Optimization of Adaptive Cruise Control under Uncertainty

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Abstract: With the recent developments of autonomous vehicles, extensive studies have been conducted about Adaptive Cruise Control (ACC), which is an essential component of advanced driver-assistant systems (ADAS). The safety assessment must be performed on the ACC system before its commercialization. The validation process is generally conducted via simulation due to insufficient on-road data and the diversity of driving scenarios. Our paper aims to develop an optimization-based reference generation model for ACC, which can be used as a benchmark for assessment and evaluation. The model minimizes the difference between the actual and reference inter-car distance, while respecting constraints about vehicle dynamics and road regulations. ACC sensors can be impacted by external factors such as weather and produce inaccurate data. To handle the uncertainty involved, we also propose a chance-constrained stochastic model to reach results with a high level of confidence. Our numerical results illustrate that the stochastic model outperforms the deterministic model on randomly generated driving scenarios.

1 INTRODUCTION

During the past two decades, there has been an increasing trend towards autonomous driving in both industry and research, which led to many technological advances and commercial successes. Autonomous vehicle applications, e.g., advanced driver assistance systems (ADAS) are extensively incorporated into modern cars to enhance safety and improve driving comfort. The most basic feature of ADAS is adaptive cruise control (ACC), which has been the focus of researchers and has been actively studied.

Since 1966, ACC aims to keep a safe distance with a leading vehicle by adjusting the vehicle's speed and acceleration (Levine and Athans, 1966). This functionality relies both on sensor information about the location and the motion of the vehicle ahead and on a controller to regulate the spacing between the vehicles. An ACC-equipped vehicle drives at a preset speed until a leading car is detected by the sensors, then switches to the distance regulation mode by activating the ACC controller, which calculates the safety distance and controls the operation.

Various approaches are applied to achieve the objective of designing an ACC that most closely matches the human expert driving behavior in terms of maneuvering vehicle speed according to different driving conditions with respect to traffic regulations and comfortable driving. These ACC systems target different objectives and are designed under different standards. Therefore, we need a thorough validation process to ensure the safety of those ACC systems and also assess their performance before making them commercially available. The result of the validation and evaluation also allows us to identify potential areas for improvement by identifying current weaknesses. Due to the fact that the real road tests, which are time consuming and costly, cannot cover a large number of driving scenarios, we carry out the validation process within a simulator, which can generate driving scenarios. The driving scenarios include the motion state of the vehicles at each sampling time.

As part of functional testing of ADAS, the goal of the ACC validation is to determine whether the right decision was made, a critical accident was avoided, and identify potential flaws. The validation process begins with our model. In our model, each driving scenario serves as an input, and the

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reference commands are calculated by solving an optimization problem. Then we can analyze the actual commands by comparing them to our generated reference commands. This process is illustrated in Figure 1. Generating reference trajectories is a typical motion planning problem, and there are many ways to solve it, including sampling-based methods, graph-based methods, and optimization-based methods. Among those methods, the optimization approach fits our needs well, since we can improve comfort and security of the vehicle by limiting jerks, acceleration, speed, and relative position, and reach a reasonable distance ahead by minimizing the value of an objective function. Furthermore, this approach gives us a lot of flexibility in tailoring objectives and constraints according to various driving scenarios and requirements. These facts led us to devise an ACC reference generation model based on optimization.

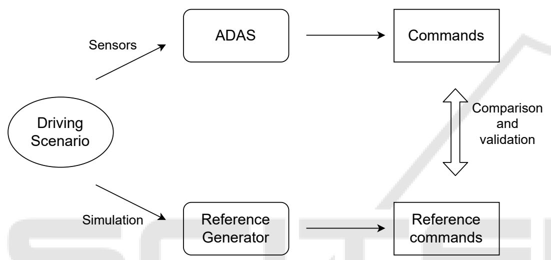

Figure 1: Validation process of ADAS.

As part of an ACC system, various types of sensors may be employed, such as cameras, lidar, radar etc. Sensor performance is highly influenced by a variety of factors, including the maintenance state and environmental conditions (Rasshofer et al., 2011). There is an inherent level of inaccuracy in sensor data which must be accounted for when computations follow. In order to deal with sensor uncertainty, we develop a chance constrained stochastic programming model and compare its simulation results with the deterministic model.

The main contribution of this paper is to study and compare deterministic and stochastic optimization models for ACC reference generation. Using the optimization framework, we solve a quadratic programming (QP) problem in order to come-up with the best command to optimize the distance between two vehicles while satisfying all the problem constraints. Moreover, we present a comprehensive comparison of the results obtained with our generated driving data that simulates realistic driving scenarios to demonstrate the benefit of the stochastic models.

