

Outpatient Diversion using Real-time Length-of-Stay Predictions

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Abstract: In this work, we show how real-time length-of-stay (LOS) predictions can be used to divert outpatients from their assigned facility to other facilities with lesser congestion. We illustrate the implementation of this diversion mechanism for two primary health centers (PHCs), wherein we divert patients from their assigned PHC to the other PHC based on their predicted LOSs in both facilities. We develop a discrete-event simulation model of patient flow operations at these two PHCs in an Indian district and observe significantly longer LOSs at one of the PHCs due to disparities in the patient loads across both PHCs. We first determine the expected LOS of the patient at the point in time at which they are expected to arrive at a PHC using system state information recorded at the current time at the PHC in question. The real-time LOS predictions are generated by estimating patient wait times on a real-time basis at the queueing subsystems within the PHC. We then divert the patient to the appropriate PHC on the basis of the predicted LOS estimates at both PHCs, and show through simulation that the proposed framework leads to more equitable utilization of resources involved in provision of outpatient care.

1 INTRODUCTION

Patient length-of-stay (LOS) is an important indicator of the efficiency of hospital management systems (Ayyoubzadeh et al., 2020). Long LOSs and overcrowding are important operational issues facing healthcare facilities around the world (Wachtel and Elalouf, 2020), including in India (Sharma et al., 2021). This in turn leads to both operational and clinical issues, such as denial of hospital admissions, cancellation of surgeries, and higher likelihoods of developing healthcare acquired infections (Zhou et al., 2019; Arefian et al., 2019).

Congestion and longer LOSs at specific facilities within a healthcare facility network, which in turn lead to inequitable utilization of facilities across the network, may occur due to patient perceptions of better quality of care at certain facilities within the network (Liu et al., 2018). Healthcare administrators may adopt different strategies to relieve congestion such as capacity replanning (Whiteside et al., 2020), patient diversion (Marquinez et al., 2021), and early discharges for medically stable patients. However, increasing resource capacity is often difficult due to the capital investment required and the attendant

Table 1: Key acronyms and notation.

Acronyms / Notation	Corresponding constituent words
LOS	Length-of-stay
PHC	Primary health center
DES	Discrete event simulation
δ_x	Travel time to reach PHC x
ρ	Resource utilization
$\Delta\rho$	Change in resource utilization
w	Wait time in a queue
Δw	Change in queue wait time
$G(\cdot)$	Queueing system service time <i>cdf</i>

medical training required when personnel numbers are increased. Hence, patient diversion is often the preferred alternative to mitigate overcrowding. In this study, we describe an approach towards using real-time LOS predictions to facilitate diversion of outpatients across primary healthcare facilities. While we demonstrate our approach via the diversion of outpatients across two primary healthcare centers (PHCs) in the Indian context, our approach can be used in any setting where patient diversion is being considered due to high LOSs, or more generally, where significant disparities in resource utilization in a healthcare network is observed.

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In India, PHCs are the first point of contact for the patients with a formally trained doctor and provide outpatient care, and limited emergency and inpatient care (Shoab and Ramamohan, 2021). The Indian government has recently undertaken an initiative to expand and upgrade existing PHCs (Blanchard et al., 2021), and hence it is reasonable to assume an increase in the patient load at these PHCs given the established link between increased quality of care, improved infrastructure and higher demand (Rao and Sheffel, 2018). Based on previous work regarding modeling PHC operations (Fatma et al., 2020; Shoab and Ramamohan, 2021), we developed a discrete-event simulation (DES) of medical care operations of two PHCs in an Indian district. Under conditions of high outpatient demand, the DES outcomes indicated significant disparity in outpatient LOSs across both facilities, and this would likely extend to the entire public health network in the district. Further, we also noted that under these high demand conditions, patient wait times to receive service from the doctor did not increase significantly due to the very high service rates of the doctor. Instead, the impact of high demand was reflected in wait times at supporting subsystems in the PHC such as the pharmacy and the laboratory, which in turn yielded high LOSs in the PHCs as a whole. For example, average LOSs were approximately 60 minutes, in comparison to average total service times of less than 10 minutes across all services that a patient may avail of in a typical PHC visit. Thus, implementing diversion on the basis of wait times in this context did not appear appropriate; hence, we considered diversion based on patient LOS as a whole in the facility.

The diversion mechanism that we propose utilizes real-time LOS predictions to inform the diversion decision. Diversion mechanisms based on real-time wait time or ‘delay’ predictions for emergency patients have been discussed in previous work (Fatma and Ramamohan, 2021a); however, in this work, we use real-time delay estimates to calculate real-time LOS predictions that in turn determine the diversion decision. The real-time delay estimates, and in turn the real-time LOS predictions, are generated at the point when the patient decides to visit the facility in question (as opposed to when the patient arrives at the facility), implying that an information technology (IT) system, including a web/smartphone application, to convey this information to this patient would be required. Examples of such IT systems can be found in the United Kingdom’s National Health Service (Mustafee and Powell, 2020). In this paper, we develop a mechanism for patient diversion based on such real-time LOS predictions (including a discussion of how such

LOS predictions can be generated) and demonstrate via DES its impact on the operational outcomes of a healthcare facility network. Our study provides the operational basis that an IT system such as that described in Mustafee and Powell (2020) can leverage to implement real-time LOS-based diversion.

We now discuss the literature relevant to this study, and describe our research contributions with respect to the literature.

