Optimizing Heterogeneous Maritime Search Teams using an Agent-based Model and Nonlinear Optimization Methods

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Abstract: This paper introduces a new search planning methodology, nicknamed Pathfinder, that can optimize heterogeneous teams of mobile and stationary searchers. Unlike previously developed search methods, the new methodology applies an Agent-Based Model (ABM) to simulate target movement and behavior then uses nonlinear optimization methods to find optimal search plans for complex teams of searchers. We describe initial target location with a probability distribution influenced by evidence and environmental data. The ABM models target movement based on environmental and behavioral factors. Then, Pathfinder suggests a search plan that maximizes the probability of target detection and satisfies searcher requirements.

1 INTRODUCTION

Search Theory was initially developed during World War 2 by B.O. Koopman to assist with creating optimal search strategies to find German U-boats (Koopman, 1946 (declassified in 1958)) Search Theory has advanced significantly in the past 75 years to include most elementary searcher types and target types (see, (Stone, Royset, & Washburn, Optimal Search for Moving Targets, 2016)) for a comprehensive review. Historically, the Office of Naval Research (ONR) has been a driving force in Search Theory research in the United States. The USCG was one of the first organizations to deploy a computerized methodology for search and rescue (SAR) operations called Computer-Assisted Search Planning System (CASP) (Richardson & Discenza, 1980). This used a Monte Carlo particle method to model targets. In 2003 the USCG started development of the Search and Rescue Optimal Planning System (SAROPS) (Kratzke, Stone, & Frost, 2010) to replace CASP. SAROPS was a significant improvement over the USCG's CASP system by incorporating a custom numerical search technique to find search plans and improving the Monte Carlo particle method used to model targets. This methodology has been operational since January 2007.

In more recent times, more methodologies in Search Theory were developed that accommodate more search scenarios. One of them is the genetic simulated annealing algorithm (GSAA). (Ai, Li, Gao, Xu, & Shang, 2019) Another one is based on branch-and-bound algorithms. (Sato & Royset, 2010) There is also a new interactive heuristic-based optimization model (Abi-Seid, Morin, & Nilo, 2019) created to assist SAR operations in Canada. Nonlinear optimization has been applied to search theory research, in particular, finding hidden objects (El-Hadidy & Alfreedi, 2020). At the same time nonlinear optimization techniques have not yet been fully utilized to optimize heterogeneous teams of mobile and stationary searchers.

An important component in any search theory methodology, that optimizes search plans to find a mobile target, is how it models target movement. Traditionally diffusion processes have been popular see (Lin & Goodrich, 2010) and (Eagle, 1984). Currently a particle method is used by SAROPS. (Kratzke, Stone, & Frost, 2010). Some research has been done to apply an Agent-Based Model (ABM) to

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model wilderness searches (Mohibullah & Julie, 2013). In addition, some case studies have been done to apply agent-based simulations to maritime search operations as a way to improve verification and validation methods. (Onggo & Karatas, 2016)

Most relevant research in adopting ABM in maritime environments is focused on military and security applications. This includes port security (Harris, Dixon, Dunn D.L., & Romich, 2013) and using UAVs for surface surveillance (Steele, 2004). In addition, several papers have been published in regards to force protection simulations including (Walton, Paulo, McCarthy, & Vaidyanathn, 2005) and (Sullivan, 2016). Finally, there has been several papers published on the use of ABM and counter-piracy operations (Dabrowski & Villiers, 2015) and (Marchione, Johnson, & Wilson, 2014). A common issue encountered by this line of research is in verification and validation of these models. For example, in analysing strategies to protect merchant ships from pirate attack (Deraeve, Anderson, & Low, 2010).

In the past 20 years, Search Theory has been used multiple times to find missing aircraft. The search for Air France, which was lost in June 2009, was found using Search Theory. (Stone, In Search of Air France Flight 447, 2011) The flight recorders were recovered in May 2011. Currently Search Theory is used every day by the USCG to find missing persons along the US coastlines using SAROPS.