The remainder of the paper is organized as follows. Section II discusses different ACC algorithms under study. Section III describes our ACC validation model. In Section III, we present our numerical experiments and compare our different approaches. Conclusions are provided in Section IV.

2 LITERATURE REVIEW

ACC has been the subject of numerous studies, from its system design to its on the ground validation.

Many ACC systems employ optimal control methods (Chehardoli, 2020; Kim, 2012; Zhu et al., 2018). However, model predictive control (MPC) has also gained popularity since 2010 (Takahama and Akasaka, 2018; Li et al., 2010; Naus et al., 2010). A wide variety of papers has studied ACC from multiple perspectives, including driving modeling (Seppelt and Lee, 2015), energy-optimized driving models (Weißmann et al., 2018), string stability (Liang and Peng, 1999), and collision avoidance (Lunze, 2018).

Validating the functionality of autonomous driving is also an important task, not only for ACC but also other modules which need assessments. In (Lattarulo et al., 2017), the authors present a global framework of testing methodology for the evaluation of path planning and control algorithms, including a unified test architecture and validation process. Other similar works include (Lattarulo et al., 2018) , (Alnaser et al., 2019).

Aside from the overall testing framework, individual functionality like ACC should also be carefully examined. In (Mehra et al., 2015), an experimental platform is presented for the validation and demonstration of an optimization-based ACC controller whilst (Djoudi et al., 2020) present a simulation based tool chain for reference generation and test analysis. Several other insightful works on testing and validating adaptive cruise control can be found in (Schmied et al., 2015; Shakouri et al., 2015).

3 PROBLEM FORMULATION

3.1 Overview

In this section, we describe the modeling of the ACC driving scenario and the formulation of the related optimization problem. A typical ACC driving scenario includes two cars driving simultaneously in a single lane, namely, the ego car and the target car. The ego car is equipped with an ACC system whilst the target car is the leading car positioned ahead. Figure 2 illustrates the driving scenario, as well as the states of two cars at moment t_i. The purpose of our ACC reference generation is to generate a sequence of acceleration commands, i.e., the decision variables in our optimization problem. The objective of the ego car is to keep a distance from the target car with respect to different constraints, e.g., vehicle dynamics, driving comfort, and road regulations.

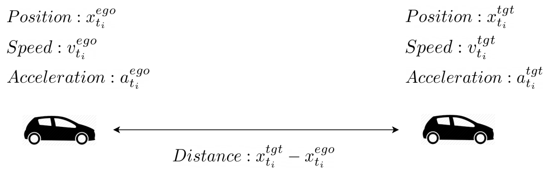

Figure 2: ACC driving scenario at moment t_i.

Suppose that the total duration of a driving scenario is Ts composed of n sampling time dt, i.e. $T = ndt$ with a corresponding timestamp $[t_0, t_1, ...t_i, \dots \; t_n]$ where $t_{i+1} = t_i + dt, \;\; \forall i \in \{0, 1, \dots \; n-1\}$. At each moment t_i, the ACC of the ego car uses sensors to gather the information from the target car and generates the acceleration commands. In the following, we list the parameters and the decision variable used in our model. The input parameters are given by the ego car sensors, and the decision variable represents the ACC optimal commands. The parameters of the ego car are the initial position $x_{t_0}^{ego}$, the initial velocity $v_{t_0}^{ego}$ whilst the parameters of the target car is composed of the position vector $X_T^{tgt} = (x_{t_1}^{tgt}, x_{t_2}^{tgt}, \dots \; x_{t_n}^{tgt})^T$, the velocity vector $V_T^{tgt} = (v_{t_0}^{tgt}, v_{t_1}^{tgt}, \dots \; v_{t_{n-1}}^{tgt})^T$ and the acceleration vector $A_T^{tgt} = (a_{t_0}^{tgt}, a_{t_1}^{tgt}, \dots \; a_{t_{n-1}}^{tgt})^T$ in the whole driving scenario. The decision variables is the ACC ego car acceleration commands vector $A_T^{ego} = (a_{t_0}^{ego}, a_{t_1}^{ego}, \dots \; a_{t_{n-1}}^{ego})^T$.

