_k = \eta_k + (\dot{L}/L)$, $\dot{z} = s_z f(k,z) - \eta_z z$. Initial and final states of the system are describes by formulas: $k(t_0) = k_0,\ \ z(t_0) = z_0;\ k(t_T) = k_T,\ \ z(t_T) = z_T$.

Moreover $k_T = k^*(t_T),\ z_T = z^*(t_T)$, where are

k^*, z^* – the values of variables on the trajectory of sustainable balanced growth. If during the planning period T, the socio-economic system manages to enter the trajectory of sustainable balanced growth, then it remains on it until the end of the period T.

11. The criterion of optimality management is the unit (per person) discounted maximum consumption accumulated over the entire planning period $[t_0, t_T]$:

$$Cr = \int_{t_0}^{t_T} \left(s_c \bar{f}(k,z)\lambda + c_{\min}\right) e^{-\delta(t-t_0)} dt \to \max_{s\in\Omega} \qquad (2)$$

The set of admissible management has the form:

$$\Omega = \left\{ (s_l) = (s_k, s_z) : s_l \in [0,1], \sum_l s_l = 1 - s_c \right\} \qquad (3)$$

12. Information passport of the problem (initial information) has the form: $\{\tilde{\eta}_1, \eta_2, \bar{f}, c_{\min}, \lambda, \delta, T\}$.

The formulated problem is the task of estimating trajectories of sustainable development by the socio-economic system, taking into account the effective volume of labor resources.

2.2 Solving Problem Algorithm

We talk about two parts of the optimal trajectory of the system. The first section is a transition period until a stable growth trajectory is reached, the second section is movement along this trajectory. The studied model belongs to the class of RKK models of economic dynamics. The RKK-model is a model based on the ideas of F. Ramsey, D. Kass and T. Koopmans, presented in the works (Ramsey, 1928; Cass, 1965; Koopmans, 1965). An adapted form of models of this class can be found in (Belenky, 2007; Ketova, 2013).

The solving problem algorithm is to estimate trajectories of sustainable economic development. This is the optimal trajectory of the socio-economic system, which brings the system onto the second trajectory is trajectory of balanced sustainable growth.

Let us denote a vector of phase variables is $x = (k, z)$, a vector of dual variables is $\psi = (\psi_k, \psi_z)$, a vector of management variables is $s = (s_c, s_k, s_z)$. The Hamiltonian $H(\psi, s, x, t)$ of this problem has the form:

$$\begin{aligned} H(\psi,s,x,t) &= \left[(1 - s_k - s_z)\bar{f}(k,z)\lambda + c_{\min}\right]e^{-\delta t} \\ &+ \psi_k \left[s_k \bar{f}(k,z) - \tilde{\eta}_k k\right] + \psi_z \left[s_z \bar{f}(k,z) - \eta_z z\right] \end{aligned} \qquad (4)$$

We ignore the existence of phase restrictions. The above condition $0 < c_{\min} < \lambda f$ allows the socio-economic system to be in the area where $\bar{f}(k,z) > 0$.

The necessary conditions of the Pontryagin maximum principle (Pontryagin, 1961; Intriligator, 2002) in relation to the problem (items 1-12) are as follows:

- For each fixed $t \in [0, T]$:

$$\begin{aligned} s_k(t) &= \arg\max_{(s_k,s_z)\in\Omega} H(\psi,s,x,t) \\ s_z(t) &= \arg\max_{(s_k,s_z)\in\Omega} H(\psi,s,x,t) \end{aligned} \qquad (5)$$

- Dual variables must satisfy the system of differential equations:

$$\dot{\psi}_k = -\frac{\partial H}{\partial k}, \ \dot{\psi}_z = -\frac{\partial H}{\partial z} \qquad (6)$$

- Phase variables must satisfy systems of differential equations with boundary conditions:

$$\begin{aligned} \dot{k} = \frac{\partial H}{\partial \psi_k}, \ \dot{z} = \frac{\partial H}{\partial \psi_z}, \\ k(0) = k_0, \ \ z(0) = z_0; \ k(T) = k_T, \ \ z(T) = z_T \end{aligned} \qquad (7)$$