2 LITERATURE REVIEW

In this section, we focus on the literature associated with real-time LOS prediction and patient diversion. As part of the literature review, we also discuss studies that highlight the importance of information sharing across facilities in the network wherein diversion is implemented. We describe the research contributions of this work at the end of each subsection.

2.1 Length-of-Stay Prediction

The importance of LOS prediction at healthcare facilities and their subunits such as the emergency department is reflected in the considerable body of literature devoted to this topic (Song et al., 2015; Alsinglawi et al., 2020; Shi et al., 2021; El-Bouri et al., 2021). From the perspective of the methods used for LOS prediction, it appears that data mining and statistical learning methods have been most widely used (Verburg et al., 2014; Barnes et al., 2016; Turgeman et al., 2017; Gutiérrez et al., 2021; Shaaban et al., 2021). A review conducted by El-Bouri et al. (2021) found that multiple machine learning techniques such as deep neural networks, random forest trees, regression techniques and decision trees among others have been used to predict the patients LOS. The authors then conclude that the LOS predictions were found to be useful in planning patient admission schedules and, as required, the preparation of post-discharge care.

LOS prediction studies in the literature have primarily focused on inpatients and emergency patients using statistical learning techniques. The typical methodology for LOS prediction for these patient types involved the following steps: (a) recording clinical information of patients such as medication history at the time of admission, patient age, and the number of comorbidities, (b) identifying the set of predictor variables among the data available for LOS prediction, (c) training and validating the statistical learning methods on the dataset constructed via mining the available data (Baek et al., 2018; Aghajani and Kar-

gari, 2016; Hijry and Olawoyin, 2020). The majority of authors concluded that factors such as severity of the disease, recency of diagnosis and type of surgery, patient age, number of comorbidities, surgery type, number of days of hospitalisation before surgery, etc., were significantly correlated with patient LOS. These studies did not determine the effect of operational variables such as the number of patients in the queue for system resources, the elapsed service time of patients currently receiving care, etc., on patient LOS.

In this context, we generate LOS predictions via a methodology grounded in queueing theory and real-time delay prediction that estimates patient LOSs in the PHC as a function of operational ‘system state’ variables such as the number of patients in the queue at each queueing system within the PHC (e.g., the queueing system represented by the doctor and the laboratory), the elapsed service times of patients currently receiving service at each queueing subsystem, arrival and service rates of patients in the PHC, and the average travel time of patients to the PHC from their point of origin. If the patient decides to leave for the PHC at time t , then the LOS prediction is generated for the time of arrival of the patient in the system - i.e., at time $t + \delta$, where δ is the travel time to the PHC from the patient’s location.

We note here that because we use real-time delay predictions to generate the LOS predictions, our approach can be extended to multi-class multi-server systems for which real-time delay predictors have previously been developed, including certain systems with non-stationary arrival and service processes (Ibrahim, 2018). Further, in general, our approach is likely to require fewer variables to be recorded for generating the real-time delay estimates in comparison to statistical learning predictors, and its development and deployment will not require construction of a training and validation dataset. We also note that the majority of previous work focuses on generating LOS predictions for patients at a single unit of a healthcare facility, such as the emergency department, and do not generate predictions for patients that receive service from multiple units within a given facility. In comparison, our approach generates LOS predictions at multiple units within the same facility; that is, total predicted LOS is the summation of time spent at each PHC subsystem.

2.2 Patient Diversion

There are multiple studies considering patient diversion as a means to reduce congestion at healthcare facilities. In a review by Li et al. (2019), the authors find that patient diversion was initiated based

on multiple measures of congestion, such as the average wait time of patients before receiving care, the number of patients waiting to receive care, the number of occupied beds in a ward, etc. From a methodological standpoint, the approaches used to optimize various aspects of the diversion process include a Markov decision process formulation to maximize the cost-effectiveness of diversion policies for ICU patients in a public-private hospital network (Marquinez et al., 2021), a simulation optimization approach to minimize the time to receive care (including wait times and travel times between facilities) (Ramirez-Nafarrate et al., 2011), and a mixed integer programming approach to minimize the number of patients waiting for care across the network (Nezamoddini and Khasawneh, 2016). Fatma and Ramamohan (2021a) evaluated via DES the effectiveness of patient diversion strategies on multiple operational outcomes such as resource utilizations, wait times, and the proportion of patients whose wait time before receiving care exceeds a threshold duration. The proportion of patients leaving without receiving care has also been an operational outcomes of interest in multiple studies (Morley et al., 2018).

An important consideration in implementing a diversion mechanism within a healthcare facility network is information sharing across the network regarding operational variables at each facility that are used to determine diversion decisions. In this regard, there is considerable support in the literature regarding implementing centralized diversion policies across healthcare facilities, because diverting patients without considering the operational information at other facilities can yield worse operational outcomes for both patients and the healthcare facilities across the network, such as increased wait times for the diverted patient, and increased crowding at healthcare facilities in the network (Deo and Gurvich, 2011; Li et al., 2019; Shi et al., 2021). This has been further emphasized in the recently published literature, wherein Shi et al. (2021) find that communication and coordination among healthcare facilities are crucial to relieve congestion across all facilities in a healthcare network, and diversion decisions must be made using information available from electronic management systems networked across all facilities under consideration (Adjerid et al., 2018; Dong et al., 2019; Piermarini and Roma, 2021). In this context, our study also proposes a diversion mechanism for outpatients that implicitly assumes the presence of a centralized IT system that maintains information regarding the operational status - in terms of system state variables such as the number of patients waiting and elapsed service times in various queues within the facility -

that is available to all facilities within the network.