We propose a new search planning methodology based on an Agent-Based Model (ABM) and nonlinear optimization techniques. Pathfinder introduced in this paper strives to advance search planning by focusing on 4 core areas:

1. optimizing heterogeneous teams of mobile and stationary searchers
2. modeling target behavior
3. inherent searcher safety
4. enhance future research, training, and appropriations

Pathfinder incorporates an ABM that can model target movement based on behavioral factors, besides environmental factors. This is important because the behavior of missing targets could fall into different scenarios such as, for example dropping an anchor or clinging onto a buoy (Adlerstein, 2019). Therefore, by incorporating various scenarios in the ABM, Pathfinder can model a more realistic target movement. The second important feature of Pathfinder is that it employs nonlinear optimization methods to find optimal search plans based on modeled target movement. Furthermore, employment of nonlinear optimization methods allows us to optimize teams of heterogeneous searchers, including stationary and mobile searchers together, among other benefits, including not constraining Pathfinder to using ladder pattern search plans and rectangular search areas. The ladder pattern searches may include regions of least concern (Kratzke, Stone, & Frost, 2010) and thus can be less efficient. Pathfinder finds search paths that maximize the probability of detection (POD) with the flexibility of adding constraints and penalties, that help find search paths that are realistic and easy to implement. This flexibility of the methodology allows searchers to focus on regions of high POD. In addition, Pathfinder can strengthen the optimization model to address concerns and demands of naval pilots and SAR personnel. Another important feature of Pathfinder is that it can accommodate searcher safety. For example, additional modifications can guarantee that searchers do not come within a dangerous distance from each other. Finally, since Pathfinder can efficiently simulate thousands of different scenarios, it can be used for SAR research, training, appropriations, and evaluation of new equipment and techniques.

We organized the paper as follows. The next section reviews a search scenario that will help explain Pathfinder, Section 3 reviews Pathfinder, Section 4 discusses results, Section 5 provides concluding remarks, and Section 6 discusses how Pathfinder could be further improved and transformed into a new life saving application.

2 SEARCH SCENARIO

Imagine being a search manager that creates, implements, and manages SAR operations. It is a normal summer day at a popular beach location, warm with clear skies. There is a strong wind due north at 10knots. This day the currents are strong, with a west to east flow. Then a distress signal was received and a search operation needs to be launched.

The call is from a recreational boat with people on board that have possibly experienced a health issue. This boat is not far from the coastline and the caller indicates they are heading to a pier along the coastline. The call was interrupted and further communication attempts were unsuccessful. We estimate the boat is no longer being actively sailed and may have lost power. The last known location was estimated by triangulating the emergency radio call. Suppose the search assets available, and modeled in Pathfinder, are short range assets like the MH-60 "Jayhawk" (Pike, n.d.) and search boats like the 47-foot Motor Lifeboat. (MLB). (Motor Lifeboat (MLB), n.d.). We also include a theoretical "smart buoy" which represents a stationary searcher. We will demonstrate the feasibility and benefits of this new methodology by finding optimal search plans for this search scenario.

3 DETAILED DESCRIPTION OF PATHFINDER AND ITS COMPONENTS

There are two principal components of Pathfinder. The optimization model and the ABM. The ABM simulates a large number of possible scenarios of a target trajectory, and then sends the information to the optimization model, which then creates optimal search plans for the search operation. Figure 1 shows the relationship of these components to search operations and data.

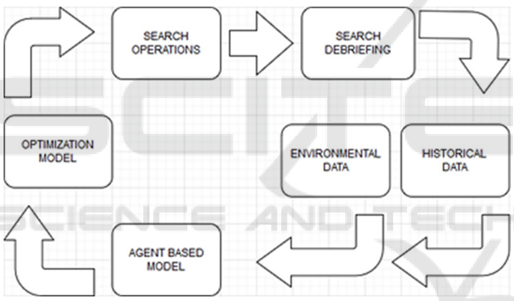

Figure 1: The two principal components of Pathfinder, ABM and optimization model, and their relationship with available data and search operations.