Introducing a replacement for the vector of dual variables is $\pi = (\pi_k, \pi_z)$

$$\pi_k = \psi_k e^{\delta \cdot t}, \ \pi_z = \psi_z e^{\delta \cdot t}, \qquad (8)$$

we transform (4) to the form:

$$\begin{aligned} H(\pi,s,x,t) &= \left[(1 - s_k - s_z)\bar{f}(k,z)\lambda + c_{\min}\right]e^{-\delta t} + \\ &+ \pi_k e^{-\delta t}\left[s_k \bar{f}(k,z) - \tilde{\eta}_k k\right] + \\ &+ \pi_z e^{-\delta t}\left[s_z \bar{f}(k,z) - \eta_z z\right] \end{aligned} \qquad (9)$$

Condition (5) in relation to (9) is written as:

$$\arg\max_{\bar{s}\in\Omega} s_k(\pi_k - \lambda) + s_z(\pi_z - \lambda) \qquad (10)$$

whence we define the management parameters – $s_k(t)$ and $s_z(t)$.

The quantity present in (10) is a function of time, therefore takes place quasi-stationary nature of the optimal trajectory manifests itself.

The system of equations (6), taking into account the replacement (8), can be written in the formula (11):

$$\begin{cases} \dot{\pi}_k = (\delta + \tilde{\eta}_k)\pi_k - \\ \qquad - [(1 - s_k - s_z)\lambda + s_k \pi_k + s_z \pi_z] f_k^{/}(k, z), \\ \dot{\pi}_z = (\delta + \eta_z)\pi_z \\ \qquad - [(1 - s_k - s_z)\lambda + s_k \pi_k + s_z \pi_z] f_z^{/}(k, z) \end{cases} \tag{11}$$

and system (7) takes the form:

$$\begin{cases} \dot{k} = s_k \bar{f}(k, z) - \tilde{\eta}_k k, \\ \dot{z} = s_z \bar{f}(k, z) - \eta_z z. \end{cases} \tag{12}$$

The trajectory of balanced sustainable growth is determined from the conditions:

$$\pi_k = \pi_z = \lambda, \tag{13}$$

$$\dot{\pi}_k = \dot{\pi}_z = \dot{\lambda}, \tag{14}$$

Substituting (13), (14) in (11), (12), we find the parameters of trajectories of balanced sustainable growth k^*, z^*, s_k^*, s_z^*:

$$\begin{cases} f_k^{/}(k^*, z^*) = \delta + \tilde{\eta}_k - (\dot{\lambda}/\lambda), \\ f_z^{/}(k^*, z^*) = \delta + \eta_z - (\dot{\lambda}/\lambda), \\ s_k^* = (\tilde{\eta}_k k^* + \dot{k}^*) / \bar{f}(k^*, z^*), \\ s_z^* = (\eta_z z^* + \dot{z}^*) / \bar{f}(k^*, z^*). \end{cases} \tag{15}$$

To estimate the optimal trajectory of the socio-economic system until the trajectory of balanced sustainable growth is reached, nonstationary equations (11) and (12) are used, which are solved in reverse time using the numerical modified Euler method with correction (Kalitkin, 2011). Based on the initial values of the variables $k(t_0) = k_0$, $z(t_0) = z_0$, when problem solving the time point t^* is selected at which the optimal trajectory of the socio-economic system enters the trajectory of balanced sustainable growth. In parallel, when solving, the values of the variables $\{\pi_k, \pi_z\}_t$ and $\{s_k, s_z\}_t$ are restored. At the final stage, the problem of optimal distribution of investments is solved in a straightforward way.

3 RESULTS OF ESTIMATION SUSTAINABLE DEVELOPMENT TRAJECTORIES OF THE REGIONAL SOCIO-ECONOMIC SYSTEM ON THE EXAMPLE OF THE UDMURT REPUBLIC

Statistical data of socio-economic indicators of UR are presented on the website of the Federal State Statistics Service (http://www.gks.ru) in the Official Statistics section (subsection “National Accounts”, subsection “Population” and subsection “Entrepreneurship”) and on the website of the Federal Treasury (http://www.roskazna.ru) in the Budget Execution section.