Given the decision variable and the initial state of the ego car, we can derive the velocity and the position of the ego car by the the equations of motion. The ego car velocity $v_{t_{i+1}}^{ego}$ at time t_{i+1} is given by the velocity at the previous sample time $v_{t_i}^{ego}$ and the acceleration $a_{t_i}^{ego}$:

$$v_{t_{i+1}}^{ego} = v_{t_i}^{ego} + a_{t_i}^{ego} dt. \tag{1}$$

The velocity for the whole driving scenario can be written in matrix form as

$$V_T^{ego} = \begin{pmatrix} v_{t_0}^{ego} \\ \vdots \\ v_{t_i}^{ego} \\ \vdots \\ v_{t_{n-1}}^{ego} \end{pmatrix} = \begin{pmatrix} v_{t_0}^{ego} \\ \vdots \\ v_{t_0}^{ego} + \sum_{k=0}^{k=i-1} a_{t_k}^{ego} dt \\ \vdots \\ v_{t_0}^{ego} + \sum_{k=0}^{k=n-2} a_{t_k}^{ego} dt \end{pmatrix} = dt\, \mathcal{K}_n A_T^{ego} + v_{t_0}^{ego} \mathbb{1}_n, \tag{2}$$

where $\mathcal{K}_n \in \mathbb{R}^{n \times n}$ and $\mathbb{1}_n \in \mathbb{R}^{n \times 1}$

$$\mathcal{K}_n = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & 0 & \dots & 0 & 0 \\ 1 & 1 & 0 & \dots & 0 & 0 \\ & \vdots & & \ddots & & \vdots \\ 1 & 1 & 1 & \dots & 0 & 0 \\ 1 & 1 & 1 & \dots & 1 & 0 \end{pmatrix} \tag{3}$$

$$\mathbb{1}_n = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}. \tag{4}$$

Similarly, the ego car position at time t_{i+1} is given by

$$x_{t_{i+1}}^{ego} = x_{t_i}^{ego} + v_{t_i}^{ego} dt + \frac{1}{2} a_{t_i}^{ego} dt^2. \tag{5}$$

The corresponding matrix format for all time steps is

$$X_T^{ego} = \begin{pmatrix} x_{t_1}^{ego} \\ \vdots \\ x_{t_i}^{ego} \\ \vdots \\ x_{t_n}^{ego} \end{pmatrix} = \begin{pmatrix} x_{t_0}^{ego} + v_{t_0}^{ego} dt + \frac{1}{2} a_{t_0}^{ego} dt^2 \\ \vdots \\ x_{t_0}^{ego} + \sum_{k=0}^{k=i-1} v_{t_k}^{ego} dt + \frac{1}{2} \sum_{k=0}^{k=i-1} a_{t_k}^{ego} dt^2 \\ \vdots \\ x_{t_0}^{ego} + \sum_{k=0}^{k=n-1} v_{t_k}^{ego} dt + \frac{1}{2} \sum_{k=0}^{k=n-1} a_{t_k}^{ego} dt^2 \end{pmatrix} = dt\, \mathcal{M}_n V_T^{ego} + \frac{1}{2} dt^2 \mathcal{M}_n A_T^{ego} + x_{t_0}^{ego} \mathbb{1}_n, \tag{6}$$

where $\mathcal{M}_n \in \mathbb{R}^{n \times n}$,

$$\mathcal{M}_n = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 1 & 1 & 0 & \dots & 0 \\ 1 & 1 & 1 & \dots & 0 \\ & \vdots & & \ddots & \vdots \\ 1 & 1 & 1 & \dots & 1 \end{pmatrix}. \tag{7}$$

We use Equation (2) to rewrite Equation (6) in terms of the initial position, the initial velocity and the acceleration vector, i.e.,

$$\begin{aligned} X_T^{ego} &= dt\, \mathcal{M}_n V_T^{ego} + \frac{1}{2} dt^2 \mathcal{M}_n A_T^{ego} + x_{t_0}^{ego} \mathbb{1}_n \\ &= dt\, \mathcal{M}_n (dt\, \mathcal{K}_n A_T^{ego} + v_{t_0}^{ego} \mathbb{1}_n) \\ &\quad + \frac{1}{2} dt^2 \mathcal{M}_n A_T^{ego} + x_{t_0}^{ego} \mathbb{1}_n \\ &= dt^2 (\mathcal{B}_n + \frac{1}{2} \mathcal{M}_n) A_T^{ego} + v_{t_0}^{ego} dt\, \mathcal{C}_n \\ &\quad + x_{t_0}^{ego} \mathbb{1}_n, \end{aligned} \tag{8}$$

where $\mathcal{B}_n = \mathcal{M}_n \cdot \mathcal{K}_n \in \mathbb{R}^{n\times n}$ and $\mathcal{C}_n = \mathcal{M}_n \cdot \mathbb{1}_n \in \mathbb{R}^{n\times 1}$.

