Optimal Models for Distributing Vaccines in a Pandemic

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Abstract: Distributing vaccines among a massive population is one of the challenging tasks in a pandemic. Therefore, health care organizations need to optimize the assignment of vaccination appointments for people while considering their priorities and preferences. In this paper, we propose two optimal vaccine distribution models as Integer Linear Programming (ILP) models; namely, Priority-based Model (PM) and Priority & Preference-based Model (PPM), to maximize the distribution of vaccines among a given population. In PM, we divide the people among several priority groups and ensure maximum vaccine distribution among the higher priority groups. However, along with the priority groups, PPM also considers a list of preferred vaccine distribution centers and time slots for each person. Thus, this model maximizes vaccine distribution among the higher priority groups by assigning appointments in their desired location and time. We analyzed the performance of our proposed models on a randomly generated dataset. In addition, we also performed a case study for our proposed models on the COVID-19 vaccination dataset from Thunder Bay, Canada. In both experiments, we show that PPM outperforms PM in full-filling people's preferences while maximizing the distribution of vaccines among the higher priority groups.

1 INTRODUCTION

A pandemic creates massive challenges for health care organizations, including inadequate capacity, financial loss, and resource management. Recently, the COVID-19 pandemic shows us how a pandemic can cause an array of acute challenges, from providing enough protective equipment to adjusting hospital's capacity by reducing financial loss (Ali and Alharbi, 2020; Xiong et al., 2020; Nicole et al., 2020). As of September 2021, more than 230 million cases are reported worldwide, along with 4.7 million deaths (Worldometers, 2021). Health care specialists are urging people to get vaccinated to get over the pandemic. However, as most countries have already started to vaccinate people, it becomes challenging to distribute vaccines among a massive population with limited resources. People can have different preferences of locations and timing for getting their vaccines; moreover, our vaccine distribution centers have limited capacity. Therefore, assigning appointments for vaccination becomes challenging for health care organizations. Mahzabeen et al. (Emu et al., 2021) propose several vaccines distributing models using Constraint Satisfaction Programming (CSP). These models do not consider people's preferences for setting up appointments in their preferred vaccine distribution centers at their preferred time.

In this paper, we propose two optimal vaccine distribution models, namely,

- Priority based Model (PM)
- Priority & Preference based Model (PPM),

to optimize the distribution of vaccines in a given geographical region. We formulate both models as Integer Linear Programming (ILP) models and show that these problems are NP-hard. In PM, our goal is to maximize the distribution of vaccines among the people with higher priority and maximize the minimum resource utilization of any distribution centers among all vaccine distribution centers. In PPM, we aim to maximize the number of vaccinated people with higher priority assigned to one of their preferred vaccine distribution centers and time slots. We also aim to maximize the minimum resource utilization of any distribution center among all distribution centers. We perform a random experiment and a case study on these models and show how these models provide the most optimal vaccine distribution. For the case study, we use COVID-19 vaccination data obtained from Thunder Bay, Ontario, Canada. Our experimental analysis shows that PPM assigns most people to

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their preferred distribution centers and time slot. Our models also maximize the minimum resource utilization of any distribution centers among all distribution centers. Our vaccine distribution models can optimally distribute vaccines or any medical equipment among a massive population.

The rest of the paper is organized the follows. In Section 2, we discuss the related works in the literature. The system model and optimal vaccine distribution models are discussed in Section 3 and 4, respectively. Section 5 discusses the experimental analysis on our proposed models. Finally, we conclude the paper in Section 6.

2 RELATED WORKS

Vaccines are one of the most effective ways to prevent a sudden outbreak and develop immunity against certain infectious diseases (Tan et al., 2012). The overall supply chain of vaccines can be categorized into four broad categories: product, production, allocation, and distribution (Duijzer et al., 2018). The importance of strategic spatiotemporal vaccine distribution to control the spread of highly infectious diseases has been proven undeniable through existing literature (Grauer et al., 2020). The final step of vaccine distribution involves various decisions such as inventory control, location of vaccine stockpiles, logistics related to the point of dispensing, staffing levels, routing, and scheduling, etc. (Duijzer et al., 2018). Often, the operations research perspective is adopted to achieve optimal vaccine distribution schemes. Most of the operations research-based models have been developed using Quadratic Programming (QP), Integer Linear Programming (ILP), Mixed Integer Linear Programming (MILP), Constraint Optimization (CO) frameworks (Emu et al., 2021).

