A Queueing Analysis of Multi-type Servers and Multi-type Customers System based on Gas Stations

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Keywords: Queueing Model, GI/M/1-type Markov Chain, Gas Station, Performance.

Abstract: Nowadays, cars are essential to life, and most cars used in society need refueling. In gas stations, an odd phenomenon often occurs where the server (refueling lane) is available, but the service is not available, and this is due to the mismatch between the type of the customer (car) and the type of the server. In this paper, we model some types of the system of gas stations as queueing models and analyze them. In addition, we derive performance measures and compare these types of systems. Some counter-intuitive results emerge in this study.

1 INTRODUCTION

Nowadays, many people worldwide use cars, and transporting by cars is essential for their lives. There are over 80 million cars in Japan, and over 60 million are passenger vehicles (Ministry of Land, Infrastructure, Transport and Tourism, Japan, 2021). Recently, zero-emission vehicles are developing, such as Electric Vehicles (EVs) and Fuel Cell Vehicles (FCVs), but the number of such vehicles is still low. In Japan, the number of EVs and FCVs is about 130 thousand at the end of FY (Fiscal Year) 2019 (Next Generation Vehicle Promotion Center, Japan, 2020). Accordingly, most cars on the streets are powered by engines which need to be refueled. Generally, people refuel cars at a Gas Station (GS), and some unusual phenomena occur.

Typically, each refueling machine installed at a GS has two servers (refueling lanes), and each lane can provide refueling service independently. Hereafter, we refer the server to as a service lane in a refueling machine. Most cars have a fuel door on either the left or right side, so the two lanes are for cars with a left-side fuel door and a right-side fuel door. In reality, although many GSs have equal numbers of lanes for left and lanes for right, the left-right ratio of fuel door position is not always 1:1 (it varies by country and region). For example, there are many more vehicles with left-side fuel doors than those with right-side ones in Japan. This can cause the blocking phenomenon even when some servers in the system are available like Figure 1. The disadvantages of this phenomenon have been studied using a queueing model (Mélange et al., 2011).

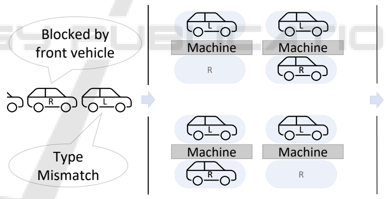

Figure 1: Queueing phenomenon that often occurs in GS.

On the other hand, refueling machines with long hoses can provide service regardless of the position of the fuel door. Theoretically, the aforementioned phenomenon cannot happen at GSs that install this type of machine, so this type of machine is preferred in terms of operational efficiency. However, it is difficult to replace all machines due to various constraints such as costs or safety.

From these backgrounds, our research focuses on analyzing the queueing system with multi-type servers and multi-type customers like GSs. In this research, we consider three types of GSs. In the first type, all machines have a regular hose (dedicated use). In the second type, all machines have a long hose (shared use). Finally, a third type is a hybrid form of the above two types. Then, we model them as queueing systems and analyze the difference of per-

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Machida, Y. and Phung-Duc, T.
A Queueing Analysis of Multi-type Servers and Multi-type Customers System based on Gas Stations.
DOI: 10.5220/0010816100003117
In *Proceedings of the 11th International Conference on Operations Research and Enterprise Systems (ICORES 2022)*, pages 145-152
ISBN: 978-989-758-548-7; ISSN: 2184-4372

formances between each type of system.

The structure of this paper is organized as follows. In Section 2, we explain the queueing models for three types of GS systems. In Section 3, we present an analysis of the proposed models in detail. In Section 4, we derive the stability conditions for some systems. In Section 5, we introduce some performance measures. In Section 6, we show several numerical examples. Finally, in Section 7, we conclude this paper and discuss future works.

2 QUEUEING MODELS

In this section, we model three types of GS systems as queueing models. A GS may provide various services, but here we assume that a GS provides only refueling service. There are two types of servers in a GS: dedicated server (regular hose) and shared server (long hose). In this paper, we consider three types of systems, i.e., All-Shared servers (AS), All-Dedicated servers (AD), and Shared-and-Dedicated Mix (SDM). In all models, customers whose cars are equipped with a fuel door on the left side (type-L customer) and right side (type-R customer) arrive at the system according to Poisson processes with rates λ_L and λ_R. Moreover, service times follow the exponential distribution with a mean of $1/\mu$. After the service, the lane becomes empty. In order to analyze models, we set necessary assumptions. First, we assume that the arrival intervals of customers and the service times of each server are independent of each other. Second, the order of services is assumed to be FCFS.

2.1 All-Shared Servers System (AS)

First of all, we describe a queueing model of a GS that adopts an AS system. AS means that all servers in the system can provide service for all customers, so all customers are indiscriminate in this type of system. There is no need to distinguish between different types of customers in this system, so the arrivals of arbitrary customers (type-L or type-R) follow a Poisson process with rate λ $(= \lambda_L + \lambda_R)$. The schematic of the model is shown in Figure 2.

If the number of servers is c, a GS that adopts the AS system can be modeled as an M/M/c queueing system.

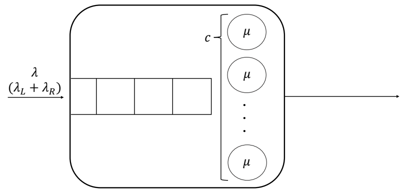

Figure 2: The queueing model of AS system GS.

