Moving Gap Solitons in Semilinear Coupled Bragg Gratings with a Phase Mismatch

Shuvashis Saha and Javid Atai
School of Electrical and Information Engineering, The University of Sydney, NSW 2006, Australia

Keywords: Moving Gap Solitons, Bragg Gratings, Phase Mismatch, Dual-core System.

Abstract: We consider the existence and stability of the moving gap solitons in a semilinear coupler where one core has Kerr nonlinearity and the other is linear and both cores are equipped with a Bragg grating with a phase mismatch between them. We analyze the effect of the phase mismatch and the soliton velocity on the existence and stability of moving gap solitons. It is found that larger phase mismatch leads to the expansion of the stability region of the moving gap solitons.

1 INTRODUCTION

Nonlinear photonic structures equipped with fiber Bragg gratings (FBGs) have attracted much interest due to their potential applications in the slow light applications (Aceves and Wabnitz, 1989; de Sterke and Sipe, 1994; Sukhorukov and Kivshar, 2006). One of the most interesting features of FBGs is that there is a band gap in their linear spectrum where no linear waves can propagate. Moreover, a strong effective dispersion is generated due to the cross-coupling between forward and backward propagating waves (de Sterke and Sipe, 1994). This strong effective dispersion can be counterbalanced by the nonlinearity of the medium when the pulse intensity is sufficiently high giving rise to solitary waves known as gap solitons (GSs). A very important property of GSs is that they can propagate with any velocities ranging from zero to the speed of light in the medium (Aceves and Wabnitz, 1989). Experimentally GSs were observed in a $\sim 6\,cm-$long FBGs (Eggleton et al., 1996). Thus far, moving gap solitons with a velocity in excess of 23% of the speed of light in the medium have been observed experimentally (Mok et al., 2006).

The properties of GSs have been studied theoretically for different photonic structures and nonlinear media such as cubic-quintic nonlinear media (Dasanayaka and Atai, 2013; Islam and Atai, 2014), photonic crystals (Skryabin, 2004; Atai et al., 2006; Neill and Atai, 2007), nonuniform Bragg gratings (Atai and Malomed, 2005; Ahmed and Atai, 2017), waveguide arrays (Mandelik et al., 2004; Dong et al., 2011) and dual-core fibers with gratings in one or both cores (Mak et al., 1998; Atai and Malomed, 2005; Baratali and Atai, 2012).

Semilinear coupled systems exhibit superior switching characteristics and support a wide range of GSs (Chowdhury and Atai, 2017). It has been demonstrated that very slow GSs can be generated in a grating-assisted semilinear coupler (Atai and Malomed, 2000; Shnaiderman et al., 2011). In the case of dual-core systems made of coupled identical Bragg gratings, both symmetric and asymmetric GSs exist in the system (Mak et al., 2004). However, in the presence of a finite phase shift between the gratings, the symmetric GSs are transformed into quasi-symmetric ones (Tsofe and Malomed, 2007).

In this paper we analyze the existence and dynamics the moving GSs in a semilinear coupled system where both cores are equipped with a BG but there is a phase mismatch between the gratings and one core is linear while other one has Kerr type nonlinearity.

2 THE MODEL

The system model which was introduced in Ref. (Saha and Atai, 2021) is given by the following coupled partial differential equations:

$$\begin{aligned}
iu_t + iu_x + \left[|v|^2 + \frac{1}{2}|u|^2\right]u + v + \kappa\phi &= 0 \\
iv_t - iv_x + \left[|u|^2 + \frac{1}{2}|v|^2\right]v + u + \kappa\psi &= 0 \\
i\phi_t + ic\phi_x + \psi e^{i\theta/2} + \kappa u &= 0 \\
i\psi_t - ic\psi_x + \phi e^{-i\theta/2} + \kappa v &= 0
\end{aligned} \quad (1)$$

Saha, S. and Atai, J.
Moving Gap Solitons in Semilinear Coupled Bragg Gratings with a Phase Mismatch.
DOI: 10.5220/0010819900003121
In *Proceedings of the 10th International Conference on Photonics, Optics and Laser Technology (PHOTOPTICS 2022)*, pages 38-41
ISBN: 978-989-758-554-8; ISSN: 2184-4364