Sharon et al. (Hovav and Tsadikovich, 2015) propose a mathematical model to improve the overall supply chain by optimizing inventory management of influenza vaccines. With the help of the Lagrangian approach and branch-and-bound techniques, a research study has been conducted to factor in environmental considerations for the cold supply chain of vaccines (Saif and Elhedhli, 2016). The authors propose a hybrid optimization-simulation tool to reduce the effects of refrigerant gases and carbon emissions caused by the preservation of vaccines as much as possible (Saif and Elhedhli, 2016). Lin et al. have developed a policy-based model for taking intermediary decisions on the transportation of vaccines from distributors to retailers (Lin et al., 2020).

An equity constraints-based framework using the Gini index has been studied to distribute Influenza vaccines optimally (Enayati and Özaltın, 2020). The authors justify the model implications and the scalability of the model on larger instances through extensive simulation studies (Enayati and Özaltın, 2020). Another research study has developed a simulation tool to optimize the average waiting time of individuals to expedite mass vaccination rate (Gupta et al., 2013). Furthermore, Rajan et al. formulated a stochastic genetic algorithm for deriving optimal vaccine distribution strategies that have been proven to demonstrate 85% more efficacy compared to random vaccination schemes (Patel et al., 2005). Recently, some of the research studies have made an effort to maintain transparency, data integrity, and immutability using blockchain framework for vaccine rollouts (Antal et al., 2021). This research study highly emphasized the employment of smart contracts to enable awareness among network peers (Antal et al., 2021).

To the best of our knowledge, existing literature studies ignore the preferences of individuals for the vaccine distribution decision-making process. In this paper, we propose an ILP based vaccine distribution model that simultaneously prioritizes individual preferences and resource utilization of vaccine distribution centers. Such a model can be generalized and adapted for sudden pandemic and epidemic urgency situations that may arise in the future. Moreover, the convenience caused by incorporating the preferences of the people alongside demographics may even further diminish vaccine hesitancy and accelerate vaccination rates.

3 SYSTEM MODEL

In this section, we discuss the system model for the vaccine distribution problem. We are given a set $\mathcal{E} = \{e_1, e_2, \ldots, e_n\}$ of n people required to be vaccinated. We use a set $\mathcal{P} = \{p_1, p_2, \ldots, p_n\}$ of n non-zero positive integers to specify the priority of people for vaccination purpose, where p_i defines the priority level of a person $e_i \in \mathcal{E}$. The higher values of p_i indicate higher priority. A person with higher priority is desired to get faster vaccination service. Let $\mathcal{N}$ represent the total number of available vaccines. We are given a set $\mathcal{H} = \{h_1, h_2, \ldots, h_m\}$ of m vaccine distribution centres (DCs), i.e., hospitals. Let $\mathcal{T}$ be a set of time slots. We denote $\mathcal{T}_j$ as a set of available time slots provided by the DC h_j where $\mathcal{T}_j \subseteq \mathcal{T}$. Therefore, we can infer that $\mathcal{T} = \cup_{j=1}^{m} \mathcal{T}_j$. Let $\mathcal{B} = \{b_1, b_2, \ldots, b_m\}$ be a set of m positive integers where b_j specifies the number of people can be vaccinated in each time slot in h_j. Each person $e_i \in \mathcal{E}$ provides a list $\mathcal{L}_i$ of preferred

DCs including a set of preferred time slots from the corresponding DCs. Hence, each element in $\mathcal{L}_i$ includes a DC $h_j \in \mathcal{H}$ and a set $\mathcal{T}_j^i \subseteq \mathcal{T}_j$ of preferred time slots from h_j, i.e., each element of $\mathcal{L}_i$ is defined as $\{h_j \in \mathcal{H}, \{\mathcal{T}_j^i \subseteq \mathcal{T}_j\}\}$. In this paper, we consider the solution of the vaccine distribution problem in a binary decision variable as $x_{i,j,k} \in \{0,1\}$ where,

$$x_{i,j,k} = \begin{cases} 1, & \text{a person } e_i \text{ is assigned to } h_j \text{ at } t_k \in \mathcal{T}_j; \\ 0, & \text{otherwise.} \end{cases}$$