2.2 All-Dedicated Servers System (AD)

Next, we describe a queueing model of a GS that adopts an AD system. In the AD system, all servers are either for type-L customers or for type-R customers. Thus, this system has two types depending on how customers line up, queue-divided type and queue-combined type.

2.2.1 Queue-divided AD System (AD-D)

In this system, customers search for the server that matches the position of the fuel door on their car and start getting service. In the AD-D type, if no server is available, customers line up separately for each type of their fuel door side. It means that there are two adjacent independent systems. One contains all servers serving type-L customers, and the other contains all servers serving type-R customers and each system behaves like an AS system.

If the numbers of servers in each system are c_L and c_R $(c_L + c_R = c)$, a GS that adopts the AD-D system can be modeled as two separate M/M/c_L and M/M/c_R queueing systems.

2.2.2 Queue-combined AD System (AD-C)

Then what if all customers line up together in one queue? In this type of system, we can observe two curious phenomena. First, in this type of system, with customers in the queue, the server that does not match the customer in the queue head may remain empty and unused even if the customer behind the head matches it. Second, from the situation mentioned above, when the server that matches the customer at the head of the queue becomes empty, customers behind the head of the queue might enter the servers simultaneously. In other words, more than two customers might enter the servers simultaneously in this type of system. In this type of system, customers arrive at the system according to a Poisson process with rate λ $(= \lambda_L + \lambda_R)$, and the probabilities that a type-L or type-R customer is at the head of the queue are λ'_L $(= \lambda_L/\lambda)$ and λ'_R $(= \lambda_R/\lambda)$, respectively. The schematic of this model is shown in Figure 3.

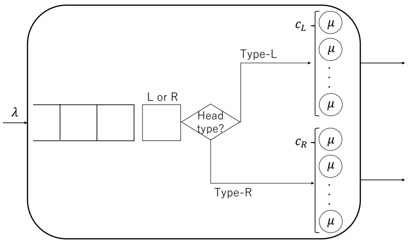

Figure 3: The queueing model of AD-C system GS.

In order to analyze this model, we provide the necessary settings. The numbers of type-L, type-R servers are c_L, c_R $(c_L + c_R = c)$. We define $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$, $S_L := \{0, 1, ..., c_L\}$, $S_R := \{0, 1, ..., c_R\}$ $(c_L + c_R = c)$, $S_H := \{0, 1\}$, $S_D^* := S_L \times S_R \times S_H \times \mathbb{N}_0$. Let $C_L(t)$ and $C_R(t)$ respectively denote the numbers of cars staying in type-L and type-R servers in the system at time t, where $C_L(t) \in S_L,\ C_R(t) \in S_R$. The type of the fuel door of the customer at the head of the queue at time t is $H(t) \in S_H$. Denote by $L(t)$ the number of customers in the queue at time t, where $L(t) \in \mathbb{N}_0$. After all, we define $X_D(t) := (L(t),\ C_L(t),\ C_R(t),\ H(t))$. Since S_D^* includes states that $X_D(t)$ cannot reach, we define S_D as the subset of S_D^* that excludes unreachable states. In this way, we can see $\{X_D(t);\ t \geq 0\}$ is an irreducible Markov chain on the state space S_D. Based on the above settings, we analyze this model in Section 3.

2.3 Shared-and-Dedicated-Mix System (SDM)

Finally, we describe a queueing model of a GS that adopts the SDM system. SDM means that some servers in the system are shared, and the others are dedicated. There is a study on the system of this mechanism modeled as a loss system for the type that uses a dedicated server first (D-first) (Kawashima, 1985). We consider two types of this system about priority disciplines for server usage: D-first and shared server first (S-first).

In order to analyze these models, we provide the necessary settings. The numbers of type-L, type-R, shared servers are c_L, c_R, c_S $(c_L + c_R + c_S = c)$. We define $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$, $S_L := \{0, 1, ..., c_L\}$, $S_R := \{0, 1, ..., c_R\}$, $S_S := \{0, 1, ..., c_S\}$ $(c_L + c_R + c_S = c)$, $S_H := \{0, 1\}$, $S_M^* := S_L \times S_R \times S_S \times S_H \times \mathbb{N}_0$. Let $C_L(t)$, $C_R(t)$ and $C_S(t)$ respectively denote the numbers of cars staying in type-L, type-R and shared servers in the system at time t, where $C_L(t) \in S_L$, $C_R(t) \in S_R$, $C_S(t) \in S_S$. The type of the fuel door of the customer at the head of the queue at time t is $H(t) \in S_H$. Denote by $L(t)$ the number of customers in the queue at time t, where $L(t) \in \mathbb{N}_0$. After all, we define $X_M(t) := (L(t),\ C_L(t),\ C_R(t),\ C_S(t),\ H(t))$. Since S_M^* includes states that $X_M(t)$ cannot reach, we define S_M as the subset of S_M^* that excludes unreachable states. In this way, we can see $\{X_M(t);\ t \geq 0\}$ is an irreducible Markov chain on the state space S_M. Based on the above settings, we analyze this model in later sections. The difference between the two types of this system is mentioned in in Section 3.