3.1 Domain

We need to discuss a few fundamental definitions. Pathfinder uses a two-dimensional domain to model the search area.

$$\Omega \in R^2$$

Searchable subdomains are constructed to limit searchers from areas they are not allowed, such as foreign or restricted territories. We define this area, Ω_s, such that

$$\Omega_s \subseteq \Omega.$$

In our case the searchable sub domain is the same as the domain. In addition, the domain is a coastline that is mostly maritime environment that is 1,000 km^2.

We define our searcher paths $z_t^k, t = 1, . . , T$ for searcher $k = 1, \ldots, K$ and target paths $u_t^g, t = 1, \ldots, T$ for target g of G targets to satisfy the following.

$$u^g \in \Omega$$
$$z^k \in \Omega_s$$

3.2 Prior Distribution

To describe the position of the target before the search starts we use a initial probability distribution $\theta(x)$, which could be based, in particular, on a targets last known location. To add more accuracy and flexibility, we use regions defined as $R_i \subseteq \Omega$ with probabilities a_i. This is useful when there are several sources of information, evidence, and the chance of error. These regions satisfy the probability that the target is in the domain M.

$$\sum_{i=0}^{m} a_i \int_{R_i} \theta(x)dx = M \; and \; \sum_{i=0}^{m} a_i = 1 \qquad (1)$$

For example, in our scenario, we have two circular regions representing the last known location at the center of the domain. A large 40% region with a radius of 4.15 km, where there is a 40% chance the target was in that region at $t = 0$. Then a smaller 50% region within it with a radius of 1.65 km where there's is a 50% chance the target was there at $t = 0$. The rest of the domain falls within a 10% region where there is a 10% chance that a target is there. The 40% and 50% regions will reside in the center of the search domain.

3.3 Agent-based Model

This prior distribution is used in an ABM to model target movement. This model uses numerous independent agents that are affected by environmental factors, behavioral factors, and hazards.

First, environmental factors are wind and currents that are in our search area. The wind the currents in our example will push these agents north then east. The ABM uses equations from the USCG (USCG, 2013) to calculate leeway speed and can incorporate the Rayleigh Method (Kratzke, Stone, & Frost, 2010) in the future. The ABM also incorporates hazards such as rocks and etc. that agents must navigate around. In our example, there are no hazards to navigate around.

There are also behavioral factors. These behavioral factors depend on survival modes to model target movement. When most people are lost they rely on a survival strategy to survive or find their way home. These include overdue, travel aid, route finding, wandering, and staying put. In our example, overdue, travel aide, wandering, and stay put are seen. When a target is overdue, it is not lost at all and are simply late getting home or their next waypoint. The travel aide mode is when a target has some travel aids and has the ability to eventually self-rescue. This mode relies on the theory of "bounded rationality" (Simon, 1982) According to this theory, rationality is bounded due to data and mental capabilities. Thus, a missing person's idea of a path home is more accurate as they approach future waypoints. Wandering is when a target does not have travel aids and wonders the domain. Finally, the stay put mode is when a target decides to stay where they are. In the case of a boat this could be implemented by using an anchor or beaching the boat. The ABM provides us with our estimate target paths u_t^g $t = 1, \ldots, T$ that will be used to optimize search plans.

This is unlike current methodologies that implement particle methods. Particle methods use particles that move based on environmental factors and hazards. Therefore, particle methods cannot model intelligent agents that can make decisions. With an ABM, the agents are intelligent and can think and make decisions. Thus, we can model behavioral factors. The ABM can also model targets that decide to change survival modes and target type changes. Therefore, an ABM can model far more target types accurately than a particle method.

In our example the ABM is using 501 agents to model target movement. Such a number provides sufficient map coverage and also shows a variety of target behaviors while being within the technical capability of the test system.