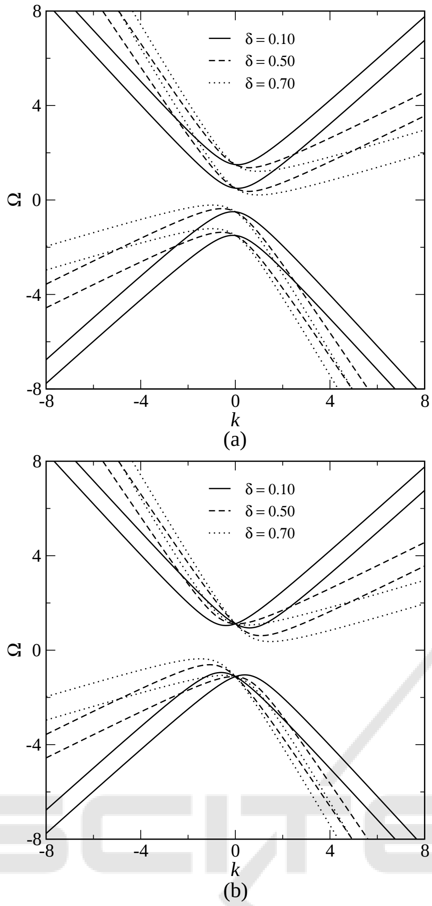

Figure 1: Examples of the spectra generated by the dispersion relation for $\kappa = 0.5$, $c = 1$, and different values of soliton velocity δ; (a) $\theta = 0$ and (b) $\theta = 2\pi$.

where u and v represent the forward and backward waves in the nonlinear core and ϕ and ψ are their counterparts in the linear core. κ accounts for the linear coupling coefficient between two cores. Relative group velocity in the nonlinear core is set to 1 and c denotes the relative group velocity mismatch. The phase mismatch between two Bragg gratings is denoted by θ.

To determine the moving GS solutions, Eqs. (1) need to be transformed to the moving frame using the transformation $\{X,T\} = \{x - \delta t, t\}$. δ accounts for the normalized velocity of moving solitons ($\delta = 1$ corresponds to velocity of light in the medium). The transformation gives rise to the following set of partial differential equations in the moving reference frame:

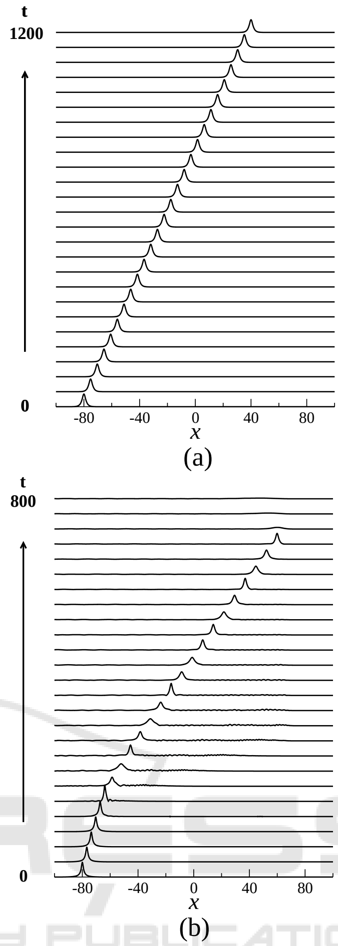

Figure 2: Examples of the evolution of moving gap solitons for zero phase mismatch ($\theta = 0$). (a) Stable soliton corresponding to $\Omega = 0.20$ and (b) Unstable soliton corresponding to $\Omega = -0.45$. The other parameters are $\kappa = 0.5$, $c = 1.0$ and $\delta = 0.1$. Only the u components are shown.

$$
\begin{aligned}
iu_T + i\,(1-\delta)\,u_X + \left(|v|^2 + \frac{1}{2}|u|^2\right) u + v + \kappa\phi &= 0 \\
iv_T - i\,(1+\delta)\,v_X + \left(|u|^2 + \frac{1}{2}|v|^2\right) v + u + \kappa\psi &= 0 \\
i\phi_T + i\,(c-\delta)\,\phi_X + \psi e^{i\theta/2} + \kappa u &= 0 \\
i\psi_T - i\,(c+\delta)\,\psi_X + \phi e^{-i\theta/2} + \kappa v &= 0
\end{aligned}
\tag{2}
$$

To determine the linear spectrum in which the moving soliton solutions may reside, we substitute the plane wave solutions of $\{u, v, \phi, \psi\} \sim \exp(ikX - i\Omega T)$ into the linearized form of Eqs. (2), which gives the following dispersion relation:

$$
\Omega^4 + 4\Omega^3\delta k - \Omega^2 c^2 k^2 + 6\Omega^2\delta^2 k^2 - \Omega^2 k^2 - 2\Omega^2 + k^2
$$
$$
-2\Omega^2\kappa^2 - 2\Omega c^2\delta k^3 + 4\Omega\delta^3 k^3 - 2\Omega\delta k^3 - 4\Omega\delta k\kappa^2
$$