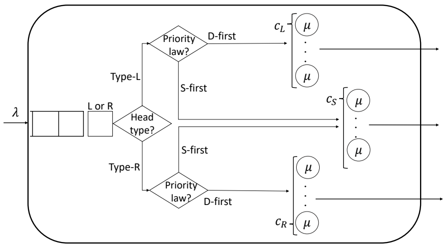

Figure 4: The queueing model of SDM system GS.

3 QUEUEING ANALYSIS

In this section, we define the infinitesimal generators for the two models mentioned above: the AD-C system and the SDM system, and describe the analysis of these models. For the other models, the solution method already exists so that we will derive the performance measures in the later section.

3.1 AD-C System

We construct the transition matrix by separating the change in the number of customers in the queue. Then, we represent the infinitesimal generator Q_D in (1), where O is a zero matrix of appropriate size.

$$
Q_D = \begin{array}{c} \\ \mathcal{L}_0^D \\ \mathcal{L}_1^D \\ \mathcal{L}_2^D \\ \mathcal{L}_3^D \\ \vdots \\ \mathcal{L}_l^D \\ \mathcal{L}_{l+1}^D \\ \vdots \end{array}
\begin{array}{c}
\begin{array}{cccccccc} \mathcal{L}_0^D & \mathcal{L}_1^D & \mathcal{L}_2^D & \mathcal{L}_3^D & \cdots & \mathcal{L}_l^D & \mathcal{L}_{l+1}^D & \cdots \end{array} \\
\begin{pmatrix}
B_0 & C_0 & O & O & \cdots & O & O & \cdots \\
B_1 & A_1 & A_0 & O & \cdots & O & O & \cdots \\
B_2 & A_2 & A_1 & A_0 & \cdots & O & O & \cdots \\
B_3 & A_3 & A_2 & A_1 & \cdots & O & O & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
B_l & A_l & A_{l-1} & A_{l-2} & \cdots & A_1 & A_0 & \cdots \\
O & A_{l+1} & A_l & A_{l-1} & \cdots & A_2 & A_1 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\end{array}. \quad (1)
$$

In Q_D, l represents the number of type-L and type-R servers, whichever is greater, plus one. $\mathcal{L}_0^D, \mathcal{L}_k^D$ $(k \geq 1)$ are the sets given as follows.

$$
\begin{aligned}
\mathcal{L}_0^D := &\{(0,0,0)\} \cup \{(0,0,1)\} \cup ... \cup \{(0,0,c_R)\} \cup \{(0,1,0)\} \\
&\cup ... \cup \{(0,c_L,c_R)\}. \\
\mathcal{L}_k^D := &\{(k,0,0,0)\} \cup \{(k,0,0,1)\} \cup \{(k,0,1,0)\} \cup ... \cup \{(k,0,c_R,1)\} \\
&\cup \{(k,1,0,0)\} \cup ... \cup \{(k,c_L,c_R,1)\}.
\end{aligned}
$$

In $\mathcal{L}_k^D$, k corresponds to the number of customers in the queue. Therefore, the block matrices B_0 and A_1 represent the state transition when the number of customers in the queue does not change. The block matrices C_0 and A_0 represent the state transition when the number of customers in the queue increases by one. A_k $(2 \leq k \leq l+1)$ represents the state transition when the number of customers in the queue decreases by $k-1$. Finally, the block matrix B_k $(1 \leq k \leq l)$ represents the state transition when the number of customers in the queue decreases from k to zero. For the elements of each matrix, please refer to the Appendix.

Next, we compute the stationary distribution of (1). Because $\{X_D(t) \in S_D;\ t \geq 0\}$ defined in the previous section is a continuous-time Markov chain of GI/M/1-type, we calculate the stationary distribution by referring to the method shown in (Adan et al., 2017). We define the stationary distribution $\pi_{i,j,k,l}^D$ of $X_D(t)$ for $(i,j,k,l) \in S_D$ as follows.

$$\pi_{i,j,k,l}^D = \lim_{t\to\infty} P(L(t)=i,\ C_L(t)=j,\ C_R(t)=k,\ H(t)=l),$$
$$\pi_{0,j,k}^D = \lim_{t\to\infty} P(L(t)=0,\ C_L(t)=j,\ C_R(t)=k).$$

3.2 SDM System

We consider two priorities in this type of system: D-first and S-first, but these only make a difference in B_0. The differences in B_0 in consideration of the transition matrix. The differences are explained in the Appendix. Finally, we represent the infinitesimal generator Q_M in (2), where O is a zero matrix of appropriate size.

$$Q_M = \begin{array}{c} \\ \mathcal{L}_0^M \\ \mathcal{L}_1^M \\ \mathcal{L}_2^M \\ \mathcal{L}_3^M \\ \vdots \\ \mathcal{L}_l^M \\ \mathcal{L}_{l+1}^M \\ \vdots \end{array} \begin{array}{c} \begin{array}{cccccccc} \mathcal{L}_0^M & \mathcal{L}_1^M & \mathcal{L}_2^M & \mathcal{L}_3^M & \cdots & \mathcal{L}_l^M & \mathcal{L}_{l+1}^M & \cdots \end{array} \\ \begin{pmatrix} B_0 & C_0 & O & O & \cdots & O & O & \cdots \\ B_1 & A_1 & A_0 & O & \cdots & O & O & \cdots \\ B_2 & A_2 & A_1 & A_0 & \cdots & O & O & \cdots \\ B_3 & A_3 & A_2 & A_1 & \cdots & O & O & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ B_l & A_l & A_{l-1} & A_{l-2} & \cdots & A_1 & A_0 & \cdots \\ O & A_{l+1} & A_l & A_{l-1} & \cdots & A_2 & A_1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \end{array}. \quad (2)$$