Table 1 and Table 2 present statistical data of UR for the period 2000-2019 for all macroeconomic indicators in comparable values, that are involved in the model and are described in the first part of this article.

The parameters, presented in the information passport of the optimal management problem of the socio-economic system of the region are to be determined. They were calculated for the period 2000-2019 based on statistical data on UR (economic indicators are given to comparable prices in 2019) in accordance with the algorithm for identifying unknown parameters (Ketova and Rusyak, 2009). The following values are obtained: $\tilde{\eta}_k = 0.03$, $\eta_z = 0.07$, $Y = F(K, Z) = 0.72 K^{0.54} Z^{0.46}$. Discount factor is $\delta = 0.05$, planning period is $T = 10$ years, $c_{\min} = 0.4Y$.

Table 1: Macroeconomic indicators of the UR for the period 2000-2019, measured in billion rubles, are expressed in current year prices.

t	Y	K	Z	I	E	C
2000	53	184	192	10	5	38
2001	65	221	248	12	6	47
2002	78	255	263	15	9	54
2003	89	279	283	18	11	60
2004	100	315	287	22	14	65
2005	140	368	346	27	20	93
2006	165	394	403	34	27	103
2007	206	484	417	45	40	121
2008	243	553	488	54	45	145
2009	231	592	455	41	45	145
2010	275	650	434	51	48	175
2011	336	697	432	62	56	218
2012	373	817	421	64	70	238
2013	405	870	435	83	80	242
2014	450	975	371	92	84	275
2015	518	1041	358	82	90	346
2016	532	1170	353	87	92	352
2017	552	1246	336	84	89	380
2018	631	1363	315	97	103	432
2019	694	1458	346	101	110	464

Table 2: Social indicators of the UR for the period 2000-2019, measured in thousands of people.

t	P	L
2000	1591.8	809.0
2001	1583.1	799.8
2002	1573.2	821.2
2003	1564.6	808.3
2004	1557.7	803.0
2005	1550.1	823.0
2006	1542.2	846.5
2007	1535.8	838.4
2008	1530.6	839.4
2009	1526.7	854.8
2010	1522.8	831.9
2011	1519.2	837.7
2012	1517.9	829.7
2013	1517.4	828.4
2014	1517.3	822.8
2015	1517.3	820.5
2016	1517.0	806.5
2017	1514.9	788.7
2018	1510.2	781.2
2019	1505.2	788.6

For determining the predicted share of the economically active population of the UR in the total population, forecast of their dynamics is built for the period 2020-2035, based on the results of solving the problem of demographic dynamics (Ketova, 2013). The size of the general and economically active population groups of the UR are presented in Figure 2, a, the dynamics of the share of the economically active population is shown in Figure 2, b.

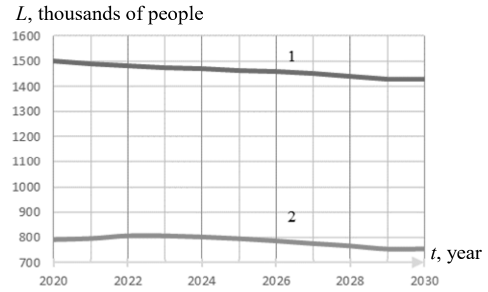

Figure 2, a: Forecast dynamics of the population of UR for the period 2020-2035: total number (1), economically active population (2).

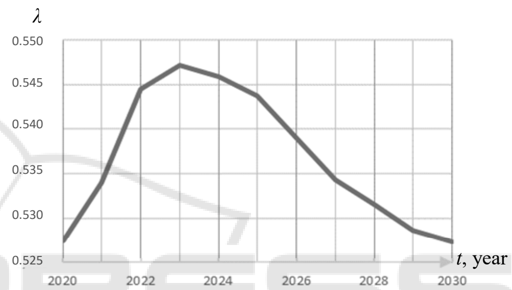

Figure 2, b: Forecast dynamics of the economically active population share in the total population of the UR for the period 2020-2035.