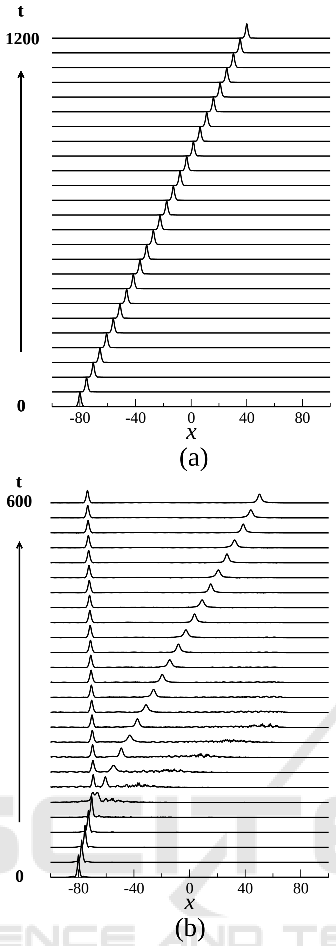

Figure 3: Examples of the evolution of moving gap solitons for $\theta = 2\pi$. (a) Stable soliton corresponding to $\Omega = 0.20$ and (b) Unstable soliton corresponding to $\Omega = -0.70$. The other parameters are $\kappa = 0.5$, $c = 1.0$ and $\delta = 0.1$. Only the u components are shown.

$$-4\Omega\delta k - c^2\delta^2k^4 + c^2k^4 + c^2k^2 - 2ck^2\kappa^2 + \delta^4k^4 + \kappa^4 - \delta^2k^4 - 2\delta^2k^2\kappa^2 - 2\delta^2k^2 - 2\cos\left(\frac{\theta}{2}\right)\kappa^2 + 1 = 0, \quad (3)$$

where k denotes the wave number and Ω is the frequency detuning in the moving reference frame and it is related to the frequency detuning (ω) in the stationary frame by $\Omega(k) = \omega(k) - \delta k$. Fig. 1 shows the dispersion relation curves for different values of δ, when $c = 1$, $\kappa = 0.5$ for $\theta = 0$ and 2π. From straightforward mathematical analysis of Eq. (3), it is concluded that when $c = 1$, the moving GS solutions exist only in the central band gap. However, there exists a critical value of soliton velocity, $\delta_{cr} < 1$ for which the central band gap completely closes.

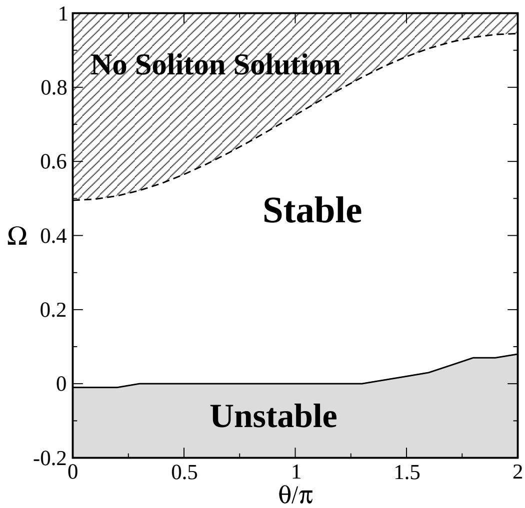

Figure 4: Stability diagram of the moving gap solitons in the (θ, Ω) plane for $\kappa = 0.5$, $c = 1.0$ and $\delta = 0.10$.

3 STABILITY OF MOVING SOLITONS

To obtain the moving soliton solutions, Eqs. (2) were solved numerically using a relaxation algorithm. We then utilized the split-step Fourier method to investigate the stability of moving GS solutions through direct simulation of Eqs. (1). Figs. 2 and 3 show some examples of the evolution of stable and unstable moving GSs for $\theta = 0$ and 2π, respectively. Our analysis shows that the model supports stable moving solitons for values of θ in the range $0 \leq \theta \leq 2\pi$. A noteworthy finding is that in the absence of the phase shift, i.e. $\theta = 0$, the unstable solitons may either decay completely (Fig. 2(b)) or shed some energy in the form of radiation and evolve to another stable moving soliton. However, as is shown in Fig. 3(b), when $\theta = 2\pi$, an unstable soliton loses energy upon propagation and then splits and evolves into a quiescent and a moving soliton.

Fig. 4 summarizes the result of stability diagram on the (θ, Ω) plane when $c = 1$, $\kappa = 0.50$ and $\delta = 0.1$. A notable finding is that, increasing θ leads to the expansion of the stability region. The interplay of θ and other parameters and their effect on the stability of solitons is currently under investigation.