We note here that most studies we surveyed, as is natural, considered diversion of emergency patients. Only Nezamoddini and Khasawneh (2016) briefly discuss the effect of diverting non-emergency patients to other healthcare facilities in a multi-hospital setting. To the best of our knowledge, we did not identify other studies that considered diversion of outpatients, and in particular, we did not identify other studies that utilized real-time LOS estimates for this purpose.

As discussed above, our approach - in relation with the extant literature - also considers a centralized diversion mechanism involving information sharing. In this regard, our work is similar to the work by Fatma and Ramamohan (2021a); however, where they use real-time delay estimates to divert emergency (childbirth) patients arriving at facilities within a healthcare network seeking care, we utilize real-time delay estimates in generating LOS predictions - at the point in time when they may access the IT system associated with the healthcare network - to assign patients to a facility within the network. While this may seem like a facility assignment exercise instead of patient diversion, we consider this a diversion issue because patients are typically already assigned as belonging to the catchment area of their nearest PHC, and hence our framework determines whether they must be 'diverted' to visit a facility located farther from them on the basis of real-time LOS predictions. However, where such facility assignment is not *a priori* performed, our approach can be used for real-time facility assignment as well.

3 PRIMARY HEALTH CENTERS

In India, PHCs serve as the first point of contact between the population and the healthcare provider with the objective of making healthcare services accessible in both the rural and urban areas. PHCs are established to cover a population of 30,000 in rural areas and 20,000 in hilly, tribal, and desert areas, providing a range of essential outpatient care, and limited inpatient and childbirth care (IPHS-Guidelines, 2012).

PHCs provide six hours of outpatient care services and 24×7 emergency services including inpatient and childbirth care to the patients. PHCs have one to two doctors, a non-communicable disease (NCD) nurse for conducting lifestyle disease checks, one staff nurse per shift, a pharmacist, and a laboratory technician. Routine blood and urine investigations, and sputum tests for suspected tuberculosis cases are offered by the in-house laboratory within the PHCs. PHCs also have one labor bed for normal and

assisted deliveries for childbirth patients and four to six indoor beds for admitting emergency patients to ensure easy access to public health facilities. We refer readers to Shoaib and Ramamohan (2021) for a detailed understanding of medical care operations at PHCs.

We now briefly describe the patient flow within PHCs. Outpatients aged greater than 30 years first consult the NCD nurse before consulting the doctor. The NCD nurse checks blood pressure and blood glucose levels, body temperature, and also provides diet-related consultation to patients, as required. Other outpatients directly consult the doctor or wait in the outpatient queue if the doctor is busy with other patients. Once the outpatients complete their consultation with the doctor, approximately half the patients are sent to the in-house laboratory for routine investigations. All patients exit from PHC via the pharmacy regardless of whether they require medication, because the pharmacy also performs the administrative task of registering patient visits before their exit from the PHC. Outpatient flow through the PHC is depicted in Figure 1.

During outpatient department (OPD) hours, inpatients and childbirth patients are attended to by the doctor first and if the doctor is unavailable, they are attended to by the staff nurse. Once their consultation with the doctor is complete, inpatients are admitted to the inpatient department (IPD) where the staff nurse monitors their condition and treats accordingly. Inpatient LOSs in the IPD range from four to six hours, because patients are typically admitted for minor conditions that require monitoring and treatment for brief periods. Childbirth patients are taken to the labour room for delivery under the supervision of the doctor and the staff nurse. After delivery, childbirth patients are shifted to the IPD ward. Note that for more complex inpatient and childbirth cases requiring more intensive and specialized care, patients are referred to higher level facilities. Outside OPD hours, patients are attended to by the staff nurse alone with doctors being summoned on very rare occasions.

As documented in Shoaib and Ramamohan (2021), input parameters such as the average number of arrivals of each type of patient per day (arrival distributions were assumed to be Poisson), and the service time distribution parameters of the doctor, nurses, laboratory technicians, and the pharmacist were estimated based on data collection from in person visits to multiple PHCs. For example, the best-fit distribution for the NCD nurse service time was found to be the uniform distribution with parameters $U(2,5)$. Truncated Gaussian distributions were found to fit the service times of other

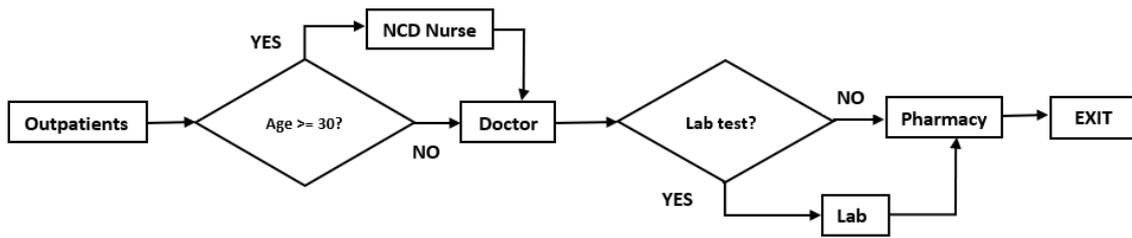


Figure 1: Outpatient flow at the PHC. Lab = laboratory.

resources - the doctor, laboratory technician, and pharmacist - best, with parameters $N(0.87, 0.21^2)$, $N(3.451, 0.873^2)$, and $N(2.084, 0.723^2)$ minutes, respectively. We refer readers to Shoaib and Ramamohan (2021) for a complete description of the parameter estimation process for other patient types served by PHCs.