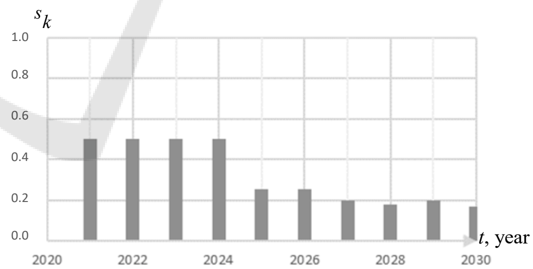

Figure 3, a: Change management parameter s_k .

Figures 3, a, b and Figures 4, a, b show some results of solving the problem of constructing trajectories of sustainable economic growth in the region. Calculations were made using comparable data for 2019. The socio-economic system reaches the trajectory of balanced sustainable growth and remains on it thanks to the management strategy (10).

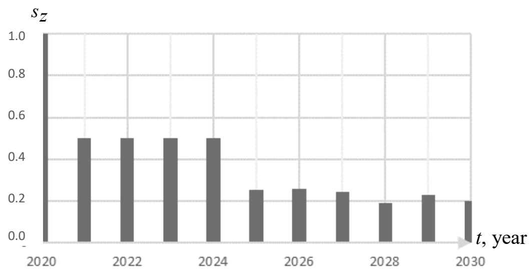

Figure 3, b: Change management parameter s_Z .

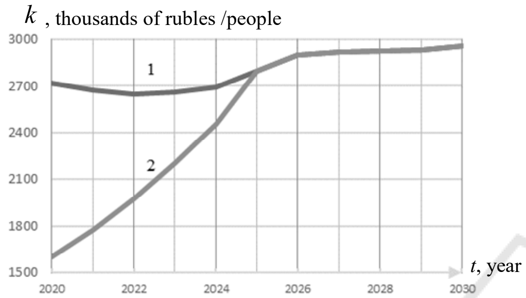

Figure 4, a: Phase coordinate trajectories k : optimal trajectory of change (1), trajectory of balanced sustainable growth (2).

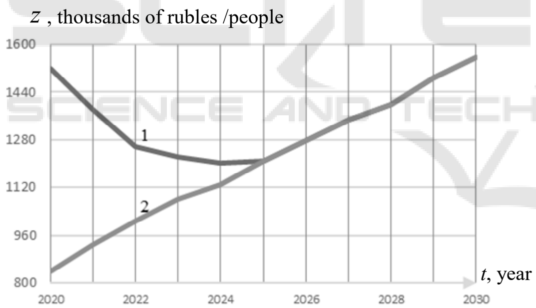

Figure 4, b: Phase coordinate trajectories z : optimal trajectory of change (1), trajectory of balanced sustainable growth (2).

Initially, the actual unit level (per employee) of the value of productive capital is above the value on the trajectory of sustainable balanced growth, which we should achieve (Fig. 4, a), the unit level of the value of the effective volume of labor resources is below the optimal values (Fig. 4, b). At the initial moment of time, the capital-labor ratio is closer to the sustainable development trajectories than labor efficiency. Implementing the strategy of managing the socio-economic system (10), we initially build up the factors farthest from the trajectory of sustainable balanced growth. At first, there is an increase in the share of investments in the development of the labor potential of the region.

In 2025, there is an exit of the socio-economic system on the sustainable balanced growth trajectory. A reduction in capital investment in factors of production begins. There is an opportunity to increase the consumption component.

Thus, as calculations are shown, in the implementation of the scenario of optimal management, the productive capital at the first stage decreases, which is explained by the need to withdraw obsolete assets that have low productivity and incur large material costs for maintenance. This policy opens up the opportunity to increase labor efficiency by 1.7 times by 2025. Since 2023, there has been an increase in productive capital. The optimal distribution of investments between the production and social spheres of the region makes it possible to increase the unit gross regional product by 2030 by 1.94 times (Figure 5, a).