In Q_M, l represents the same meaning as in Q_D. $\mathcal{L}_0^M$, $\mathcal{L}_k^M$ $(k \geq 1)$ are the sets given as follows.

$$\mathcal{L}_0^M := \{(0,0,0,0)\} \cup \{(0,0,0,1)\} \cup \ldots \cup \{(0,0,0,c_S)\} \cup \{(0,0,1,0)\}$$
$$\cup \ldots \cup \{(0,0,c_R,c_S)\} \cup \{(0,1,0,0)\} \cup \ldots \cup \{(0,c_L,c_R,c_S)\}.$$
$$\mathcal{L}_k^M := \{(k,0,0,0,0)\} \cup \{(k,0,0,0,1)\} \cup \{(k,0,0,1,0)\}$$
$$\cup \ldots \cup \{(k,0,0,c_S,1)\} \cup \{(k,0,1,0,0)\} \cup \ldots \cup \{(k,c_L,c_R,c_S,1)\}.$$

What each block of the transition matrix represents is the same as in the case of Q_D. Please refer to the Appendix for details.

Next, we compute the stationary distribution of (2). Because $\{X_M(t) \in S_M;\ t \geq 0\}$ defined in the previous section is a continuous-time Markov chain of GI/M/1-type, we calculate the stationary distribution by referring to the method shown in (Adan et al., 2017). We define the stationary distribution $\pi_{i,j,k,l,m}^M$ of $X_M(t)$ for $(i,j,k,l,m) \in S_M$ as follows.

$$\pi_{i,j,k,l,m}^M = \lim_{t\to\infty} P(L(t)=i,\ C_L(t)=j,\ C_R(t)=k,\ C_S(t)=l,\ H(t)=m),$$
$$\pi_{0,j,k,l}^M = \lim_{t\to\infty} P(L(t)=0,\ C_L(t)=j,\ C_R(t)=k,\ C_S(t)=l).$$

4 PERFORMANCE MEASURES

This paper mainly uses the average number of customers for each system as a performance measure.

A simple solution has already been shown for M/M/c type queueing systems. The average numbers of customers in the AS and AD-D systems are derived based on (Adan et al., 2017).

The average number of customers in the AD-C system $E_D(L)$ can be derived as follows.

$$E_D(L) = \sum_{j=0}^{c_L}\sum_{k=0}^{c_R}(j+k)\pi_{0,j,k}^D + \sum_{i=0}^{\infty}\sum_{j=0}^{c_L}\sum_{k=0}^{c_R}\sum_{l=0}^{1}(i+j+k)\pi_{i,j,k,l}^D.$$

The average number of customers in the SDM system $E_M(L)$ can be derived as follows.

$$E_M(L) = \sum_{j=0}^{c_L}\sum_{k=0}^{c_R}\sum_{l=0}^{c_S}(j+k+l)\pi_{0,j,k,l}^M + \sum_{i=0}^{\infty}\sum_{j=0}^{c_L}\sum_{k=0}^{c_R}\sum_{l=0}^{c_S}\sum_{m=0}^{1}(i+j+k+l)\pi_{i,j,k,l,m}^M.$$

5 NUMERICAL RESULTS

In this section, we present some numerical results based on the analysis of previous sections. Then, for all models except AS and AD-D systems, we perform Monte Carlo simulations to ensure the accuracy of the numerical results.

5.1 AS, AD-D and AD-C Systems

First of all, we present differences between AS, AD-D, and AD-C systems. We calculate the average number of customers in these systems by varying the arrival rate λ and the arrival ratio of type-L to type-R. The service rate μ is fixed at 40, the number of servers c is fixed at 8, and the numbers of L and R servers are the same. The result is shown in Figure 5.

There are some points of interest in this result.

First, in the same arrival rate of the type-L and type-R, the AS system has the highest performance, followed by the AD-D system, and the AD-C system has a considerable performance difference from the two systems mentioned above. The performance difference is caused by the blocking phenomenon in the AD-D system when there are available servers, which

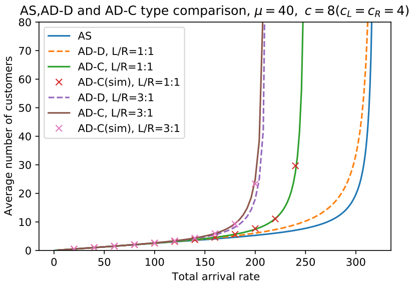

Figure 5: Comparison of AS, AD-D, AD-C systems.

does not occur in the AS system. In the AD-C system, which has a single standby queue, the phenomenon is observed more frequently, further degrading the performance of the system.

Second, if there is a significant bias in the arrival ratio of type-L to type-R, the performance difference between AD-D and AD-C becomes smaller. The more significant bias in the arrival ratio of type-L to type-R, the more the two systems will approximate a system where only one type of server is used on each side.

5.2 AD-C and SDM Systems

Next, we present differences between AD-C and two types of SDM systems. We are interested in the effect of the number of shared servers c_S in two types of SDM systems. We calculate these systems by varying λ and c_S. The service rate μ and the number of servers in the system c are the same as the comparison in Section 5.1. The results are shown in Figure 6 and Figure 7.