The focus of this analysis is outpatient flow in the PHCs given that we consider outpatient diversion based on LOS estimates for the PHCs. We approximate the outpatient flow through the PHC operations as the flow of an entity through one more M/G/1 queueing systems in series. This approximation is possible due to the very large disparity in the arrival rates between outpatients and patients requiring admission (i.e., inpatients and childbirth patients). This is discussed in further detail in Shoaib and Ramamohan (2021).

Based on the outpatient flow described in Figure 1 and input parameter estimates, we develop a discrete-event based simulation model of the patient care operations at two PHCs. The programming was done in Python using the *Salabim* package on an Intel i7 64-bit Microsoft Windows operating system with 16 gigabytes of memory. *Salabim* is a discrete event simulation package in python developed by (van der Ham, 2018). The package provides animation, queues, states, monitors for data collection and presentation, and simulation event tracing functionality. A single replication of the simulation involved a run-time horizon of 365 days with a warm-up period of 180 days. Each replication required approximately 4 minutes of computational run-time. We collect data from 40 replications and report the means and standard deviations of key performance measures at both PHCs in Table 2.

From Table 2, we observe that the average patient LOS at PHC₂ is significantly higher than that of PHC₁. We also note a similar trend in other performance measures such as resource utilizations and average wait time for all the staffs involved in providing medical care to outpatients at both PHCs. We also see that the individual wait times at each queueing subsystem within PHC₂ do not appear to be prohibitively long in themselves; however, as the over-

Table 2: PHC simulation outcomes.

Outcomes*	PHC ₁ ⁺ (9/1/1/1/1)	PHC ₂ ⁺ (2/1/1/1/1)
ρ_{doc}	0.392 (0.003)	0.935 (0.003)
ρ_{NCD}	0.512 (0.006)	1.475 (0.019)
ρ_{phar}	0.383 (0.003)	1.705 (0.007)
ρ_{lab}	0.317 (0.005)	1.417 (0.019)
w_{opd}	0.036 (0.001)	0.235 (0.002)
w_{phar}	0.243 (0.005)	27.93 (0.618)
w_{lab}	0.593 (0.012)	17.47 (0.419)
w_{NCD}	0.629 (0.023)	30.12 (0.830)
LOS	7.180 (0.033)	57.31 (1.154)

⁺Outpatient interarrival time (in minutes)/number of doctors/number of NCD nurse/number of laboratory technicians/number of pharmacists. *Resource utilizations (ρ) are dimensionless fractions, wait times (w), and average lengths of stay (LOS) are in minutes.

* ρ_{doc} : doctor’s utilization, ρ_{NCD} : NCD nurse’s utilization, ρ_{phar} : pharmacist’s utilization, ρ_{lab} : laboratory technician’s utilization, w_{opd} : OPD queue wait time, w_{phar} : pharmacy queue wait time, w_{lab} : laboratory queue wait time, w_{NCD} : NCD nurse queue wait time.

all LOS in the facility is considerable given the significantly lower service times (for example, the average consultation time with the doctor is less than one minute) at these individual subsystems within the PHC. These observations motivated the development of a diversion mechanism for outpatients based on LOS predictions, which we now describe.

4 OUTPATIENT DIVERSION MECHANISM

The diversion mechanism for outpatients that we develop is depicted in Algorithm 1. We describe this mechanism using the case of a specific patient whose assigned PHC is PHC₁. We consider one other PHC, which we call as PHC₂ (located farther from the patient than PHC₁), as a potential destination for diversion; however, our approach can be extended to any number of facilities in a healthcare network. As discussed in Section 1, we assume that the patient can

access an IT system - either a web-based or smart-phone application - that communicates the LOS predictions at PHC_1 and PHC_2 , and assigns the appropriate PHC to the patient based on these predictions. We assume that the patient accesses the application at time t , implying that the LOS prediction must be generated at time t ; however, the LOS at each PHC must be estimated at the time the patient is likely to arrive at the facility in question. Thus, while the prediction is made at time t , the LOS must be estimated at time $t + \delta_1$ or $t + \delta_2$, where δ_1 and δ_2 are the travel times of the patient to PHC_1 and PHC_2 . If we denote these LOS estimates as $LOS_{t+\delta_1}$ and $LOS_{t+\delta_2}$, then the patient is assigned the facility with the smaller of $LOS_{t+\delta_1}$ and $LOS_{t+\delta_2}$.

Algorithm 1: Outpatient diversion mechanism.

- 1: At time t , estimate $LOS_{t+\delta_1}$ at PHC_1
 - 2: At time t , estimate $LOS_{t+\delta_2}$ at PHC_2
 - 3: Visit PHC with $LOS = \min\{L_{t+\delta_1}, L_{t+\delta_2}\}$
-

We note here that for the purposes of this study, we assume that the patient is required to visit the assigned PHC, and incurs some penalty if they visit any other facility in the network; however, this mechanism, if adopted, is realistically more likely to provide a choice to the patient. In such situations the probability of compliance with the diversion or facility assignment suggestion must be considered in the analysis. We do not explore this in our study, and leave it as an immediate avenue of future research.