These parameters are summarized in Table 1.

In the following, we use the position and the velocity vector of the ego car to formulate our optimization problem.

3.2 Mathematical Modeling

In the following we outline how the generation of the ACC reference can be viewed as an optimization problem.

$$\min_{A_T^{ego}} \quad ||QA_T^{ego} + P|| \tag{9}$$

$$\text{s.t.} \quad dt^2(\mathcal{B}_n + \frac{1}{2}\mathcal{M}_n)A_T^{ego} \leq X_T^{tgt} - v_{t_0}^{ego} dt\,\mathcal{C}_n - (x_{t_0}^{ego} + d_s)\mathbb{1}_n, \tag{10}$$

$$-(v_{max} + v_{t_0}^{ego})\mathbb{1}_n \leq dt\,\mathcal{K}_n A_T^{ego} \leq (v_{max} - v_{t_0}^{ego})\mathbb{1}_n, \tag{11}$$

$$-a_{max}\mathbb{1}_n \leq A_T^{ego} \leq a_{max}\mathbb{1}_n, \tag{12}$$

$$-j_{max}dt\mathbb{1}_n \leq \mathcal{D}_n A_T^{ego} \leq j_{max}dt\mathbb{1}_n. \tag{13}$$

The following part explains in detail how we derive the objective function (9) and how constraints (10, 11, 12, 13) are developed.

The objective of ACC is to maintain a safe distance between the ego car and the target car. In order to calculate the reference distance between the ego car and the target car, we define two terms: the inter-vehicle time tc (e.g., 3 seconds) for the ego car to brake safely, and the standstill distance δS to ensure there is always enough room between the two adjacent cars.

At each moment t_k, the reference distance of ACC in platoons is defined by

$$d_{t_k}^{ref} = (v_{t_{k-1}}^{ego} - v_{t_{k-1}}^{tgt})tc + \frac{1}{2}(a_{t_{k-1}}^{ego} - a_{t_{k-1}}^{tgt})tc^2 + \delta S. \tag{14}$$

So the reference distance vector in the whole driving scenario is :

$$\begin{aligned} D_T^{ref} &= tc(dt\,\mathcal{K}_n A_T^{ego} + v_{t_0}^{ego}\mathbb{1}_n - V_T^{tgt}) \\ &\quad + \frac{1}{2}tc^2(A_T^{ego} - A_T^{tgt}) + \delta S\mathbb{1}_n \\ &= (dt\cdot tc\,\mathcal{K}_n + \frac{1}{2}tc^2 I)A_T^{ego} - tcV_T^{tgt} \\ &\quad - \frac{1}{2}tc^2 A_T^{tgt} + (\delta S + v_{t_0}^{ego}tc)\mathbb{1}_n. \end{aligned} \tag{15}$$

Moreover, the current distance between the ego car and target car is

$$\begin{aligned} D_T^{vehicle} &= X_T^{tgt} - X_T^{ego} \\ &= X_T^{tgt} - [dt^2(\mathcal{B}_n + \frac{1}{2}\mathcal{M}_n)A_T^{ego} \\ &\quad + v_{t_0}^{ego} dt\,\mathcal{C}_n + x_{t_0}^{ego}\mathbb{1}_n]. \end{aligned} \tag{16}$$

By combining (16) and (15), we obtain the objective function (9):

$$\begin{aligned} &\min_{A_T^{ego}} ||D_T^{vehicle} - D_T^{ref}|| \\ &= \min_{A_T^{ego}} ||X_T^{tgt} - [dt^2(\mathcal{B}_n + \frac{1}{2}\mathcal{M}_n)A_T^{ego} \\ &\quad + v_{t_0}^{ego}dt\,\mathcal{C}_n + x_{t_0}^{ego}\mathbb{1}_n] - [(dt\cdot tc\,\mathcal{K}_n + \frac{1}{2}tc^2 I)A_T^{ego} \\ &\quad - tcV_T^{tgt} - \frac{1}{2}tc^2A_T^{tgt} + (\delta S + v_{t_0}^{ego}tc)\mathbb{1}_n]|| \\ &= \min_{A_T^{ego}} || - (dt^2\mathcal{B}_n + \frac{1}{2}dt^2\mathcal{M}_n + dt\cdot tc\,\mathcal{K}_n \\ &\quad + \frac{1}{2}tc^2 I)A_T^{ego} + X_T^{tgt} + tcV_T^{tgt} + \frac{1}{2}tc^2A_T^{tgt} - \delta S\mathbb{1}_n \\ &\quad - v_{t_0}^{ego}tc\mathbb{1}_n - x_{t_0}^{ego}\mathbb{1}_n - v_{t_0}^{ego}dt\,\mathcal{C}_n|| \\ &= \min_{A_T^{ego}} ||QA_T^{ego} + P||, \end{aligned} \tag{17}$$

