Effects of Random Delay on Travel Behavior of Subway Commuters during Peak Hour based on Equilibrium Models

Jianan Cao[a]
Beijing Jiaotong University, China

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Abstract: This paper discusses the influence of random delay on the travel behavior of subway commuters in the peak hour. We consider the situation that commuters need to take bus to complete the rest of the journey after getting off the subway. We assume that the buses have random delay T that follows a uniform distribution in the range $0 \le T \le b$. And it is found that T has an impact on the commuting model. It is shown that with the increase of b, three different scenarios emerge. The start time and end time of the peak hour, the expected value of travel cost, and the queuing time have been derived under the three scenarios. It is shown that the start time of rush hour monotonically decreases (i.e., the start time becomes earlier and earlier) and the travel cost monotonically increases with the increase of b.

1 INTRODUCTION

Subway is becoming more and more popular among people, especially commuters, as a comfortable and punctual means of transportation. However, as the number of commuters increases, the subway has become increasingly congested during the rush hour. Since Vickrey proposed the classic bottleneck model to characterize the commute behavior (Vickrey 1969), many extensions and applications of the model have been made (Arnott 1990, Laih 2004, Lindsey 2012), considering, e.g., elastic demand and general queuing networks (Braid 1989, Arnott 1993, Yang 1998), the uncertainty of road bottleneck capacity (Xiao 2015, Zhu 2019).

The bottleneck model has also been used to study the subway commuting behavior. For example, Kraus and Yoshida (Kraus 2002) investigated the optimal fare and service frequency to minimize the long-term system cost. Yang and Tang (Yang 2018) proposed a fee feedback mechanism to manage the passenger flow during peak hours and minimize the system cost while ensuring the same revenue for the authorities.

However, in the vast majority of cases, the subway does not go directly to a commuter's place of work. Passengers often need to take a bus to reach workplace after getting off the subway. During the morning rush hour, a random delay of buses is very common. Motivated by the fact, this paper studies the impact of random delay on commuters' travel behavior and travel cost.

The paper is organized as follows. Section 2 introduces the subway bottleneck model considering random delay and derives the commuter travel cost in user equilibrium. Section 3 discusses the impact of random delay on commuters' departure time choice and travel cost. Section 4 summarizes the paper.

2 THE TRAFFIC BOTTLENECK MODEL CONSIDERING RANDOM DELAY

2.1 Symbol Definition

α : the unit cost of queuing time

β : the unit cost of early arrival

γ : the unit cost of late arrival

ε : the unit cost of random delay on the bus

$q(m)$: Queuing time of taking the m th train

$e(m)$: Early arrival delay for commuters on the m th train

[a] https://orcid.org/0000-0003-1000-9538

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$l(m)$: Late arrival delay for commuters on the m th train

t_m : The moment the m th train arrives at the destination station

M : Total number of trains ($M \geq 2$)

N : Total number of commuters

s : Capacity of a train

L: Length of the peak period

h: train departure interval

T: random delay on the bus

b: Maximum random delay($b > 0$)

t^* : Commuter's work starting time

m_1 : The last train that commuters must arrive early

m_2 : The first train that commuters must be late

$E[C(t_m)]$: The expected travel cost of commuters taking train m

TTC: Expected cost of the system

AEC: Expected travel cost of the commuters

p_0 : Uniform fare on the subway

p_1 : The bus fare

2.2 Introduction of the Model

Assuming that an urban rail line connects a single origin and destination, there will be a bottleneck in the early rush hour. Every day there are a total number of N passengers who take M trains during the morning rush hour. Train departure interval is h, and each train capacity is s. Due to the limit capacity of the trains, the station becomes a bottleneck and passengers need to wait at the station. We denote the commuter's work starting time as t^*, and the time when each train arrives at the destination station as $t_m, m = 1,2,3,\ldots M$, thus the length of the morning rush hour is $L = (M-1)h$, see Figure 1.

We assume that after getting off the subway, the commuters take a bus to the workplace. The traveling time of the bus is set to T_0+T, where T_0 is free traveling time and T is random delay. Without loss of generality, we set $T_0=0$. Moreover, it is assumed that T follows a uniform distribution in the range $0 \leq T \leq b$.