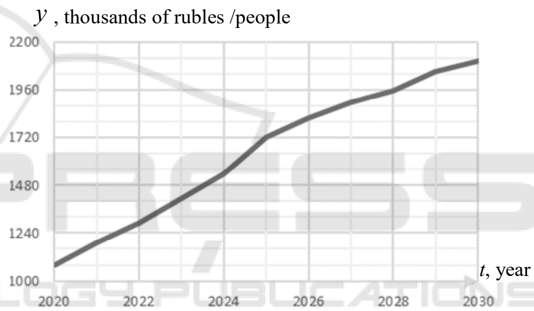

Figure 5, a: Dynamics of the unit value of gross regional product for the planning period 2020-2030.

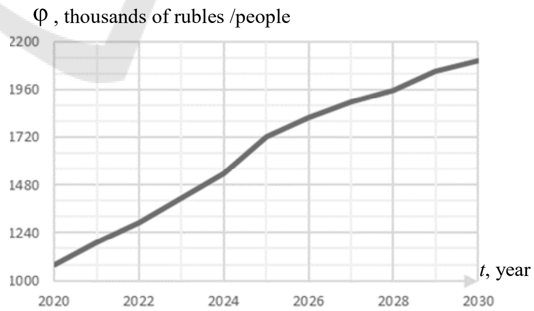

Figure 5, b: Dynamics of the accumulated unit consumption for the planning period 2020-2030.

Let's consider the time period, when the socio-economic system is located upon the trajectory of balanced sustainable growth. We see a significant increase of GRP.

This state of affairs is due to the fact that in the first years the basic production assets are actively increased and funds are invested in the labor resources of the region. Consumption in the socio-economic system is kept to a minimum. When the system reaches the trajectory of balanced sustainable growth, the rate of increase in labor productivity $y(t)$ decreases due to a decrease in the rate of growth of factors of production. The consumption in the system is increasing, unit value is $c_{\min} = 92$ thousand rubles in year (in prices of 2019).

The criterion functional (accumulated consumption per inhabitant of the region), reaches 2026 thousand rubles by the end of the planning period. (Figure 5, b).

Annual consumption is not constant over the entire planning horizon. So, until both phase coordinates enter the trajectory of sustainable balanced growth (2025), consumption is constant and amounts to 92 thousand rubles per person in year. The period of movement towards the trajectory of sustainable balanced growth is characterized by a significant share of deductions from GRP to production factors. Further, we have the opportunity to increase the share deducted for consumption (about 60% of the GRP). This ensures a rapid increase in the specific welfare.

4 CONCLUSIONS

In sum, the sustainable development trajectories of the regional socio-economic system are estimated on the example of the Udmurt Republic. The trajectory of balanced sustainable growth and the optimal trajectory of movement of the socio-economic system, which brings this system to the trajectory of balanced sustainable growth, have been constructed. The calculations are based on statistical data for the period 2000-2019. The forecast is carried out until 2030 inclusive.

The presented in the article algorithm for the construction of sustainable socio-economic development trajectories allows solving the problem of determining the optimal proportions between investments in production factors and consumption. The results of solving the problem on the example of a specific regional system made it possible to conclude that an effective balanced increase in the rate of well-being is the result of the harmonious development of productive capital and social capital. It is necessary to invest in the efficiency of the region's labor resources on a par with productive capital. These attachments must be of the same order.

As a result of solving the management problem on the example of the socio-economic system of the Udmurt Republic, the optimal values of the region's macroeconomic indicators are obtained. It is shown that the system can reach a balanced trajectory of sustainable economic growth with the implementation of the optimal control scenario by 2025, which will make it possible to increase the GRP by almost 2 times. It was revealed that at this stage, the priority is the development of the factor of efficiency of labor resources of the region, which allows achieving the fastest growth of economic indicators.

REFERENCES

Aranzhin, V. V. (2019). The relationship of wages and productivity: trends in the conditions of economy digitization. In *Russian journal of labor economics.* Vol. 6. No. 1. DOI:10.18334/et.6.1.39938. (in Russ.).

Belenky, V. Z., Ketova. K. V. (2006). The complete analytical solution of the macro-model of regional development for the exogenous demographic prognosis. In *Economics and Mathematical Methods.* Vol. 42. No. 4. (in Russ.).

Belenky, V. Z. (2007). *Optimization models of economic dynamics. Conceptual apparatus. One-dimensional models.* Nauka. Moscow. (in Russ.).