where $Q = -(dt^2\mathcal{B}_n + \frac{1}{2}dt^2\mathcal{M}_n + dt\cdot tc\,\mathcal{K}_n + \frac{1}{2}tc^2 I)$, $P = X_T^{tgt} + tcV_T^{tgt} + \frac{1}{2}tc^2A_T^{tgt} - \delta S\mathbb{1}_n - v_{t_0}^{ego}tc\mathbb{1}_n - x_{t_0}^{ego}\mathbb{1}_n - v_{t_0}^{ego}dt\,\mathcal{C}_n$ and $||\cdot||$ is Euclidean norm.

In addition to the objective function (9), we describe the following constraints

- Constraint (10) is the minimum distance constraint which aims to prevent the vehicles collisions. It is results from

$$D_T^{vehicle} \geq d_s\mathbb{1}_n. \tag{18}$$

- Constraint (11) is the maximum velocity constraint. Routes typically have a maximum velocity limit which leads to the velocity constraint. For a given a speed limit v_{max}, the constraint is deduced from

$$||V_T^{ego}||_\infty \leq v_{max}. \tag{19}$$

- Constraint (12) is the maximum acceleration constraint. Car passengers' comfort is impacted by acceleration. Vehicle maneuverings like rapid acceleration or braking should be avoided. Our model proposes an acceleration limit of a_{max} based on this motivation.

$$||A_T^{ego}||_\infty \leq a_{max}. \tag{20}$$

- Constraint (13) is the maximum jerk constraint. In jerk, we measure the acceleration variances, which significantly affect the comfort level of passengers. A maximum limit j_{max} is required for this constraint.

$$||J_T^{ego}||_\infty \leq j_{max} \tag{21}$$

Table 1: Summary of parameters.

	Symbols	Physical Meaning	Relationship
Target Car	A_T^{tgt}	Acceleration profile during simulation	$A_T^{tgt} = (a_{t_0}^{tgt}, a_{t_1}^{tgt}, \dots a_{t_{n-1}}^{tgt})^T$
	V_T^{tgt}	Speed profile during simulation	$V_T^{tgt} = (v_{t_0}^{tgt}, v_{t_1}^{tgt}, \dots v_{t_{n-1}}^{tgt})^T$
	X_T^{tgt}	Position profile during simulation	$X_T^{tgt} = (x_{t_1}^{tgt}, x_{t_2}^{tgt}, \dots x_{t_n}^{tgt})^T$
Ego Car	A_T^{ego}	Acceleration profile during simulation	$A_T^{ego} = (a_{t_0}^{ego}, a_{t_1}^{ego}, \dots a_{t_{n-1}}^{ego})^T$
	V_T^{ego}	Speed profile during simulation	$V_T^{ego} = dt\,\mathcal{K}_n A_T^{ego} + v_{t_0}^{ego}\mathbb{1}_n$
	X_T^{ego}	Position profile during simulation	$dt^2(\mathcal{B}_n + \frac{1}{2}\mathcal{M}_n)A_T^{ego} + v_{t_0}^{ego}dt\,\mathcal{C}_n + x_{t_0}^{ego}\mathbb{1}_n$
	J_T^{ego}	Jerk profile during simulation	$\mathcal{D}_n A_T^{ego}$

Since $j_{t_i} = (a_{t_i}^{ego} - a_{t_{i-1}}^{ego})/dt$, the jerk constraint can be simplified to (13) where $\mathcal{D}_n \in \mathcal{R}^{n\times n}$

$$\mathcal{D}_n = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ -1 & 1 & 0 & \dots & 0 & 0 \\ 0 & -1 & 1 & \dots & 0 & 0 \\ & \vdots & & \ddots & \vdots & \\ 0 & 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & 0 & \dots & -1 & 1 \end{pmatrix}. \tag{22}$$

Given the form of the objective function and the constraints, our model is a convex quadratic optimization problem.

In the next section, we will discuss the uncertainty involved in ACC and how to handle it by stochastic modeling with chance constraints.