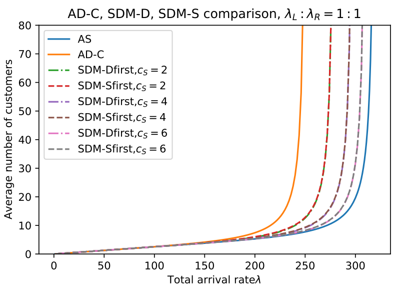

Figure 6: Comparison of AD-C system and SDM systems without bias of arrival rate.

We observe two interesting points in the results. First, according to Figure 5, Figure 6 and Figure 7, the performance of the SDM system is in the middle between the AD-C system and the AS system, and each additional shared server leads to an ever-smaller improvement of the performance of the SDM system. Second, in the two types of SDM systems, there is not much difference in performance between SDM-Dfirst and SDM-Sfirst, and in situations where the system is approaching instability, there is little difference between the two types of systems. Furthermore, there is little difference between the two types of SDM systems, whether the shared server with higher utility is used first or the dedicated server is used first in situations with no customer in the queue. Since the two types of systems are considered precisely the same when there is a queue of customers, our model, which allows for an infinite buffer, does not show a significant difference, especially when the number of customers in the queue is likely to increase. If this system is changed to the loss system with no customer in the queue or has a much larger number of servers, the differences between the two types of systems are likely to arise clearly.

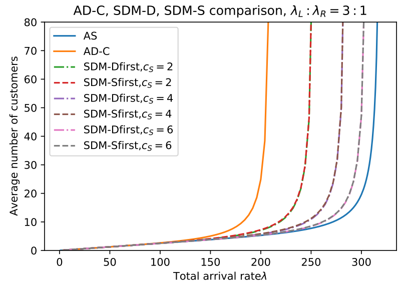

Figure 7: Comparison of AD-C system and SDM systems with the large bias of arrival rate.

5.3 Comparison in Stability Conditions of AD-C System

At last, we present a comparison of the stability conditions of the AD-C system varying the L-R ratio of the number of servers. We calculate the stability conditions of the AD-C system. In the experiment, we set $\lambda = 100$, $c = 8$ and let the arrival rate of type-L customers vary. The results are shown in Figure 8.

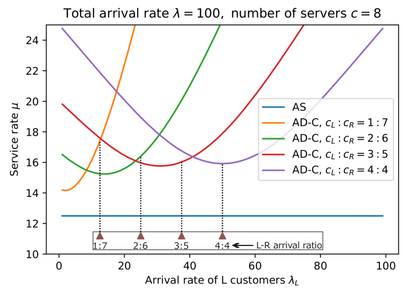

Figure 8: Comparison of L-R server ratio about AD-C system.

There are two notable points in the results.

First, the greater the bias of the L-R ratio of the number of servers, the smaller the minimum service rate required for system stability. In other words, the more biased the L-R ratio of the number of servers is, the system can be operated with less service capacity when the arrival left-right ratio is optimal. This occurs for the same reason as in the previous result: the more significant bias the L-R server ratio and arrival ratio of type-L to type-R are, the more the system behaves like a smaller AS system, hence this result.

Second, intuitively, it seems to be most efficient for the system when the L-R ratio of the number of servers and arrival ratio of type-L to type-R is the same. However, the result is often not so. In Figure 8, the lower markers represent the values at which the arrival ratios of type-L to type-R are the same as each server L-R ratio. The greater the bias in the server L-R ratio, the greater the difference between the most efficient arrival ratio of type-L to type-R and the server L-R ratio. It is thought to be caused for the risk of the minor-type customers at the head of the queue. The larger the bias in the server L-R ratio, the more likely it is that when a minor-type customer lines up at the head of the queue, many major-type customers will line up behind. In order to avoid this waste, it is thought to be desirable that the arrival ratio of type-L to type-R is larger bias than the server L-R ratio, and in such a situation, major-type customers are more likely to be at the head of the queue, making it difficult for the aforementioned risk to occur.

6 CONCLUSION

We have modeled the systems with multi-type servers and multi-type customers based on gas stations by queueing systems. In this paper, we have evaluated some differences among the systems. First, we compared an AS system and the two types of AD systems. Second, we observed the impact of the number of shared servers in the two types of SDM systems. Third, we considered the effect of servers and arrival left-right ratio in the AD-C system.

Finally, we consider the future works. In this study, we have modeled the systems of gas stations in simplified situations. However, in reality, the gas station systems have much more complexity, so one may consider incorporating more complex and realistic situations. First, for example, some customers can get service from all types of servers, like motorcycles. Second, in the point of view about queue creation, there exists a system that is neither AD-D nor AD-C, where customers are divided by their type in the middle of the queue. Studies of similar systems exist (Mélange et al., 2020), but there is no mention of multiple servers' cases. Second, due to the small size of the gas station site, there are incidents where customers cannot leave after the service and customers in the queue cannot enter the available servers. This phenomenon has already been studied earlier (Teimoury et al., 2011; Jiang, 2018), but their models are single-row services, which are more straightforward than the actual GS multi-row service. In addition, there are various kinds of factors that seem to affect a gas station system. For example, customers also enter the system for other services such as car washing, then join the queue for fueling afterward. In this case, the new system may be modeled as a tandem queue where multiple types of services are available, and more variables need to be added to capture such complexity.