In this model, a passenger on the m th subway will encounter a queuing time on the station $q(m)$, a uniform fare on the subway p_0, the bus fare p_1, a random delay T, an early arrival time $e(m)$ or a late arrival time $l(m)$. His/her total travel cost can be expressed as follows:

$$c(m) = \alpha q(m) + \beta e(m) + \gamma l(m) + \varepsilon t(m) + p_0 + p_1 \quad (1)$$

Here α, β, γ, ε are the unit cost of queuing time at the station, arriving early, arriving late and random delay on the bus, respectively. It is assumed that $\beta < \alpha < \varepsilon < \gamma$.

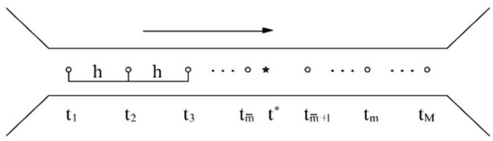

Figure 1: Bottleneck model of subway in rush hours.

2.3 Three Scenarios

In user equilibrium, commuters on the first train and the last train do not encounter queues at the subway station, and commuters on each train have the same expected travel cost. Commuters on the m th train may face three possible arrival states: always arrive early, always arrive later, and arrive either early or late. The expected travel cost in the three states is as follows:

Expected travel cost for commuters who always arrive early:

$$\begin{aligned} E[C(t_m)] &= -\int_0^b \beta(t_m + T)\frac{1}{b}dt + \int_0^b \varepsilon\frac{T}{b}dt \\ &+\alpha q(m) + p_0 + p_1 \\ &= -\beta t_m - \beta\frac{b}{2} + \varepsilon\frac{b}{2} + \alpha q(m) + p_0 + p_1 \end{aligned} \quad (2)$$

Expected travel cost for commuters who always arrive late:

$$\begin{aligned} E[C(t_m)] &= \int_0^b \gamma(t_m + T)\frac{1}{b}dt + \int_0^b \varepsilon\frac{T}{b}dt \\ &+\alpha q(m) + p_0 + p_1 \\ &= \gamma t_m + \gamma\frac{b}{2} + \varepsilon\frac{b}{2} + \alpha q(m) + p_0 + p_1 \end{aligned} \quad (3)$$

Expected travel cost for commuters who arrive either early or late:

$$E[C(t_m)] = -\int_0^{-t_m} \frac{\beta}{b}(t_m+T)dt + \int_{-t_m}^{b} \frac{\gamma}{b}(t_m+T)dt$$
$$+\int_0^b \frac{\varepsilon}{b}Tdt + \alpha q(m) + p_0 + p_1 \quad (4)$$
$$= \frac{\beta+\gamma}{2b}t_m^2 + \gamma t_m + \frac{b}{2}(\varepsilon+\gamma) + \alpha q(m) + p_0 + p_1$$

3 IMPACT OF RANDOM DELAY ON COMMUTERS' BEHAVIOR

In equilibrium state, the value of maximum delay b has a significant effect. With the increase of b, there emerge three scenarios.

3.1 Scenario 1

when $0 \le b < \frac{2\beta}{\beta+\gamma}(M-1)h$, Scenario 1 emerges. In Scenario 1, commuters on train $1 \sim m_1$ always arrive early, commuters on train $m_2 \sim M$ always arrive late, and commuters on train $m_1+1 \sim m_2-1$ may arrive either early or late. The schematic diagram of Scenario 1 is shown in Fig.2.

In order to simplify the calculation, t^* is set as 0 in this paper. In Scenario 1, the peak starts and ends at:

$$\begin{cases} E[C(t_1)] = E[C(t_M)] \\ t_M - t_1 = (M-1)h \end{cases} \rightarrow \begin{cases} t_1 = -\frac{\gamma}{\beta+\gamma}(M-1)h - \frac{b}{2} \\ t_M = \frac{\beta}{\beta+\gamma}(M-1)h - \frac{b}{2} \end{cases} \quad (5)$$

In Scenario 1, m_1 is the last train that commuters must arrive early, m_2 is the first train that commuters must be late. The range of m_1, m_2 can be expressed as follows:

$$\begin{cases} t_1 + (m_1-1)h + b < 0 \\ t_1 + m_1 h + b > 0 \\ t_1 + m_1 h < 0 \end{cases} \rightarrow \begin{cases} m_1 > \frac{\gamma}{\beta+\gamma}(M-1) - \frac{b}{2h} \\ m_1 < \frac{\gamma}{\beta+\gamma}(M-1) - \frac{b}{2h} + 1 \\ m_1 < \frac{\gamma}{\beta+\gamma}(M-1) + \frac{b}{2h} \end{cases} \quad (6)$$

$$\begin{cases} t_1 + (m_2-2)h < 0 \\ t_1 + (m_2-2)h + b > 0 \\ t_1 + (m_2-1)h > 0 \end{cases} \rightarrow \begin{cases} m_2 > \frac{\gamma}{\beta+\gamma}(M-1) + \frac{b}{2h} + 1 \\ m_2 > \frac{\gamma}{\beta+\gamma}(M-1) - \frac{b}{2h} + 2 \\ m_2 < \frac{\gamma}{\beta+\gamma}(M-1) + \frac{b}{2h} + 2 \end{cases} \quad (7)$$