3.3 Stochastic Model

The above presented model is deterministic, i.e., the input parameters are known in advance. However, in real life autonomous vehicle problems the parameters are unknown, and may include different sources of noise from external factors like weather. Results are highly dependent upon the quality of input data. Consequently, the parameters can be better modeled by random variables, which provides through probability distributions more robust solutions. In the following, we model the ACC problem by chance constrained problem. We suppose that the target car's position information $x_{t_i}^{tgt}$ obtained from the ego car's sensor include some noise, and follows a normal distribution $x_{t_i}^{tgt} \sim N(\mu_i, \sigma_i^2)$. As a result, X_T^{tgt} follows multivariate normal distribution $X_T^{tgt} \sim N(\mu_T, \Sigma_T)$, where

$$\mu_T = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{pmatrix} \tag{23}$$

and

$$\Sigma_T = \begin{pmatrix} \sigma_1^2 & 0 & \dots & 0 \\ 0 & \sigma_2^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_n^2 \end{pmatrix}. \tag{24}$$

The objective function for this stochastic optimization problem is

$$\begin{aligned} &\min_{A_T^{ego}} ||\mathbb{E}(D_T^{vehicle} - D_T^{ref})|| \\ &= \min_{A_T^{ego}} ||\mu_T + tcV_T^{tgt} + \frac{1}{2}tc^2 A_T^{tgt} - \delta S\mathbb{1}_n \\ &\quad - v_{t_0}^{ego} tc\mathbb{1}_n - x_{t_0}^{ego}\mathbb{1}_n - v_{t_0}^{ego} dt\,\mathcal{C}_n - [dt^2\mathcal{B}_n \\ &\quad + \frac{1}{2}dt^2\mathcal{M}_n + dt\cdot tc\,\mathcal{K}_n + \frac{1}{2}tc^2]A_T^{ego}|| \\ &= \min_{A_T^{ego}} ||QA_T^{ego} + P'||, \end{aligned} \tag{25}$$

where $P' = \mu_T + tcV_T^{tgt} + \frac{1}{2}tc^2 A_T^{tgt} - \delta S\mathbb{1}_n - v_{t_0}^{ego} tc\mathbb{1}_n - x_{t_0}^{ego}\mathbb{1}_n - v_{t_0}^{ego} dt\,\mathcal{C}_n$.

The minimum distance constraint (18) for each moment t_i can be expressed as a chance constraint (Prékopa, 2013) with a given a threshold α, i.e.,

$$\begin{aligned} &\mathbb{P}(D_{t_i}^{vehicle} \geq d_s) \geq \alpha, \ \forall t_i \\ &= \mathbb{P}(x_i^{tgt} \geq x_{t_i}^{ego} + d_s) \geq \alpha \\ &= \mathbb{P}(\frac{x_i^{tgt} - \mu_i}{\sigma_i} \geq \frac{x_{t_i}^{ego} + d_s - \mu_i}{\sigma_i}) \geq \alpha \\ &= F_N(\frac{x_{t_i}^{ego} + d_s - \mu_i}{\sigma_i}) \leq 1 - \alpha \\ &= x_{t_i}^{ego} + d_s \leq \mu_i + \sigma_i F_N^{-1}(1-\alpha), \end{aligned} \tag{26}$$

where F_N^{-1} is the quantile function of standard normal distribution.

For the whole driving scenario, the minimum distance constraint in a matrix form is

$$\begin{aligned} &dt^2(\mathcal{B}_n + \frac{1}{2}\mathcal{M}_n)A_T^{ego} + v_{t_0}^{ego} dt\,\mathcal{C}_n + \\ &(x_{t_0}^{ego} + d_s)\mathbb{1}_n \leq \hat{X}_T^{tgt}, \end{aligned} \tag{27}$$

where

$$\hat{X}_T^{tgt} = \begin{pmatrix} \mu_1 + \sigma_1 F_N^{-1}(1-\alpha) \\ \mu_2 + \sigma_2 F_N^{-1}(1-\alpha) \\ \vdots \\ \mu_n + \sigma_n F_N^{-1}(1-\alpha) \end{pmatrix}. \quad (28)$$

Since only the minimum distance constraint is related to the position of the target car X_T^{tgt}, all other constraints remain unchanged.

4 NUMERICAL EXPERIMENTS

Numerical simulations aim to compare the deterministic model and stochastic models on different randomly generated instances. Considering the sensor error for the ego car, we generate driving scenarios with different configurations, including the target car's trajectory profile and ego car's initial state. Next, we formulate the optimization problems based on the generated parameters and employ a QP solver to obtain the results (Goldfarb and Idnani, 1983). Finally, we compare the statistical results of the two models to illustrate the stochastic model's effectiveness.