Caron, J., Fally, T., Markusen. J. (2020). Per capita income and the demand for skills. In *Journal of International Economics*, 123, 103306.

Cass, D. (1965). Optimum Growth in an Aggregative Model of Capital Accumulation. In *Rev. of Econ. Studies*. Vol. 32.

Da Silveira, J. J., Lima, G. T. (2021). Wage inequality as a source of endogenous macroeconomic fluctuations. *Structural Change and Economic Dynamics*, No. 56.

Drobot, E. V., Makarov, I. N., Yarikova, E. V. (2019). Spatial development of Russia: problems of differentiation in the conditions of globalization. In *Journal of International Economic Affairs*, Vol. 9. No. 4. DOI:10.18334/eo.9.4.41347. (in Russ.).

Galiullin, Kh. Ya., Ermakov, G. P., Simonova, M. V. (2017). Concept of labor efficiency. In *Russian journal of labor economics,* Vol. 4. No. 3. DOI:10.18334/et.4.3.38263. (in Russ.)

Heil, M. (2020). How does finance influence labour market outcomes? A review of empirical studies. In *Journal of Economic Studies*, No. 47(6).

Intriligator, M. (2002). *Mathematical methods of optimization and economic theory,* Iris press. Moscow, (in Russ.).

Jaume, D. (2021). The labor market effects of an educational expansion. In *Journal of Development Economics*, No. 149, 102619.

Jung, J. H., Lim, D. G., (2020). Industrial robots, employment growth, and labor cost: A simultaneous equation analysis. In *Technological Forecasting and Social Change*, No. 159, 120202.

Kalitkin, N. N. (2011). *Numerical methods*. Nauka. Moscow. (in Russ.).

Ketova, K. V. (2007). A mathematical economic model of the manpower resource potential and cost characteristics of demographic losses. In *Expert Syst. Appl,* No. 3 (7). (in Russ.).

Ketova, K. V., Rusyak, I. G. (2009). Identification and forecast of generalized indicators of regional economic system development. In *Applied Econometrics,* No. 3.

Ketova, K. V. (2013). *Mathematical Models of Economic Dynamics*, IStU publishing house, Izhevsk. (in Russ.).

Ketova, K. V., Rusyak, I. G., Derendyaeva, E. A. (2013). Solution of the problem of optimum control regional economic system in the conditions of the scientific and technical and social and educational progress. In *Mathematical modelling*, No. 10 (25). (in Russ.).

Koopmans, T. C. (1965). On the Concept of Optimal Economic Growth. In *Pontificae Academiae Scientiarum Scripta Varia*, Vol. 28.

Laskiene, D., Pekarskiene, I., Kontautiene, R., (2021). Regional income inequality in Lithuania. In *Economy of Region*, No. 16 (4).

Park, H. W., Rieu, D. M., (2020). A Mathematical Formulation of the Dual Nature of Unproductive Labor. In *Review of Radical Political Economics*, No. 52.

Pontryagin, L. S. (1961). Mathematical theory of optimal processes, *Nauka*, Moscow. (in Russ.).

Ramsey, F. P. (1928). A Mathematical Theory of Saving. In *Econ. J.,* Vol. 38, No. 152.

Rusyak, I. G., Ketova, K. V. (2008). Analysis of Demographic Losses Economic Characteristics. In *Tomsk State University Journal,* No. 310. (in Russ.).

Shumilina, V. E., Tsvil, M. M. (2019). Statistical modeling and forecasting of the index of labor productivity in the Russian Federation. In *Bulletin of Eurasian Science*, No. 1. (in Russ.).

Smirnov, M. A., Sannikov, O. V. (2008). Labor productivity, wages and economic efficiency of labor. In *Living standards of the population of Russian regions*, No. 2. (in Russ.).

Tavani, D., Zamparelli, L., (2021). Labor-augmenting technical change and the wage share: new microeconomic foundations. In *Structural Change and Economic Dynamics*, No. 56.

Varlamova. J., Larionova. N. (2020). Labor Productivity in the Digital Era: A Spatial-Temporal Analysis. In *International Journal of Technology*, No. 11 (6).