4 OPTIMAL VACCINE DISTRIBUTION MODELS

In this paper, we formulate the vaccine distribution research problem as an Integer Linear Programming (ILP) model. A DC h_j can give vaccine to at most $\mid \mathcal{T}_j \mid \times b_j$ number of people. We define the resource utilization of a DC as the utilization rate of its total capacity. Therefore, the resource utilization $\mathcal{R}_j$ of a DC h_j is

$$\mathcal{R}_j = \frac{\sum_{i=1}^{n} \sum_{t_k \in \mathcal{T}_j} x_{i,j,k}}{\mid \mathcal{T}_j \mid \times b_j}. \tag{1}$$

Our primary goal is to maximize the number of overall vaccinated people and maximize the minimum resource utilization of any DC among all DCs. However, in many cases, we aim to maximize the distribution of vaccines among the people with higher priority. Moreover, we may want to maximize the number of vaccinated people assigned to one of their preferred DCs and time slots. Thus, we propose two different vaccine distribution ILP models: Priority-based Model (PM) and Priority & Preference-based Model (PPM).

4.1 Priority based Model (PM)

In this model, our goal is to maximize the distribution of vaccines among the people with higher priority and maximize the minimum resource utilization of any DC among all DCs. Therefore, we formulate the following optimization problem:

$$\textbf{MAX} \quad \left(\min_{j=1}^{m} \mathcal{R}_j + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{t_k \in \mathcal{T}_j} p_i \times x_{i,j,k} \right) \tag{2}$$

subject to,

$$\sum_{j=1}^{m} \sum_{t_k \in \mathcal{T}_j} x_{i,j,k} \leq 1 \; ; \quad \forall e_i \in \mathcal{E} \tag{3}$$

$$\sum_{i=1}^{n} x_{i,j,k} \leq b_j \; ; \quad \forall h_j \in \mathcal{H} \text{ and } \forall t_k \in \mathcal{T}_j \tag{4}$$

$$\sum_{i=1}^{n} \sum_{t_k \in \mathcal{T}_j} x_{i,j,k} \leq \mid \mathcal{T}_j \mid \times b_j \; ; \quad \forall h_j \in \mathcal{H} \tag{5}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{t_k \in \mathcal{T}_j} x_{i,j,k} \leq \mathcal{N} \tag{6}$$

$$x_{i,j,k} = \{0,1\}; \quad \forall e_i \in \mathcal{E}, \forall h_j \in \mathcal{H} \text{ and } \forall t_k \in \mathcal{T}_j \tag{7}$$

The constraint in equation (3) ensures that each person can take at most one vaccine. The constraint in equation (4) ensures that in a DC h_j, at most b_j people can be vaccinated in each time slot. In equation (5), we ensure that each DC h_j can give vaccine to at most $\mid \mathcal{T}_j \mid \times b_j$ people. The constraint in equation (6) ensures that the total vaccine distribution should not be more than the total number of available vaccines.

4.2 Priority & Preference based Model (PPM)

In this model, our goal is to maximize the number of vaccinated people with higher priority assigned to one of their preferred DCs and time slots and maximize the minimum resource utilization of any DC among all DCs. We define a preference score function f which takes a person e_i, a DC h_j and a time slot t_k as input and calculates the preference score for assigning the person e_i to the DC h_j at the time slot t_k. The definition of the function f is

$$f(e_i, h_j, t_k) = \begin{cases} 2 + p_i^2, & \text{if } h_j \text{ is in } \mathcal{L}_i \text{ and } t_k \in \mathcal{T}_j^i; \\ 1 + p_i^2, & \text{if } h_j \text{ is in } \mathcal{L}_i \text{ and } t_k \notin \mathcal{T}_j^i; \\ p_i^2, & \text{otherwise.} \end{cases}$$

We formulate the following optimization problem

$$\textbf{MAX} \left(\min_{j=1}^{m} \mathcal{R}_j + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{t_k \in \mathcal{T}_j} f(e_i, h_j, t_k) \times x_{i,j,k} \right) \tag{8}$$

subject to the equations (3)-(7).