3.4 Detection Function

Next, we model the probability that a searcher at z_t^k will detect a target at u_t^g at time t. This is implemented using a detection function, which depends on several factors including time, distance, visibility, and properties of the target. Some previous methodologies use the idea of sweep widths, lateral ranges, etc. See (Frost & Stone, 2001). In Pathfinder, we use a modification of the inverse Nth power law (Iida, 1993) below, because it gives us a lot of flexibility.

$$\Gamma\left(u_t^g, z_t^k, \Delta t\right) = 1 - \exp\left(-\Delta t \, \frac{\alpha\left(z_t^k, \tau\left(z_t^k\right), u_t^g, v\right)}{\left|u_t^g - z_t^k\right|}^{n\left(z_t^k, \tau\left(z_t^k\right), u_t^g, v\right)}\right) \quad (2)$$

$$n(*) > 0, \alpha(*) > 0, \Delta t > 0, z_t^k \in \Omega_s, u_t^g \in \Omega$$

This function depends on time step Δt, target type u_t^g, searcher type z_t^k, visibility v, terrain type $\tau(z_t^k)$, and the parameters $\alpha(*)$ and $n(*)$. For notational simplicity we also define the probability of not detecting a target as below:

$$\bar{\Gamma}\left(u_t^g, z_t^k, \Delta t\right) = 1 - \Gamma\left(u_t^g, z_t^k, \Delta t\right) \quad (3)$$

Since our example is a marine search operation, we used data for a missing boat (N. C. Edwards, 1981) and found some of these values.

For example, for a USCG Point Class cutter searching for a 16-foot boat or orange life raft in a maritime terrain, n and α were found as $\alpha = 0.413$ and $n = 5.955$. Likewise, for a USCG HH-52 helicopter searching for a 16-foot boat or orange life raft in a maritime terrain, n and α were found as $\alpha = 0.471$ and $n = 3.656$.

Both of these search assets are retired by the USCG so future data collection and analysis is needed.

3.5 Optimization Model

The objective of the Pathfinder methodology is to find optimal searcher paths, $z_t^k, t = 1, \ldots, T$, that maximize the POD. These paths depend on target paths from the ABM, $u_t^g, t = 1, \ldots, T$, and the detection function. We call a collection of searcher paths a search plan. This POD function is as follows:

$$F(z_t^k) = 1/|G| \sum_{t=1}^{\frac{T}{\Delta t}} \sum_{k=1}^{|K|} \sum_{g=1}^{|G|} \Gamma\left(u_t^g, z_t^k, \Delta t\right) \prod_{j=1}^{t-1} \bar{\Gamma}\left(u_j^g, z_j^k, \Delta t\right) \quad (4)$$

This objective function is a modification of the objective function found in (Ding & Castanon, 2018) and follows the theory in (Przemieniecki, 2000) page 277.

To make the objective function produce realistic search trajectories, we incorporate three penalty terms for fuel, momentum, and center-of-mass. The fuel penalty below is used to make more cost-effective search trajectories and cut down on suboptimal waypoints.

$$P_F(z_t^k) = \sum_{k=1}^{|K|} \sum_{t=1}^{\frac{T}{\Delta t}} P_k^F \left\|z_t^k - z_{t-1}^k\right\|_2 \quad (5)$$

$$where\ P_k^F \leq 0\ \ for\ searcher\ k$$

The following is the momentum penalty. This penalty reduces zig-zagging and generally smooths paths and make them easier to follow.

$$P_M\left(z_t^k\right) = \sum_{k=1}^{|K|} \sum_{t=1}^{\frac{T}{\Delta t}} \frac{M_k\, P_k^M \left\| z_{t+1}^k - 2z_t^k - z_{t-1}^k \right\|_2}{\Delta t} \qquad (6)$$

$$where\ \ M_k\ > 0\ for\ searcher\ k$$

$$and\ \ P_k^M \leq 0\ for\ searcher\ k$$

Finally, the center-of-mass penalty eliminates erratic search trajectory and helps the nonlinear optimization model converge to a solution.

$$P_{CM}\left(z_t^k\right)\ = \sum_{k=1}^{|K|} \sum_{t=1}^{T/\Delta t} P_k^{CM} \| z_t^k - avg(u_t^g) \|_2 \qquad (7)$$

$$where\ P_k^{CM} \leq 0\ for\ searcher\ k$$

$$where\ avg(u_t) = \frac{1}{G} \sum_{g=1}^{G} u_t^g$$

With these 3 penalty terms we have the following Pathfinder's optimization model with the positive weights w_F, w_M, and w_{CM}.