ACKNOWLEDGEMENTS

The research of the second author is supported in part by JSPS KAKENHI Grant Number 21K11765.

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APPENDIX

We describe each of the block matrices used in the infinitesimal generators defined in Section 3. In the following, the element in the $i-$th row from the top and $j-$th column from the left of a block matrix is called the (i, j) element. For instance, the (i, j) element of a block matrix A is denoted as $A_{i,j}$. Note that the matrix components of the undefined part in each block matrix are all zero.

First, we describe each of the block matrices used in the infinitesimal generator about the AD-C system.

A_0 is a $((c_L+1)\times(c_R+1)\times 2)$-order square matrix that represents the transition of the number of customers in the queue from i to $i+1$ $(i \geq 1)$. Thus, each element $(A_0)_{i,j}$ is defined as follows.

Table 1: Matrix components of A_0.

$(A_0)_{i,j}$	i	j
λ	$(m+1)\times(c_R+1)\times 2$	$(m+1)\times(c_R+1)\times 2$
λ	$(c_L(c_R+1)+n)\times 2+1$	$(c_L(c_R+1)+n)\times 2+1$
		if $(0\leq m\leq c_L,\ 0\leq n\leq c_R)$

B_0 is a $((c_L+1)\times(c_R+1))$-order square matrix that represents the transition of states when the number of customers in the queue is zero. Each element $(B_0)_{i,j}$ is defined as follows.

Table 2: Matrix components of B_0.

$(B_0)_{i,j}$	i	j
λ_L	$(c_R+1)\times m+n+1$	$(c_R+1)\times(m+1)+n+1$
$(m+1)\mu$	$(c_R+1)\times(m+1)+n+1$	$(c_R+1)\times m+n+1$
		if $(0\leq m\leq c_L-1,\ 0\leq n\leq c_R)$
λ_R	$(c_R+1)\times m+n+1$	$(c_R+1)\times m+n+2$
$(n+1)\mu$	$(c_R+1)\times m+n+2$	$(c_R+1)\times m+n+1$
		if $(0\leq m\leq c_L,\ 0\leq n\leq c_R)$

Denoting $I := \{1,2,...,(c_L+1)\times(c_R+1)\}$, $J := \{1,2,...,(c_L+1)\times(c_R+1)\times 2\}$, $i\in I$, we define the diagonal components of B_0 as follows.

$$(B_0)_{i,i} = -\left(\sum_{j\in I\setminus\{i\}}(B_0)_{i,j}+\sum_{j\in J}(C_0)_{i,j}\right).$$

C_0 is a matrix of size $((c_L+1)\times(c_R+1))\times((c_L+1)\times(c_R+1)\times 2)$ that represents the transition of the number of customers in the queue from zero to 1. Each element $(C_0)_{i,j}$ is defined as follows.

Table 3: Matrix components of C_0.

$(C_0)_{i,j}$	i	j
λ_L	$c_L\times(c_R+1)+n+1$	$(c_L(c_R+1)+n)\times 2+1$
λ_R	$(m+1)\times(c_R+1)$	$(m+1)\times(c_R+1)\times 2$
		if $(0\leq m\leq c_L,\ 0\leq n\leq c_R)$

A_1 is a $((c_L+1)\times(c_R+1)\times 2)$-order square matrix that represents the transition of states when the number of customers in the queue is greater than or equal to 1. Each element$(A_1)_{i,j}$ is defined as follows.

Table 4: Matrix components of A_1.

$(A_1)_{i,j}$	i	j
$(m+1)\mu$	$(m+2)\times(c_R+1)\times 2$	$(m+1)\times(c_R+1)\times 2$
$(n+1)\mu$	$(c_L(c_R+1)+(n+1))\times 2+1$	$(c_L(c_R+1)+n)\times 2+1$
		if $(0\leq m\leq c_L-1,\ 0\leq n\leq c_R-1)$

We define the diagonal components of A_1 when the number of customers in the queue is l as follows.

$$(A_1)_{i,i} = -\left(\sum_{j\in I}(B_l)_{i,j}+\sum_{k=2}^{l}\sum_{j\in J}(A_k)_{i,j}+\sum_{j\in I\setminus\{i\}}(A_1)_{i,j}+\sum_{j\in J}(A_0)_{i,j}\right).$$

$A_k(k\geq 2)$ is a $((c_L+1)\times(c_R+1)\times 2)$-order square matrix that represents the transition that the number of customers in the queue decreases by $k-1$ but not to zero. Note that the probability that a customer in the queue is type-L is λ_L/λ $(:= \lambda'_L)$ and the probability that a customer in the queue is type-R is λ_R/λ $(:= \lambda'_R)$. Each element $(A_k)_{i,j}$ is defined as follows.

Table 5: Matrix components of A_k.

$(A_k)_{i,j}$	i	j
$c_L\mu\lambda'_L\lambda'^{k-2}_R$	$(c_L(c_R+1)+(n+1))$ $\times 2-1$	$(c_L(c_R+1)+(k+n-1))$ $\times 2-1$
$c_L\mu\lambda'^{k-1}_R$	$((c_L+1)(c_R+1)+(2-k))$ $\times 2-1$	$(c_L+1)\times(c_R+1)\times 2$
$c_R\mu\lambda'_R\lambda'^{k-2}_L$	$(m+1)\times(c_R+1)\times 2$	$(k+m-1)\times(c_R+1)\times 2$
$c_R\mu\lambda'^{k-1}_L$	$(c_L-k+3)\times(c_R+1)\times 2$	$(c_L+1)\times(c_R+1)\times 2-1$
		if $(0\leq m\leq c_L+2-k,\ 0\leq n\leq c_R+2-k)$

B_k $(k\geq 1)$ is a matrix of size $((c_L+1)\times(c_R+1)\times 2)\times((c_L+1)\times(c_R+1))$ that represents the transition of the number of customers in the queue from k to zero. Each element $(B_k)_{i,j}$ is defined as follows.