4.3 The Complexity of the PM and PPM Optimization Problems

The PPM optimization problem is NP-hard, and we can obtain the hardness result by reduction from the Generalized Assignment Problem (GAP), which is NP-hard (Özbakir et al., 2010). In the generalized assignment problem, we have given a set of n items and m bins with a fixed capacity. For each bin, each item has a profit and weight. The goal is to find the maximum possible profit from a feasible item-bin assignment. In a viable solution, the total weight of assigned items in a bin must not exceed its capacity. If the profit and weight of an item do not vary between different bins, then the generalized assignment problem is reduced to the multiple knapsack problem. Therefore, our proposed PM problem can also be shown NP-hard by reduction from the multiple knapsack problem, which is NP-hard (Chekuri and Khanna, 2005). Consequently, we can derive the following theorem.

Theorem 1. *The PM and PPM optimization problems are NP-hard.*

5 EXPERIMENTAL EVALUATION

In this section, we demonstrate the experimental results of our proposed models in two different scenarios. In the following, we show the performances of our models on randomly generated datasets and a real-world dataset from Thunder Bay, Ontario, Canada. In all cases, we compare the results between our two proposed models and show how PPM optimizes both priority and preference in each assignment. We have used Gurobi (Gurobi Optimization, LLC, 2021) optimization solver to solve the proposed ILP models.

5.1 Random Simulation

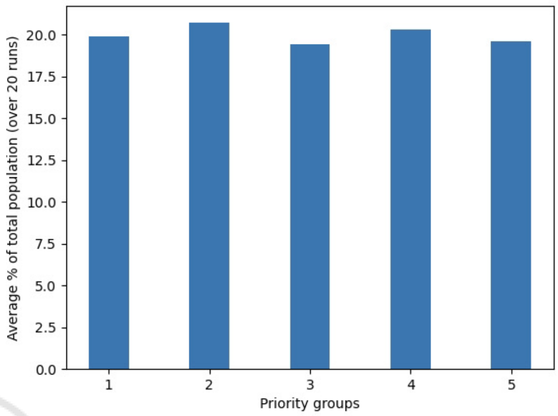
In this experiment, we consider 320 people ($|\mathcal{E}|$) for vaccination in 5 different vaccine DCs and the total number of available vaccines ($\mathcal{N}$) is 150. The details of each vaccine DC are given in Table 1. We consider five different priority groups $\mathcal{P} = \{1,2,3,4,5\}$ where 5 being the highest priority group and 1 being the lowest priority group. To show the impact of the population distribution and preference distribution, we use two different distribution models in our random simulation; namely,

- **Random-Simulation-1 (RS-1):** Population and preference lists of vaccine DCs are distributed using uniform random distribution.
- **Random-Simulation-2 (RS-2):** Population is distributed into the five groups using the normal distribution. Preference lists of vaccine DCs are generated using a poisson like distribution where h_3 and h_4 are likely to be chosen by most people.

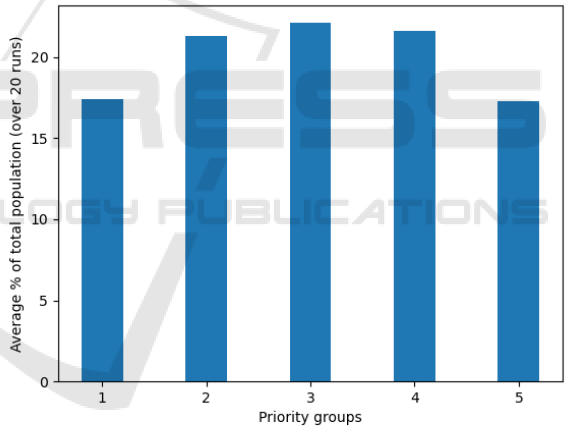
In both models, the preference of time slots is chosen using a uniform random distribution. Figure 1 shows the distribution of 300 people among 5 priority groups for RS-1 and RS-2.

Table 1: No of time slots and maximum no of vaccination in each time slot of vaccine DCs in the random simulation.