When $2 \le M \le 1 + \frac{\beta+\gamma}{2\beta}$:

If $0 \le b < h$, the range of m_1, m_2 can be expressed as follows:

$$\frac{\gamma}{\beta+\gamma}(M-1) - \frac{b}{2h} < m_1 < \frac{\gamma}{\beta+\gamma}(M-1) + \frac{b}{2h} \quad (8)$$

$$\frac{\gamma}{\beta+\gamma}(M-1) - \frac{b}{2h} + 2 < m_2 < \frac{\gamma}{\beta+\gamma}(M-1) + \frac{b}{2h} + 2 \quad (9)$$

If $h \le b < \frac{2\beta}{\beta+\gamma}(M-1)h$, the range of m_1, m_2 can be expressed as follows:

$$\frac{\gamma}{\beta+\gamma}(M-1) - \frac{b}{2h} < m_1 < \frac{\gamma}{\beta+\gamma}(M-1) - \frac{b}{2h} + 1 \quad (10)$$

$$\frac{\gamma}{\beta+\gamma}(M-1) + \frac{b}{2h} + 1 < m_2 < \frac{\gamma}{\beta+\gamma}(M-1) + \frac{b}{2h} + 2 \quad (11)$$

When $M > 1 + \frac{\beta+\gamma}{2\beta}$:

The range of m_1, m_2 can be expressed as follows:

$$\frac{\gamma}{\beta+\gamma}(M-1) - \frac{b}{2h} < m_1 < \frac{\gamma}{\beta+\gamma}(M-1) + \frac{b}{2h} \quad (12)$$

$$\frac{\gamma}{\beta+\gamma}(M-1) - \frac{b}{2h} + 2 < m_2 < \frac{\gamma}{\beta+\gamma}(M-1) + \frac{b}{2h} + 2 \quad (13)$$

In user equilibrium, the expected system cost and the expected travel cost of the commuters are:

$$TTC = \left[\theta(M-1)h + \varepsilon\frac{b}{2}\right]N \quad (14)$$

$$AEC = \theta(M-1)h + \varepsilon\frac{b}{2} + p_0 + p_1 \quad (15)$$

Where $\theta = \beta\gamma/(\beta+\gamma)$ is a constant. The queuing time encountered by commuters taking service run m should satisfy the following formula:

$$q(m) = \begin{cases} \frac{\beta}{\alpha}(m-1)h & m \in (1, m_1] \\ -\frac{\beta+\gamma}{2b\alpha}\left[-\frac{\gamma}{\beta+\gamma}(M-1)h - \frac{b}{2} + (m-1)h\right]^2 + \frac{\gamma}{\alpha}(M-m)h & m \in [m_1+1, m_2-1] \\ \frac{\gamma}{\alpha}(M-m)h & m \in [m_2, M) \end{cases} \quad (16)$$

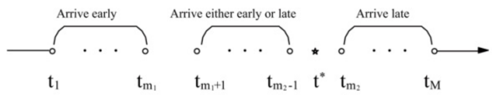

Figure 2: Arrival situation of commuters in peak period in the first case.

3.2 Scenario 2

When $\frac{2\beta(M-1)h}{\beta+\gamma} \le b < \frac{(\beta+\gamma)(M-1)h}{2\beta}$, there are potentially two Scenarios 2 and 2′. In Scenario 2, commuters on train $1 \sim m_1$ always arrive early, while commuters taking train $m_1+1 \sim M$ may arrive early or late, as shown in Fig.3. In Scenario 2′, the commuters taking train $1 \sim m_2$ arrive early or late, and the commuters taking train $m_2+1 \sim M$ always arrive late. In the Appendix, we show that Scenario 2' cannot exist.