Table 6: Matrix components of B_k.

$(B_k)_{i,j}$	i	j
$c_L\mu\lambda'^{k-1}_R$	$(c_L(c_R+1)+(n+1))$ $\times 2-1$	$c_L(c_R+1)+n+k-1$
$c_R\mu\lambda'^{k-1}_L$	$(m+1)\times(c_R+1)\times 2$	$(m+k)(c_R+1)$
		if $(0\leq m\leq c_L+1-k,\ 0\leq n\leq c_R+1-k)$

Second, we describe each block matrices used in the infinitesimal generator about two types of SDM systems.

A_0 is a $((c_L+1)\times(c_R+1)\times(c_S+1)\times 2)$-order square matrix that represents the transition of the number of customers in the queue from i to $i+1$ $(i\geq 1)$. Each element $(A_0)_{i,j}$ is defined as follows.

Table 7: Matrix components of A_0.

$(A_0)_{i,j}$	i	j
λ	$(m+1)\times(c_R+1)$ $\times(c_S+1)\times 2$	$(m+1)\times(c_R+1)$ $\times(c_S+1)\times 2$
λ	$(c_L(c_R+1)+n)\times(c_S+1)$ $\times 2+1$	$(c_L(c_R+1)+n)\times(c_S+1)$ $\times 2+1$
		if $(0\leq m\leq c_L,\ 0\leq n\leq c_R)$

B_0 is a $((c_L+1)\times(c_R+1)\times(c_S+1))$-order square matrix that represents the transition of states when the number of customers in the queue is zero. There are two types in the SDM system. The only difference between the two types is the components of B_0. Each element of B_0 of D-first $(B_0^D)_{i,j}$ and S-first $(B_0^S)_{i,j}$ are defined as follows.

Table 8: Matrix components of B_0^D.

$(B_0^D)_{i,j}$	i	j
λ_L	$(l\times(c_R+1)+m)\times(c_S+1)+n+1$	$((l+1)\times(c_R+1)+m)\times(c_S+1)+n+1$
$(l+1)\mu$	$((l+1)\times(c_R+1)+m)\times(c_S+1)+n+1$	$(l\times(c_R+1)+m)\times(c_S+1)+n+1$
	if $(0\leq m\leq c_L-1,\ 0\leq n\leq c_R,\ 0\leq n\leq c_S)$	
λ_R	$(l\times(c_R+1)+m)\times(c_S+1)+n+1$	$(l\times(c_R+1)+m+1)\times(c_S+1)+n+1$
$(m+1)\mu$	$(l\times(c_R+1)+m+1)\times(c_S+1)+n+1$	$(l\times(c_R+1)+m)\times(c_S+1)+n+1$
	if $(0\leq m\leq c_L,\ 0\leq n\leq c_R-1,\ 0\leq n\leq c_S)$	
λ_L	$(c_L\times(c_R+1)+m)\times(c_S+1)+n+1$	$(c_L\times(c_R+1)+m)\times(c_S+1)+n+2$
λ_R	$(l\times(c_R+1)+c_R)\times(c_S+1)+n+1$	$(l\times(c_R+1)+c_R)\times(c_S+1)+n+2$
λ	$(c_L\times(c_R+1)+c_R)\times(c_S+1)+n+1$	$(c_L\times(c_R+1)+c_R)\times(c_S+1)+n+2$
$(n+1)\mu$	$(l\times(c_R+1)+m)\times(c_S+1)+n+2$	$(l\times(c_R+1)+m)\times(c_S+1)+n+1$
	if $(0\leq m\leq c_L,\ 0\leq n\leq c_R,\ 0\leq n\leq c_S-1)$	

Table 9: Matrix components of B_0^S.

$(B_0^S)_{i,j}$	i	j
λ_L	$(l\times(c_R+1)+m)\times(c_S+1)+c_S+1$	$((l+1)\times(c_R+1)+m)\times(c_S+1)+c_S+1$
$(l+1)\mu$	$((l+1)\times(c_R+1)+m)\times(c_S+1)+n+1$	$(l\times(c_R+1)+m)\times(c_S+1)+c_S+1$
	if $(0\leq m\leq c_L-1,\ 0\leq n\leq c_R,\ 0\leq n\leq c_S)$	
λ_R	$(l\times(c_R+1)+m)\times(c_S+1)+c_S+1$	$(l\times(c_R+1)+m+1)\times(c_S+1)+c_S+1$
$(m+1)\mu$	$(l\times(c_R+1)+m+1)\times(c_S+1)+c_S+1$	$(l\times(c_R+1)+m)\times(c_S+1)+c_S+1$
	if $(0\leq m\leq c_L,\ 0\leq n\leq c_R-1,\ 0\leq n\leq c_S)$	
λ	$(l\times(c_R+1)+m)\times(c_S+1)+n+1$	$(l\times(c_R+1)+m)\times(c_S+1)+n+2$
$(n+1)\mu$	$(l\times(c_R+1)+m)\times(c_S+1)+n+2$	$(l\times(c_R+1)+m)$ $times(c_S+1)+n+1$
	if $(0\leq m\leq c_L,\ 0\leq n\leq c_R,\ 0\leq n\leq c_S-1)$	