$\mathcal{H}$:	h_1	h_2	h_3	h_4	h_5
$\mid \mathcal{T}_j \mid$	20	15	20	25	10
b_j	2	1	3	2	1



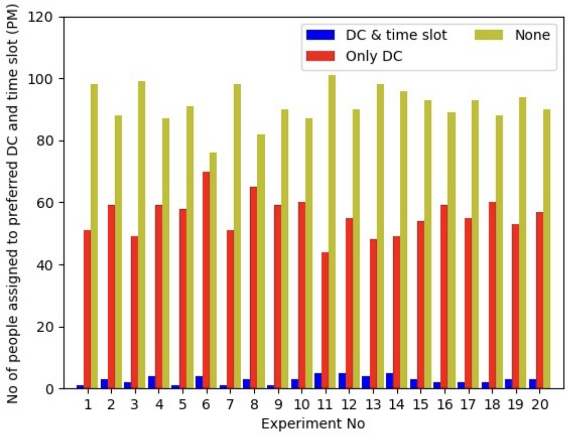
(a) RS-1



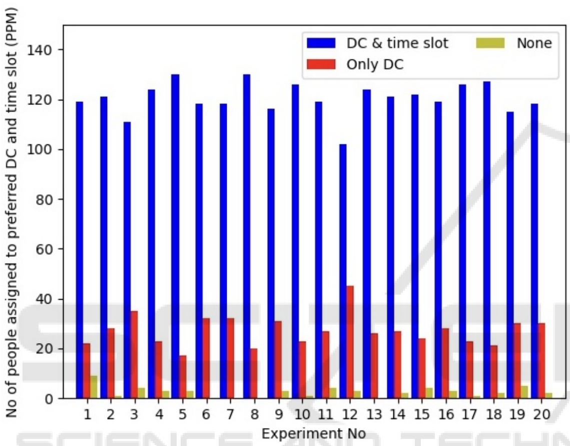
(b) RS-2

Figure 1: Distribution of 300 people among 5 priority groups in the random simulation.

Figure 2 and 3 show the distribution of assigned people to their preferred DC and time slot by finding an optimal solution for both PPM and PM for RS-1 and RS-2. In both random simulations, PPM assigns around 80% of people to their preferred DC and time slot because the model considers each person's priority level and preference list in the objective function. However, the priority-based model (PM) only considers the priority level of each people in the objective function. Therefore, in this model, nearly 3% of people are assigned to their preferred DC and time slot, and most of the people are assigned to DCs and time

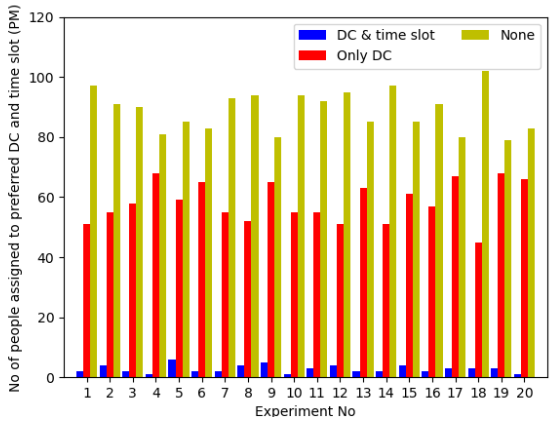


(a) RS-1 (PM)

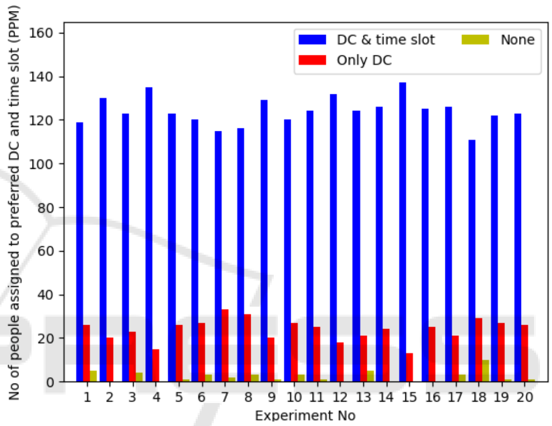


(b) RS-1 (PPM)

Figure 2: No of people assigned to their preferred DC and time slot in RS-1 (total 150 vaccinated people).



(a) RS-2 (PM)



(b) RS-2 (PPM)

Figure 3: No of people assigned to their preferred DC and time slot in RS-2 (total 150 vaccinated people).