Another consideration here is that the diversion decision can be based on whether the sum of travel time to the PHC under consideration plus predicted LOS at the PHC is less than the corresponding quantity at the other PHCs in the network. We do not consider this case in our current version of the LOS-based diversion mechanism; this is because we wish to introduce this real-time LOS prediction based diversion mechanism in its simplest form (i.e., based only on predicted LOS at each facility). However, the diversion mechanism in Algorithm 1 can easily be modified to take such additional considerations into account, and we intend to explore this specific consideration in future work. Note also that the estimates of travel times used in the above algorithm can be considered to be average values; however, the actual values used in the diversion algorithm can be real-time estimates obtained from applications such as Google Maps depending upon the application process interface links that can be integrated into the IT system used for deployment of the diversion mechanism.

It is evident that the key steps within the diversion mechanism in Algorithm 1 are the estimation of the

LOSs $L_{t+\delta_1}$ and $L_{t+\delta_2}$. We now describe how these LOS predictions are generated.

4.1 Real-time LOS Prediction

As discussed in Section 3, we approximate the outpatient care process at the PHC as receiving care from one or more M/G/1 systems in series. The four possible M/G/1 system pathways for an outpatient in the PHC have also been depicted in Figure 1. We estimate the LOS in the PHC as the sum of the LOSs in each of the M/G/1 subsystems where the patient is likely to receive service.

$$LOS_{phc} = LOS_{doc} + LOS_{NCD} + LOS_{lab} + LOS_{pharmacy} \quad (1)$$

The terms in equation 1 are self-explanatory. We note here that for LOS_{phc} to be as accurate as possible, the LOSs associated with the laboratory and NCD nurse queuing systems must be weighted with the probabilities that the patient avails of service from these subsystems. However, we do not do this because of precedence in the literature regarding providing quantiles higher than the 50th quantile of the distribution of the predicted delay - or in this case, the predicted LOS - to increase customer satisfaction in queues (Whitt, 1999; Ibrahim, 2018). Note that this implies that we overestimate the LOS; however, given that each of the terms in equation 1 scales in the same manner with patient load, and does so to the same extent for all facilities considered, the diversion decision itself is not affected by this overestimation. Each LOS term in the right-hand side of equation 1 is estimated as the sum of the prediction of the delay experienced by the patient at the M/G/1 system under question and the average service time at the system. This is given in equation 2 below.

$$LOS_s = d_s + E[s], \forall s \in S \quad (2)$$

In equation 2, s is a subscript indicating an M/G/1 queueing subsystem in the PHC, and $s \in S = \{doc, NCD, lab, pharmacy\}$. d_s represents the estimated delay for the patient at a given M/G/1 subsystem, and $E[s]$ represents the expected value of the service time. The estimated delay d_s is in turn estimated as a function of the number of patients ahead of the patient under consideration in the queue, and the expected remaining service time for the patient currently receiving service at the queueing system. This is expressed below.

$$d_s = L_{q(s)}E[s] + r_e, \forall s \in S \quad (3)$$

In equation 3, $L_{q(s)}$ represents the number of patients ahead of the patient under consideration in the queue at subsystem s , and r_e represents the prediction

of the remaining service time given that the elapsed service time for the patient receiving service is x . We now discuss how r_e is generated. For M/G/1 systems, the estimation of r_e is traditionally done as in equation 4. In equation 4, T is the random variable representing the service time (t is thus its realization), G is the *cdf* of the service time.

$$\begin{aligned} P(T \leq t|x) &= \frac{P(x \leq T \leq t+x)}{P(T \geq x)} \\ \implies G(t|x) &= \frac{G(t+x) - G(x)}{1 - G(x)} \end{aligned} \quad (4)$$

r_e is then estimated as the expected value of the remaining service time by calculating the *pdf* $g(t|X)$ from equation 4. However, performing this calculation can prove tedious depending upon the nature of the distribution $G(t|x)$ (Fatma and Ramamohan, 2021b). For example, computing r_e via equation 4 for the triangular distribution requires working with a piecewise continuous *cdf*, and for the Gaussian distribution requires numerical computation of the integrals involved in computation of r_e (which involve the Gaussian error function). Therefore, we propose an alternate delay predictor for such systems, which can potentially also be extended to multi-class multi-server queueing systems, that is considerably easier to compute and implement. This delay predictor is given below.

$$r_e = \begin{cases} G^{-1}(0.5) - x, & 0 \leq x < G^{-1}(0.5) \\ G^{-1}(0.75) - x, & G^{-1}(0.5) \leq x < G^{-1}(0.75) \\ \frac{(b-x)}{2}, & G^{-1}(0.75) \leq x \leq b \end{cases} \quad (5)$$

Here, r_e is estimated depending upon the range in which the elapsed service time x of the patient currently in service lies with respect to the service time distribution. $G^{-1}(0.5)$, and $G^{-1}(0.75)$ are the 50th and 75th quantiles of the service time distribution, respectively, and b is the extreme quantile (e.g., equal to $G^{-1}(0.99)$) or the upper limit of the service time *cdf* (e.g., where the *cdf* has bounded support).

r_e as estimated from equation 5 can then be used in conjunction with equations 3 and 2 to estimate the LOS at the subsystem under consideration. This computation would suffice if the LOS prediction is required to be made at time t ; however, we require LOS predictions at time $t + \delta$, where δ is the time required for the patient to arrive at the PHC. LOS predictions at time $t + \delta$ are generated as follows.