To create an ACC driving scenario, our parameters consist of two types: the parameters related to the environment and to the vehicles. The parameters related to the environment include the simulation configuration and vehicle regulations, such as the total scenario duration, velocity limit, collision avoidance limit, etc. Those parameters reflect the real life driving rules and simulation setting, therefor they are fixed during numerical experiments. The parameters related to the vehicles, such as initial position, velocity and distance, vary in each randomly generated instance due to the diversity of driving scenarios. In order to simulate real driving situations, the relationship among randomly generated vehicle parameters should be based on Newton's laws.

Parameters setup for numerical simulations are summarized in the sequel:

- Parameters related to the environment
  - Total duration of a scenario T: $2s$.
  - Sampling time step dt: $0.05s$.
  - Inter-vehicle time tc: $3s$.
  - Standstill distance σS: $3m$.
  - Minimum security distance d_s: $10m$.
  - Maximum velocity v_{max}: $30m/s$.
  - Maximum acceleration v_{max}: $5m/s^2$.
  - Maximum jerk j_{max}: $5m/s^3$.
  - Confidence level α: 0.9.
- Parameters related to the vehicles
  - Acceleration of the target car: independent random variables following a normal distribution with mean 0 and standard deviation 2, truncated from -5 to 5.
  - Initial speed of target car and ego car: independent random variables following a normal distribution with mean 15 and standard deviation 10, truncated from 5 to 25.
  - Standard deviation of target car position σ: 1.
  - Initial position of target car: random variable following a normal distribution with mean 200 and standard deviation 1.
  - Speed and position of target car: random variables following normal distributions with mean calculated by an initial value and the acceleration vector, and standard deviation 1.
  - Initial position of ego car: the initial position of the target car minus a random variable following normal distribution with mean 100 and standard deviation 20, truncated from 50 to 150.

As depicted in the previous section, each generated driving scenario is represented as a QP problem. There are several methods for dealing with this QP problem, which can mainly be divided into two categories: active-set methods and interior point methods. We use QP solver with Goldfarb–Idnani algorithm (Goldfarb and Idnani, 1983), which is a dual active set method, in order to obtain the optimal solution for our QP problem.

With the configuration above, we generate 100 random driving scenarios, which are then solved by our QP solver to obtain the results of the deterministic model and the stochastic model, respectively. Since the input parameters of the model are based on biased sensor data, it is possible that the result will violate the constraints (18) during the driving scenario. Hence, we measure a model's performance by examining its solution's number of violated constraints. The more violations of constraints there are, the worse the solution is.

Amongst 100 test driving scenario cases, we notice that only 38% of the instances are totally feasible in view of the deterministic optimal solution against 56% in view of the stochastic optimal solution with confidence level $\alpha = 0.9$.

For an in-depth analysis of constraint violations across 100 test driving scenarios, Figure 3 visualizes the constraint violation value $d_s - D_T^{vehicle}\mathbb{1}_n$, adapted from constraint (18), for the whole results of the two models. Figures 3 (a) and 3 (b) show the constraint violation value for the whole constraints whilst Figures 3 (c) and (d) show a zoom-in on a

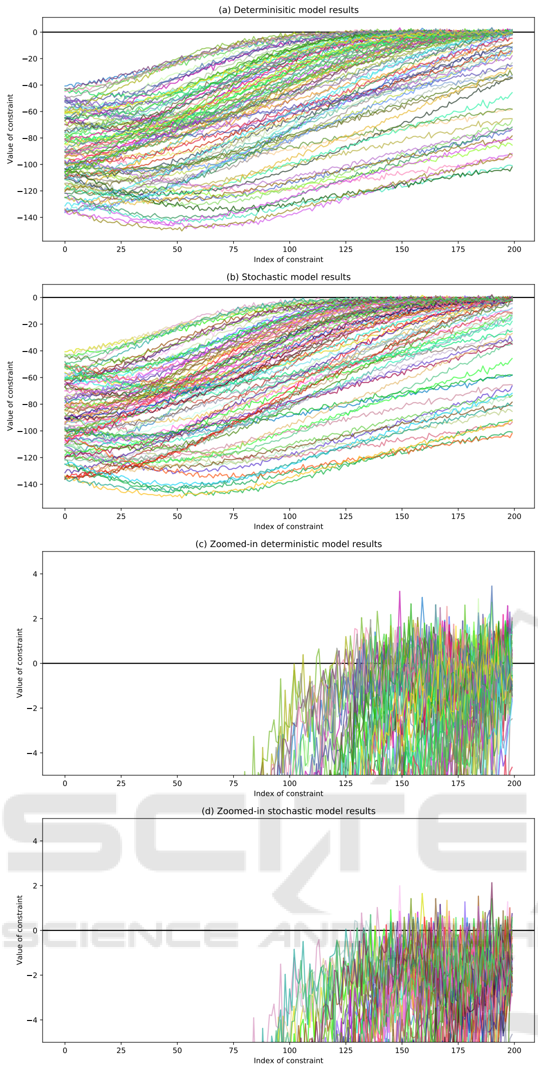

Figure 3: Constraint function values of all instances for deterministic and stochastic models.