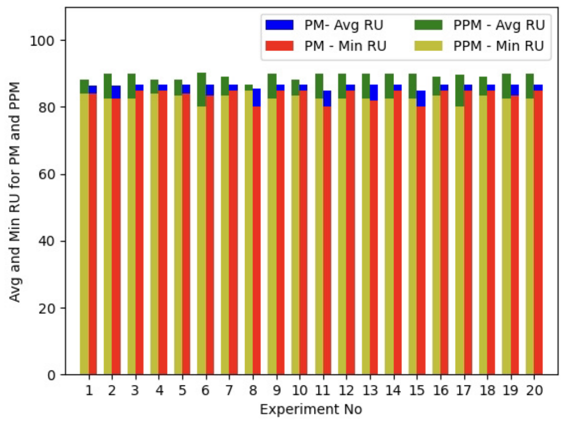
slots that are not in their preference list.

In Figure 4, we show the comparison between average and minimum resource utilization (percentage) of the vaccine DCs in each experiment for the random simulation. In both random simulations, the average resource utilization in PPM is slightly higher than the average resource utilization in PM. Moreover, the minimum and the average resource utilization rates are higher than 80% in all cases. Since the objective functions maximize the minimum resource utilization of any DC among all DCs in both PPM and PM, both the models perform similarly in terms of maximizing the minimum resource utilization.

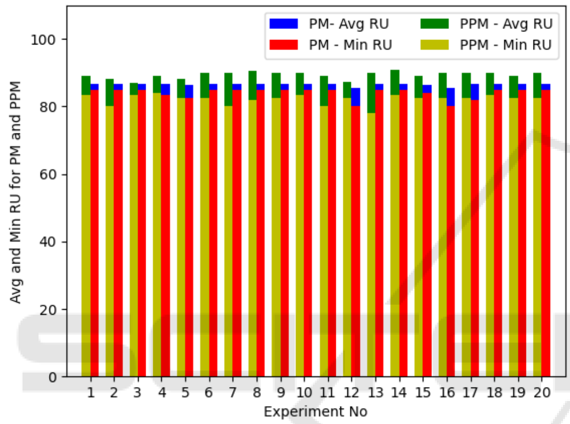
Figure 5 shows the distribution of vaccinated people among the 5 priority groups. The average percentage of vaccinated people in each group for both PM and PPM is equal. In our objective functions, the weight of priority is much higher than the weight of the preference list. Therefore, in both models, people in higher priority groups will always be vaccinated before those in lower priority groups.

5.2 Case Study - Thunder Bay

In this experiment, we consider the COVID-19 vaccine distribution dataset from Thunder Bay, Ontario, Canada. First, we use the 2016 census profile to get the number of people in different age ranges (Census, 2019). Then, we divide the people into five priority groups according to their age. In Table 2, we show the distribution of the Thunder Bay population into five priority groups. We consider 30 vaccine distribution centers where five of them are primary vaccination dispatch centers and others are local pharmacies (cov, 2020). The primary distribution centers provide vaccines to 8-9 people in each time slot, whereas local pharmacies offer 1-2 people. We assume that a total of 4000 vaccines are available for vaccination. The preference list of DCs and time slots for each person are generated using uniform random distribution.