In order to calculate the LOS prediction at time $t + \delta$, we must first estimate the effective average queue length at $t + \delta$, $L_{q(t+\delta)}$, as a function of the queue length at time t , given by $L_{q(t)}$. This can be accomplished using the average post-diversion patient

arrival and service rates, λ_e and $E[s]$, respectively. This is given below.

$$L_{q(t+\delta)} = L_q + (\lambda_e - 1) - \left\lfloor \max \left\{ \frac{\delta - r_{e(t)}}{E[s]}, 0 \right\} \right\rfloor \quad (6)$$

In equation 6, we subtract 1 from λ_e to account for the patient under consideration for diversion, and the third term on the right hand side represents the average number of patients that can be served in δ time units after taking into account the remaining service time $r_{e(t)}$ of the patient in service at time t . The average elapsed service time at time $t + \delta$, denoted by $x_{t+\delta}$, is thus estimated as the remainder of $\frac{\delta - r_{e(t)}}{E[s]}$, and the average remaining service time at $t + \delta$, denoted by $w_{e(t+\delta)}$, is estimated by inputting $x_{t+\delta}$ into equation 5. The expected delay at time $t + \delta$ for a patient arriving to subsystem s , $d_{s(t+\delta)}$, can thus be estimated by inputting $L_{q,s(t+\delta)}$ and $r_{e,s(t+\delta)}$ into equation 3, and the LOS at time $t + \delta$ can in turn be generated using the value of $d_{s(t+\delta)}$ in equation 2. This is summarized in equation 7 below.

$$\begin{aligned} LOS_{s(t+\delta)} &= d_{s(t+\delta)} + E[s] \\ \text{where } d_{s(t+\delta)} &= L_{q,s(t+\delta)} E[s] + r_{e,s(t+\delta)} \end{aligned} \quad (7)$$

The LOS at subsystem s estimated from equation 7 can be used to calculate the LOS at each of the subsystems in S , and then summed per equation 1 to yield the total LOS at the PHC. Note that the LOS prediction at each subsystem must be calculated at the point at which the predicted LOS at the previous subsystem ends. This process of successively calculating the LOSs is summarized in Algorithm 2.

Algorithm 2: Calculation of the total LOS at a PHC.

- 1: Initialize with the ordered tuple $S_{ord} = (NCD, doc, lab, pharmacy)$, $\delta =$ travel time to the PHC, current time t , $LOS_{tot} = 0$.
 - 2: **for** $s \in S_{ord}$ **do**
 - 3: Estimate LOS at s at time $t + \delta$ as $LOS_{s(t+\delta)}$.
 - 4: $LOS_{tot} = LOS_{tot} + LOS_{s(t+\delta)}$.
 - 5: $\delta = \delta + LOS_{s(t+\delta)}$
-

We must mention here that in the implementation of the above algorithm, the arrival rate at each subsystem is assumed to be the same - that is, λ_e . This is possible only because we assume that patients visit all subsystems in series (Hopp and Spearman, 2011), and we consider relaxing this assumption as an immediate avenue of future work. We describe the outcomes of implementing the diversion mechanism in the following section.

We also note here that the deployment and generation of these LOS predictions is unlikely to incur significant computational expense, regardless of the size of the facility network. This is because the generation of the LOS predictions at a facility involves just a single function evaluation, and this will remain the same regardless of the type of LOS predictor involved - for example, a queueing theory based predictor or a statistical learning based predictor. However, the specific LOS predictor to be employed must be selected taking into account the number of system state variables that are required for LOS prediction. For example, if a particular LOS predictor requires a large number of system state variables to be recorded and updated at high frequency, then the complexity of the IT system required for this purpose will also increase. On the other hand, if another LOS predictor is available that requires significantly fewer variables to be recorded at a small loss of predictor accuracy, then it might be prudent to choose this latter LOS predictor from the standpoint of ease of deployment.

Now, based on the real-time LOS estimates generated using Algorithm 2, we can implement the diversion mechanism in Algorithm 1.

5 SIMULATION EXPERIMENTS

In this section, we present results from implementing the diversion mechanism described in Algorithm 1. We compare the operational outcomes at both PHCs for three cases: (a) without diversion, (b) diversion implemented using actual real-time LOS estimates (which are available to the simulation modeler), and (c) the real-time LOS predictions generated using the methodology described in Section 4.1. Before we present the operational outcomes, we discuss the accuracy of the real-time LOS predictor.

We characterize the accuracy of the LOS predictor using the mean absolute percentage error (MAPE) score, which is given by the formula $MAPE = \frac{1}{N} \sum_{i=1}^N \left| \frac{LOS_{a,i} - LOS_{p,i}}{LOS_{a,i}} \right|$. Here $LOS_{a,i}$ is the actual LOS estimate of the i^{th} patient and $LOS_{p,i}$ is the predicted LOS estimate of the i^{th} patient and N is the number of patients in the sample. When LOS predictions are generated for the two PHCs that we consider (PHC₁ and PHC₂) in the first set of computational experiments that we describe in the following paragraphs, we find that the MAPE scores are 13.00% and 18.14% at PHC₁ and PHC₂, respectively. While this appears to indicate reasonable performance of the LOS predictor, several avenues of improvement of the predictor remain, which we discuss in the following section.