In Scenario 2, the peak starts and ends at:

$$\begin{cases} t_1 = -b-(M-1)h+\sqrt{\dfrac{2\beta(M-1)bh}{\beta+\gamma}} \\ t_M = -b+\sqrt{\dfrac{2\beta(M-1)bh}{\beta+\gamma}} \end{cases} \tag{17}$$

In Scenario 2, m_1 is the last train that commuters must arrive early. The range of m_1 can be expressed as follows:

$$\begin{cases} t_1+(m_1-1)h+b<0 \\ t_1+m_1h+b>0 \\ t_1+m_1h<0 \end{cases} \rightarrow \begin{cases} m_1 < M-\sqrt{\dfrac{2\beta(M-1)b}{(\beta+\gamma)h}} \\ m_1 > M-\sqrt{\dfrac{2\beta(M-1)b}{(\beta+\gamma)h}}-1 \\ m_1 < M+\dfrac{b}{h}-\sqrt{\dfrac{2\beta(M-1)b}{(\beta+\gamma)h}}-1 \end{cases} \tag{18}$$

When $2 \le M < \frac{\beta+\gamma}{2\beta}$:

If $\frac{2\beta(M-1)h}{\beta+\gamma} \le b < h$, the range of m_1 can be expressed as follows:

$$M-\sqrt{\frac{2\beta(M-1)b}{(\beta+\gamma)h}}-1 < m_1 < M+\frac{b}{h}-\sqrt{\frac{2\beta(M-1)b}{(\beta+\gamma)h}}-1 \tag{19}$$

If $h \le b < \frac{(\beta+\gamma)(M-1)h}{2\beta}$, the range of m_1 can be expressed as follows:

$$M-\sqrt{\frac{2\beta(M-1)b}{(\beta+\gamma)h}}-1 < m_1 < M+\frac{b}{h}-\sqrt{\frac{2\beta(M-1)b}{(\beta+\gamma)h}}-1 \tag{20}$$

When $M \ge 1+\frac{\beta+\gamma}{2\beta}$:

The range of m_1 can be expressed as follows:

$$M-\sqrt{\frac{2\beta(M-1)b}{(\beta+\gamma)h}}-1 < m_1 < M+\frac{b}{h}-\sqrt{\frac{2\beta(M-1)b}{(\beta+\gamma)h}}-1 \tag{21}$$

In user equilibrium, the expected system cost and the expected travel cost of the commuters are:

$$TTC = \left[\beta(M-1)h-\beta\sqrt{\frac{2\beta bh(M-1)}{\beta+\gamma}}+\frac{\beta+\varepsilon}{2}b\right]N \tag{22}$$

$$AEC = \beta(M-1)h-\beta\sqrt{\frac{2\beta bh(M-1)}{\beta+\gamma}}+\frac{\beta+\varepsilon}{2}b + p_0 + p_1 \tag{23}$$

The queuing time encountered by commuters taking service run m should satisfy the following formula:

$$q(m) = \begin{cases} \dfrac{\beta}{\alpha}(m-1)h & m \in (1, m_1] \\ \left[\sqrt{\dfrac{2\beta(\beta+\gamma)(M-1)h}{b}}-\dfrac{(\beta+\gamma)(M-m)h}{2b}-\beta\right]\dfrac{(M-m)h}{\alpha} & m \in [m_1+1, M) \end{cases} \tag{24}$$

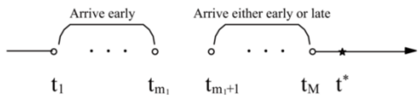

Figure 3: Arrival situation of commuters in peak period in the second case.

3.3 Scenario 3

When $b > \frac{(\beta+\gamma)(M-1)h}{2\beta}$, Scenario 3 emerges. In Scenario 3, all commuters may arrive either early or late, as shown in Fig.4. In Scenario 3, the peak starts and ends at：

$$\begin{cases} t_1 = -\dfrac{\gamma}{\beta+\gamma}b-\dfrac{(M-1)h}{2} \\ t_M = -\dfrac{\gamma}{\beta+\gamma}b+\dfrac{(M-1)h}{2} \end{cases} \tag{25}$$

In user equilibrium, the expected system cost and the expected travel cost of the commuters are:

$$TTC = \left[\frac{(\beta+\gamma)}{8b}(M-1)^2h^2-\frac{\gamma^2 b}{2(\beta+\gamma)}+\frac{b}{2}(\gamma+\varepsilon)\right]N \tag{26}$$

$$AEC = \frac{(\beta+\gamma)}{8b}(M-1)^2h^2-\frac{\gamma^2 b}{2(\beta+\gamma)}+\frac{b}{2}(\gamma+\varepsilon)+p_0+p_1 \tag{27}$$

The queuing time encountered by commuters taking service run m should satisfy the following formula:

$$q(m) = \frac{(\beta+\gamma)(M-m)(m-1)h^2}{2\alpha b} \tag{28}$$

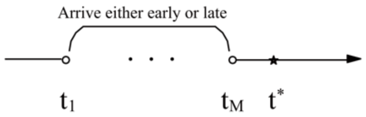

Figure 4: Arrival situation of commuters in peak period in the third case.