subset of constraints for a better readability. In Figure 3, each curve in its own color displays the constraint violation values of a driving scenario result, and the x-axis represents the index of constraints. If the value at constraint index i exceeds 0, it means that $d_s > D_{t_i}^{vehicle}$, i.e. the constraint (18) is violated at this sampling time. Figure 3 clearly indicates that the stochastic model produces fewer violations than the deterministic one.

Following the visualization of the result, we also conduct a statistical analysis of the distribution of the violated constraints number in Figure 4. We observe that the stochastic model not only produces more totally feasible solutions with 0 violations, but also yields fewer violations for cases where the solution is unfeasible.

Furthermore, keeping all other parameters

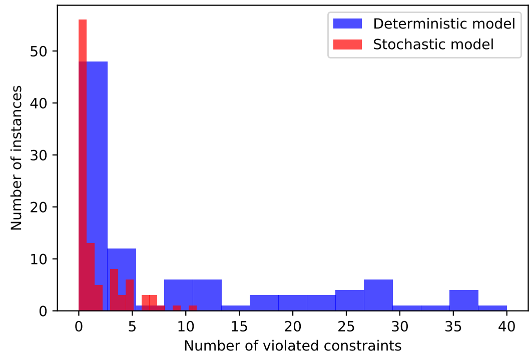

Figure 4: Histogram and cumulative histogram of number of violated constraints for two models.

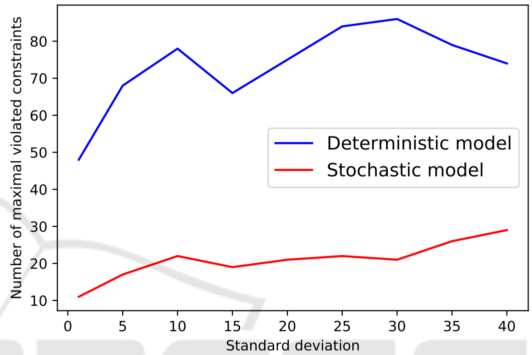

Figure 5: Maximal violated constraints under different standard deviation.

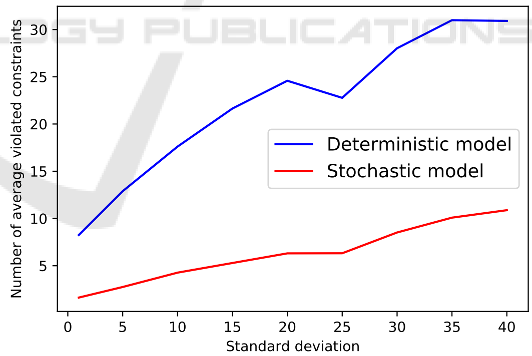

Figure 6: Average violated constraints under different standard deviation.

unchanged, we vary the standard deviation of the target car position, which depends on sensor's precision, from 1 to 40 to compare the performances of each model. The value of the standard deviation is gradually increased. We consider 100 tests for each value, and count the maximal and mean constraint violations for each model. As shown in Figure 5 and Figure 6, the stochastic model always outperforms the deterministic model by producing less constraint violations.

5 CONCLUSION AND FUTURE WORK

We presented in this paper an optimization-based approach for ACC reference generation taking into account the uncertainty associated with sensors information. As a benchmark for ACC system decision making, our optimization approach can generate a reference that meets the needs of safety, comfort, and effectiveness. According to a statistical analysis of the simulation results, our chance-constrained based stochastic model can produce more robust solutions.

For future work, we propose three open research challenges that have the merit to be addressed: development of an increasingly sophisticated vehicle model, modeling of uncertainty involving dependent random variables, and formulation of objectives that involve penalties for undesired behavior. The solution to those challenges will allow us to build a more general framework to accommodate different needs for reference generation problem. Furthermore, we will use this optimization-based reference generation framework for other autonomous driving functions, such as lane keeping assistance (LKA) and collision avoidance.

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