In Table 3, we report results from the implementation of the diversion mechanism for PHC₁ and PHC₂ with operational parameters corresponding to those described in Table 2. Given that diversion is likely to make the utilization of resources more equitable across a network of facilities, and due to space limitations, we report the mean and standard deviation of the percentage difference between operational outcomes such as resource utilizations, wait time, and average LOSs across both healthcare facilities in Table 3. We also report the percentage of patients who are diverted - that is, those who visit a PHC than their assigned PHC. We report these outcomes for the three cases discussed in the above paragraphs.

Table 3: Outpatient diversion: operational outcomes across both PHCs.

Outcomes	No diversion	With actual LOS	With predicted LOS
$\Delta_{p_{doc}}$	58.09 (0.36)	5.91 (0.03)	15.60 (0.01)
$\Delta_{p_{ncd}}$	65.26 (0.58)	7.82 (0.02)	13.50 (0.02)
$\Delta_{p_{phar}}$	77.52 (0.18)	16.33 (0.02)	22.20 (0.04)
$\Delta_{p_{lab}}$	77.59 (0.45)	10.30 (0.03)	18.02 (0.06)
$\Delta_{w_{opd}}$	84.53 (0.38)	4.07 (0.02)	46.77 (0.03)
$\Delta_{w_{phar}}$	99.12 (0.02)	51.55 (0.08)	74.28 (0.03)
$\Delta_{w_{lab}}$	96.60 (0.09)	37.95 (0.05)	79.03 (0.04)
$\Delta_{w_{ncd}}$	97.91 (0.10)	14.12 (0.10)	86.16 (0.02)
Δ_{LOS}	87.46 (0.26)	10.21 (0.04)	63.41 (0.08)
β^*	0.00 (0.00)	30.95 (2.98)	45.25 (1.48)

$\Delta_{p_{doc}}$ = difference in doctor's utilization, $\Delta_{p_{ncd}}$ = difference in NCD nurse utilization, $\Delta_{p_{phar}}$ = difference in pharmacist's utilization, $\Delta_{p_{lab}}$ = difference in laboratory technician's utilization, $\Delta_{w_{opd}}$ = difference in OPD queue wait time, $\Delta_{w_{phar}}$ = difference in pharmacy queue wait time, $\Delta_{w_{lab}}$ = difference in laboratory queue wait time, $\Delta_{w_{ncd}}$ = difference in NCD nurse queue wait time, β^* = proportion (%) of outpatients who are diverted.

It is evident from Table 3 that implementing diversion leads to significantly more equitable operational outcomes across both PHCs. Without diversion, as seen from Table 2, the LOS at PHC₂ was 57.31 minutes in comparison to an LOS of 7.18 minutes at PHC₁. When diversion is implemented using the actual real-time LOS estimates, we see that the LOS at PHC₂ reduces to 15.33 minutes whereas the LOS at PHC₁ increases to 13.75 minutes. With diversion implemented using the real-time LOS predictor, we see LOSs of 25.92 minutes at PHC₁ and 9.50 minutes at PHC₂. Thus it is clear that diversion, even using the real-time LOS predictor, yields a reduction in LOS at the more heavily utilized system.

From Table 3, we see similar trends in the other outcomes as well when diversion is implemented using the real-time LOS predictor. Overall, we see a significant decrease in the congestion in PHC₂. A key observation is that the extent to which operational out-

comes become equitable across both PHCs depend on the accuracy of the delay predictor employed - that is, the relative difference in a resource’s utilization or the wait time at a subsystem increases with a decrease in the accuracy of the delay predictor used. Another important observation is that the proportion of patients diverted also increase with decrease in delay predictor accuracy. However, this is likely due to the significant difference in the outpatient loads between the two PHCs prior to implementing diversion - that is, an average interarrival time of 9 minutes at PHC₁ compared to an average interarrival time of 2 minutes at PHC₂.

In order to check how the operational outcomes change with diversion when the difference in outpatient loads across both PHCs is not as high as in the previous case, we conducted a sensitivity analysis by changing the average interarrival times of outpatients to 2 minutes at PHC₁ and 4 minutes at PHC₂. We present the results from this sensitivity analysis results in Table 4 and observe similar trends in the operational outcomes across both PHCs as observed in Table 3. We note that the MAPE scores of the real-time LOS predictor that we propose at PHC₁ and PHC₂ were found to be 13.82% and 18.33% respectively, respectively.

Table 4: Sensitivity analysis: operational outcomes across both PHCs.

Outcomes	No diversion	With actual LOS	With predicted LOS
$\Delta_{p_{doc}}$	37.28 (0.41)	18.82 (0.05)	24.91 (0.01)
$\Delta_{p_{ncd}}$	41.92 (0.85)	19.84 (0.04)	26.68 (0.02)
$\Delta_{p_{phar}}$	49.92 (0.85)	22.96 (0.04)	34.08 (0.01)
$\Delta_{p_{lab}}$	50.02 (1.07)	24.28 (0.04)	33.29 (0.02)
$\Delta_{w_{opd}}$	60.01 (0.84)	11.52 (0.05)	17.13 (0.07)
$\Delta_{w_{phar}}$	96.19 (0.08)	46.19 (0.07)	64.84 (0.03)
$\Delta_{w_{lab}}$	87.59 (0.52)	44.81 (0.09)	71.24 (0.03)
$\Delta_{w_{ncd}}$	92.52 (0.23)	29.18 (0.07)	71.24 (0.03)
Δ_{LOS}	83.15 (0.29)	31.48 (0.07)	78.96 (0.01)
β^*	0.00 (0.00)	45.37 (0.78)	36.69 (0.10)