Based on the above formula, we can make a simple analysis of the change trend of t_1 and TTC with b value.

When $0 \le b < \frac{2\beta}{\beta+\gamma}(M-1)h$, $\frac{dt_1}{db} = -\frac{1}{2} < 0$, $\frac{dTTC}{db} = \frac{\varepsilon N}{2} > 0$. So in Scenario 1, the initial time of peak period t_1 decreases monotonically with the increase of b value, and the total system cost TTC increases monotonically with the increase of b value.

When $\frac{2\beta(M-1)h}{\beta+\gamma} \le b < \frac{(\beta+\gamma)(M-1)h}{2\beta}$:

$$\frac{dt_1}{db} = -1 + \sqrt{\frac{\beta(M-1)h}{2(\beta+\gamma)b}} \tag{29}$$

$$\frac{dTTC}{db} = \frac{\beta+\varepsilon}{2}N - \beta N\sqrt{\frac{\beta(M-1)h}{2(\beta+\gamma)b}} \tag{30}$$

When $\frac{2\beta(M-1)h}{\beta+\gamma} \le b < \frac{(\beta+\gamma)(M-1)h}{2\beta}$, $\frac{dt_1}{db} < 0$, $\frac{dTTC}{db} > 0$. So in Scenario 2, the initial time of peak period t_1 decreases monotonically with the increase of b value, and the total system cost TTC increases monotonically with the increase of b value.

When $b > \frac{(\beta+\gamma)(M-1)h}{2\beta}$, $\frac{dt_1}{db} = -\frac{\gamma}{\beta+\gamma} < 0$, $\frac{dTTC}{db} = -\frac{(\beta+\gamma)(M-1)^2h^2N}{8b^2} + \frac{N}{2}(\theta+\varepsilon) > 0$. So in Scenario 3, the initial time of peak period t_1 decreases monotonically with the increase of b value, and the total system cost TTC increases monotonically with the increase of b value.

4 CONCLUSIONS

This paper extends the bottleneck model to study the travel behavior of subway commuters during rush hours. The extended model takes into account the situation that passengers have a random delay T, which follows a uniform distribution in the range $0 \le T \le b$, to reach their workplace after getting off the subway. It is shown that with the increase of b, three different scenarios emerge. The start time and end time of the peak hour, the expected value of travel cost, and the queuing time have been derived under the three scenarios. It is shown that the start time of rush hour monotonically decreases (i.e., the start time becomes earlier and earlier) and the travel cost monotonically increases with the increase of b.

In our future work, we will consider how to manage the subway commute under random delay to lower down the travel cost of commuters.

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APPENDIX

The First Train May Arrive Early or Late, and the Last Train Is Always Late

When $\frac{2\beta(M-1)h}{\beta+\gamma} \le b < \frac{(\beta+\gamma)(M-1)h}{2\beta}$, it is also possible that the commuters taking train $1 \sim m_2$ arrive early or late, and the commuters taking train $m_2+1 \sim M$ always arrive late. In this case, the peak starts and ends at:

$$\begin{cases} t_1 = -\sqrt{\dfrac{2\beta(M-1)bh}{\beta+\gamma}} \\ t_M = (M-1)h - \sqrt{\dfrac{2\beta(M-1)bh}{\beta+\gamma}} \end{cases} \tag{31}$$

Since commuters in the first train may be early or late, and commuters in the tail train are always late, then:

$$\begin{cases} t_1 + b > 0 \\ t_M > 0 \end{cases} \rightarrow \frac{2\gamma}{\beta+\gamma}(M-1)h < b < \frac{(\beta+\gamma)}{2\gamma}(M-1)h \tag{32}$$

Because $\frac{2\gamma}{\beta+\gamma} > \frac{(\beta+\gamma)}{2\gamma}(M-1)h$, Scenario 2' cannot exist.