4 CONCLUSIONS

We have investigated the existence and stability of moving GSs in a semilinear dual-core system where one core has Kerr nonlinearity and the other is linear and both cores are equipped with BGs but there is phase shift between them. We have focused on the effect of phase mismatch and soliton velocity on the existence and stability of moving GSs. A notable finding is that the higher phase mismatch leads to the expansion of the stable regions for the moving GSs. Another finding is that for certain parameters unstable solitons may evolve into a quiescent and a moving soliton. This outcome and its prevalence is currently under investigation.

REFERENCES

Aceves, A. B. and Wabnitz, S. (1989). Self-induced transparency solitons in nonlinear refractive periodic media. *Phys. Lett. A*, 141:37–42.

Ahmed, T. and Atai, J. (2017). Bragg solitons in systems with separated nonuniform bragg grating and nonlinearity. *Phys. Rev. E*, 96:032222.

Atai, J. and Malomed, B. A. (2000). Bragg-grating solitons in a semilinear dual-core system. *Phys. Rev. E*, 62:8713.

Atai, J. and Malomed, B. A. (2005). Gap solitons in bragg gratings with dispersive reflectivity. *Phys. Lett. A*, 342:404 – 412.

Atai, J., Malomed, B. A., and Merhasin, I. M. (2006). Stability and collisions of gap solitons in a model of a hollow optical fiber. *Opt. Commun.*, 265:342–348.

Baratali, B. H. and Atai, J. (2012). Gap solitons in dual-core bragg gratings with dispersive reflectivity. *J. Opt.*, 14:065202.

Chowdhury, S. A. M. S. and Atai, J. (2017). Moving bragg grating solitons in a semilinear dual-core system with dispersive reflectivity. *Sci. Rep.*, 7:1–12.

Dasanayaka, S. and Atai (2013). Stability and collisions of moving bragg grating solitons in a cubic-quintic nonlinear medium. *J. Opt. Soc. Am. B*, 30:396–404.

de Sterke, C. M. and Sipe, J. E. (1994). Gap solitons. *Prog. Optics*, 33:203–260.

Dong, R., Rüter, C. E., Kip, D., Cuevas, J., Kevrekidis, P. G., Song, D., and Xu, J. (2011). Dark-bright gap solitons in coupled-mode one-dimensional saturable waveguide arrays. *Phys. Rev. A*, 83:063816.

Eggleton, B. J., Slusher, R. E., Krug, P. A., and Sipe, J. E. (1996). Bragg grating solitons. *Phys. Rev. Lett.*, 76:1627–1630.

Islam, M. J. and Atai, J. (2014). Stability of gap solitons in dual-core bragg gratings with cubic-quintic nonlinearity. *Laser Phys. Lett.*, 12:015401.

Mak, W. C. K., Malomed, B. A., and Chu, P. L. (1998). Solitary waves in coupled nonlinear waveguides with Bragg gratings. *J. Opt. Soc. Am. B*, 15:1685–1692.

Mak, W. C. K., Malomed, B. A., and Chu, P. L. (2004). Symmetric and asymmetric solitons in linearly coupled bragg gratings. *Phys. Rev. E*, 69:066610.

Mandelik, D., Morandotti, R., Aitchison, J. S., and Silberberg, Y. (2004). Gap solitons in waveguide arrays. *Phys. Rev. Lett.*, 92:093904.

Mok, J. T., de Sterke, C. M., Littler, I. C. M., and Eggleton, B. J. (2006). Dispersionless slow light using gap solitons. *Nature Phys.*, 2:775–780.

Neill, D. R. and Atai, J. (2007). Gap solitons in a hollow optical fiber in the normal dispersion regime. *Phys. Lett. A*, 367:73–82.

Saha, S. and Atai, J. (2021). Effect of phase mismatch between the bragg gratings on the stability of gap solitons in semilinear dual-core system. In *Proceedings of the 9th International Conference on Photonics, Optics and Laser Technology- PHOTOPTICS*, pages 36–39.

Shnaiderman, R., Tasgal, R. S., and Band, Y. B. (2011). Creating very slow optical gap solitons with a grating-assisted coupler. *Opt. Lett.*, 36:2438–2440.

Skryabin, D. V. (2004). Coupled core-surface solitons in photonic crystal fibers. *Opt. Express*, 12(20):4841–4846.

Sukhorukov, A. A. and Kivshar, Y. S. (2006). Slow-light optical bullets in arrays of nonlinear bragg-grating waveguides. *Phys. Rev. Lett.*, 97:233901.

Tsofe, Y. J. and Malomed, B. A. (2007). Quasisymmetric and asymmetric gap solitons in linearly coupled bragg gratings with a phase shift. *Phys. Rev. E*, 75:056603.