$\Delta_{p_{doc}}$ = difference in doctor’s utilization, $\Delta_{p_{ncd}}$ = difference in NCD nurse utilization, $\Delta_{p_{phar}}$ = difference in pharmacist’s utilization, $\Delta_{p_{lab}}$ = difference in laboratory technician’s utilization, $\Delta_{w_{opd}}$ = difference in OPD queue wait time, $\Delta_{w_{phar}}$ = difference in pharmacy queue wait time, $\Delta_{w_{lab}}$ = difference in laboratory queue wait time, $\Delta_{w_{ncd}}$ = difference in NCD nurse queue wait time, β^* = proportion (%) of outpatients who are diverted.

Once again, as expected, we notice that the extent to which operational outcomes become equitable across both PHCs decreases with a decrease in the accuracy of the LOS predictor employed. However, when compared to the corresponding outcome in Table 3, we notice the proportion of patients diverted actually decreases with a decrease in the accuracy of

the LOS predictor. That is, in this case, the proportion diverted decreases with the less accurate LOS predictor. Thus, our results indicate that the proportion of patients diverted across a healthcare facility network, when diverted on the basis of real-time LOS predictions, depend on both the accuracy of LOS predictor employed as well as the differences in patient loads across the facilities within the network.

Overall, it is evident that outpatient diversion using real-time LOS predictions appears to improve operational performance of a healthcare facility network from both the provider as well as patient standpoints.

We now conclude our work with a discussion of the contributions of our work and avenues for further research.

6 DISCUSSION & CONCLUSIONS

In this work, we present preliminary results from the simulated implementation of a framework for outpatient diversion using real-time LOS predictions. We emphasize here that the real-time LOS predictions and the diversion decision are made at the point of origin of the patient, as opposed to when the patient arrives at a healthcare facility. We empirically show via DES that patient diversion results in more equitable utilization of resources across the healthcare facilities that we consider when compared to the case without diversion. The diversion framework that we propose can also be repurposed for real-time facility assignment in health systems where patients are not *a priori* assigned their first point of contact for medical care.

To the best of our knowledge, our study is the first of its kind that considers real-time LOS predictions in initiating diversion across healthcare facilities. Previous studies initiated diversion using factors such as average wait time, number of beds occupied, number of patients in waiting queue. Further, we demonstrate how LOS can become a criterion for diversion especially when diversion of non-emergency patients, and in particular outpatients, is considered. Once again, we did not come across a previously published study that considered outpatient diversion. Finally, to our knowledge, our study is also the first to develop a methodology for the real-time estimation of LOSs in the near future. This in turn utilizes a novel approximation of the real-time delay predictor for M/G/1 queueing systems. In comparison to most studies in the literature that utilize LOS predictors based on statistical learning methods, our analytical approach grounded in queueing theory may be easier to program and deploy in practice. Further, our ap-

proach also opens up avenues for leveraging the substantial body of work regarding real-time delay prediction for real-time LOS-based diversion for more complex queueing systems than that encountered in this study.

A key avenue of future research involves improving the accuracy of the real-time LOS predictor that we develop in this study. While the accuracy of our real-time LOS predictor appears reasonable, with MAPEs not exceeding 20%, relaxing our assumption that all patients avail service from all subsystems within the PHC is likely to improve the accuracy of the predictor. A related direction for future work involves benchmarking our work against statistical machine learning based predictors of LOS, and assessing how the operational outcomes compare with respect to the outcomes associated with our queueing theory based predictor. Similarly, our LOS based diversion mechanism can also be compared to other diversion mechanisms, such as wait-time based mechanisms.

A key question that arises here is how the implementation of the diversion mechanism in Algorithm 1 would scale in terms of computational and operational overheads when the size of the network increases. One of the questions that must be answered prior to deploying this approach in a larger network would involve determining, for each facility in the network, the set of facilities at which real-time LOS predictions must be generated in order to determine the diversion decision. For example, in a network with 1,000 facilities, in order to determine the diversion decision for a patient, real-time LOS predictions need not be generated at each of the 1,000 PHCs (as many would be located at a significant distance from the patient's assigned PHC). Thus determining the optimal set of facilities that need to be considered as diversion destinations for a given facility in a large network becomes an important avenue of future research when considering the scaling of this approach to larger facility networks. As part of this, an immediate avenue of research we are pursuing includes expanding this diversion framework to the entire network of 9 PHCs in the Indian district that we consider, and we plan to answer the above question for this network.

Finally, another avenue of future research is extending our work to more complex queueing systems, including multi-class multi-server systems, which may be applicable in the case of diversion of inpatients or emergency patients. These more complex queueing systems can include queueing systems with non-stationary arrival and service processes. Real-time delay predictors have been developed for certain queueing systems with non-stationary arrival, service or abandonment processes (Ibrahim, 2018); and LOSs

for such systems may correspondingly be calculated in a manner similar to that which we have described in this paper. However, in case the non-stationarity is not complex - for example, increased arrival rates may be observed during the weekends, and arrival rates remain the same during all weekdays - then the methodology described for LOS prediction in this paper can directly be applied by just changing the arrival rates in the LOS predictor expressions depending upon the day of the week.

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