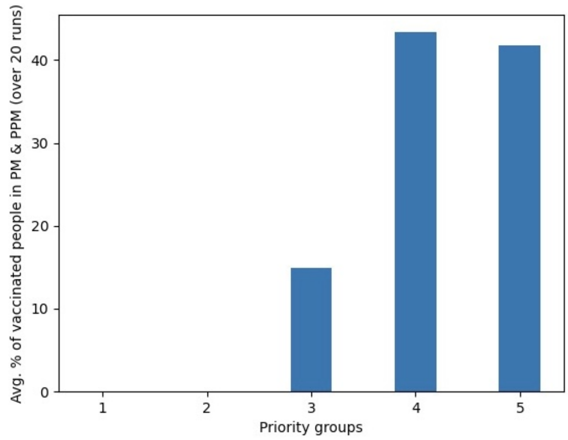


(a) RS-1

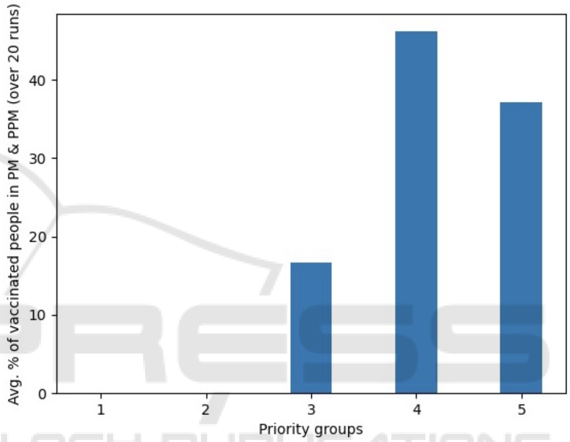


(b) RS-2

Figure 4: Comparison of average and minimum resource utilization of the vaccine DC's in random simulation.



(a) RS-1



(b) RS-2

Figure 5: Distribution of vaccinated people among the 5 priority groups.

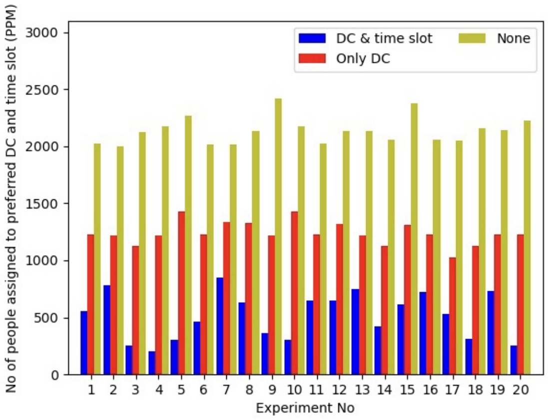
Table 2: Distribution of Thunder Bay population into five priority groups.

Group no	Age range	No of people
1	20-29	13160
2	30-39	10905
3	40-49	10990
4	50-59	14505
5	60 and above	26195

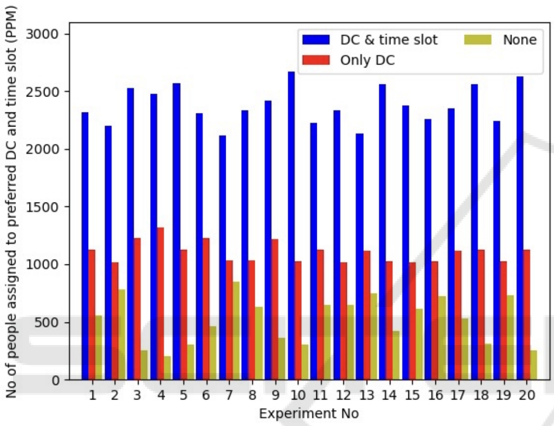
In Figure 6, we show the distribution of assigned people to their preferred DC and time slot in optimal solutions for both models. In our priority and preference-based model, around 65% of people on average are assigned to their preferred DC and time slot. However, in only the priority-based model, only 7.5% of people are assigned to their preferred DC and time slots. In this experiment, all the vaccine distribution centres are filled to 100% capacity. Therefore, their minimum and average resource utilization become 100%. Since the no of people in group 5 is much higher than the number of available vaccines, only people from that group get vaccinated.

6 CONCLUSION

This paper proposes two optimization models (PM and PPM) based on Integer Linear Programming (ILP) framework to find optimal solutions for assigning vaccination appointments for people while considering their priorities and preferences. In PM, we aim to maximize the distribution of vaccines among the people with higher priority. In PPM, our goal is to maximize the number of vaccinated people with higher priority groups by assigning appointments in their desired locations and time slots. Moreover, in both models, we maximize the minimum resource uti-



(a) PM



(b) PPM

Figure 6: No of people assigned to their preferred DC and time slot in Thunder Bay dataset.

lization of any vaccine distribution center. Experimental analysis on the random dataset and COVID-19 vaccination dataset from Thunder Bay shows that our proposed PPM outperforms PM in full-filling people's preferences while maximizing the distribution of vaccines among the higher priority groups. Although our proposed models can be adapted across various scenarios for distributing any medicine among a massive population, we have shown a minimal number of constraints in our models due to resource constraints. In the future, depending on any specific scenario, our models can be expanded by adding more real-world constraints.

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