Denoting $I:=\{1,2,...,(c_L+1)\times(c_R+1)\times(c_S+1)\}$, $J:=\{1,2,...,(c_L+1)\times(c_R+1)\times(c_S+1)\times 2\}$, $i\in I$, we define the diagonal components of B_0^D, B_0^S as follows.

$$(B_0^{D(S)})_{i,i}=-\left(\sum_{j\in I\setminus\{i\}}(B_0^{D(S)})_{i,j}+\sum_{j\in J}(C_0)_{i,j}\right).$$

C_0 is a matrix of size $((c_L+1)\times(c_R+1)\times(c_S+1))\times((c_L+1)\times(c_R+1)\times(c_S+1)\times 2)$ that represents the transition of the number of customers in the queue from zero to 1. Each element $(C_0)_{i,j}$ is defined as follows.

Table 10: Matrix components of C_0.

$(B_0)_{i,j}$	i	j
λ_L	$(c_L\times(c_R+1)+m)\times(c_S+1)+1$	$(c_L\times(c_R+1)+m)\times 2+1$
λ_R	$(l+1)\times(c_R+1)\times(c_S+1)$	$(l+1)\times(c_R+1)\times(c_S+1)\times 2$
	if$(0\leq l\leq c_L,\ 0\leq m\leq c_R)$	

A_1 is a $((c_L+1)\times(c_R+1)\times(c_S+1)\times 2)$-order square matrix that represents the transition of states when the number of customers in the queue is greater than or equal to 1. Each element $(A_1)_{i,j}$ is defined as follows.

Table 11: Matrix components of A_1.

$(A_1)_{i,j}$	i	j
$(l+1)\mu$	$(l+2)\times(c_R+1)\times(c_S+1)\times 2$	$(l+1)\times(c_R+1)\times(c_S+1)\times 2$
$(m+1)\mu$	$(c_L\times(c_R+1)+(m+2))\times(c_S+1)\times 2-1$	$(c_L\times(c_R+1)+(m+1))\times(c_S+1)\times 2-1$
	if$(0\leq l\leq c_L-1,\ 0\leq m\leq c_R-1)$	

We define the diagonal components of A_1 when the number of customers in the queue is l as follows.

$$(A_1)_{i,i}=-\left(\sum_{j\in I}(B_l)_{i,j}+\sum_{k=2}^{l}\sum_{j\in J}(A_k)_{i,j}+\sum_{j\in I\setminus\{i\}}(A_1)_{i,j}+\sum_{j\in J}(A_0)_{i,j}\right).$$

A_k $(k\geq 2)$ is a $((c_L+1)\times(c_R+1)\times(c_S+1)\times 2)$-order square matrix that represents the transition of the number of customers in the queue decreasing by $k-1$ but not to zero. We define c_{LS} and c_{RS} as the sum of the type-L servers and shared servers, type-R servers and shared servers. Each element$(A_k)_{i,j}$ is defined as follows.

Table 12: Matrix components of A_k.

$(A_k)_{i,j}$	i	j
$c_{LS}\mu\lambda'_L\lambda'^{k-2}_R$	$(c_L\times(c_R+1)+m+1)\times(c_S+1)\times 2-1$	$(c_L\times(c_R+1)+k+m-1)\times(c_S+1)\times 2-1$
$c_{LS}\mu\lambda'^{k-1}_R$	$((c_L+1)\times(c_R+1)+2-k)\times(c_S+1)\times 2-1$	$(c_L+1)\times(c_R+1)\times(c_S+1)\times 2$
$c_{RS}\mu\lambda'_R\lambda'^{k-2}_L$	$(l+1)\times(c_R+1)\times(c_S+1)\times 2$	$(k+l-1)\times(c_R+1)\times(c_S+1)\times 2$
$c_{RS}\mu\lambda'^{k--1}_L$	$(c_L+3-k)\times(c_R+1)\times(c_S+1)\times 2$	$(c_L+1)\times(c_R+1)\times(c_S+1)\times 2-1$
	if$(0\leq l\leq c_L+2-k,\ 0\leq m\leq c_R+2-k)$	

B_k $(k\geq 1)$ is a matrix of size $((c_L+1)\times(c_R+1)\times(c_S+1)\times 2)\times((c_L+1)\times(c_R+1)\times(c_S+1))$ that represents the transition of the number of customers in the queue from k to zero. Each element$(B_k)_{i,j}$ is defined as follows.

Table 13: Matrix components of B_k.

$(B_k)_{i,j}$	i	j
$c_{LS}\mu\lambda'^{k-1}_R$	$(c_L\times(c_R+1)+m+1)\times(c_S+1)\times 2-1$	$(c_L\times(c_R+1)+k+m)\times(c_S+1)$
$c_{RS}\mu\lambda'^{k--1}_L$	$(l+1)\times(c_R+1)\times(c_S+1)\times 2$	$(k+l)\times(c_R+1)\times(c_S+1)$
	if$(0\leq l\leq c_L+1-k,\ 0\leq m\leq c_R+1-k)$	