<u>*Maximize:*</u>

$$F\left(z_t^k\right) + w_F P_F\left(z_t^k\right) + w_M P_M\left(z_t^k\right) + w_{CM} P_{CM}\left(z_t^k\right) \quad (8)$$

<u>*Subject to:*</u>

- *Movement constraints on the searchers with* $\varepsilon\left(s_k, \tau(z_{t-1}^k)\right) = 0$ *implying a stationary searcher*

$$\|z_t^k - z_{t-1}^k\|_2\ \leq\ \varepsilon\left(s_k, \tau(z_{t-1}^k)\right) \geq 0, for\ k\ searcher$$

- *Initial locations constraints on the searchers*

$$z_0^k = Z_0^k\ \ for\ searcher\ k$$

- *Final locations constraints on the searchers*

$$z_T^k = Z_T^k\ \ for\ searcher\ k$$

4 DISCUSSION OF RESULTS

To examine search trajectories calculated by Pathfinder, we built a prototype to run experiments, described in figure 1. We used NetLogo (Wilensky, 1999) for the ABM module and a nonlinear solver MINOS (Murtagh & Saunders, 1978) and AMPL (AMPL Optimization inc, 2021) for the Optimization module. The computer being used is a Dell Alienware M17 with 8GB of ram and an Intel i7-9750H processor. We experiment with several search teams to find optimal solutions to the search scenario described in this paper. We were able to find optimal solutions for several scenarios. Here we demonstrate one of them.

The ABM performed as expected. The environmental factors move agents that have lost power and have not deployed an anchor, some agents are moving to their destination when they have power, and some of them employ an anchor if they are in shallow water. Of the two target types in this scenario, boat with power and a boat without power, the model shows 3 distinct behaviors a missing boat could employ. We can see these behaviors below.

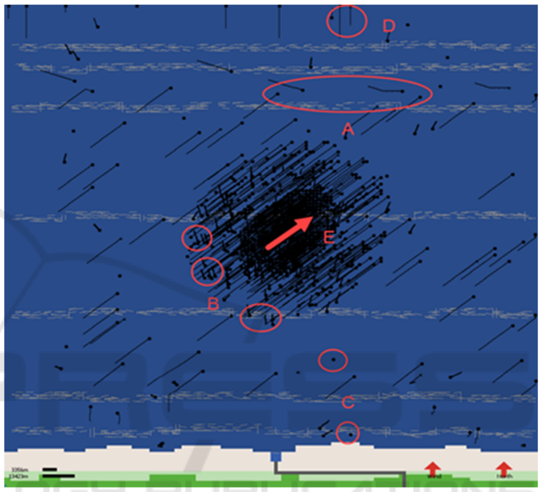

Figure 2: visualization of target behavior including A) agents being swept away by the current and wind B) Agents heading to their final destination under their own power C) agents that decided to deploy an anchor and stay put. D) agents being blown away primarily by the wind. E) the center-of-mass of target agents with direction.

Figure 2 also shows another important advantage of using an ABM to model target behavior. When searching for a missing boat, that boat may or may not have power, it may have deployed its anchor, it may have capsized, it may have sunk, there could be life rafts in the water, or the passengers may be in the water. Therefore, there are multiple target types that the SAR operations could be looking for, with several distinct behaviors each target could inhibit. Using the ABM is beneficial to search operations because it can model simultaneously all potential target types, target behaviors, and the transition of one target type to another. This ABM has the potential to be tuned and optimized by employing historical data.

Using nonlinear optimization techniques, Pathfinder can optimize a search operation with searchers with radically different capabilities. Figure

3 is a search operation optimized with Pathfinder and has 3 assets: a helicopter, a boat, and a “smart” buoy that can detect targets. Also note that there is an operations outpost from which the helicopter is operating from. This is also an asset that can detect a target while stationary, thus not optimized by pathfinder but its detection abilities considered in the model. This search plan's POD is 8.08% and shows that this new methodology can optimize teams of mobile and stationary searchers.

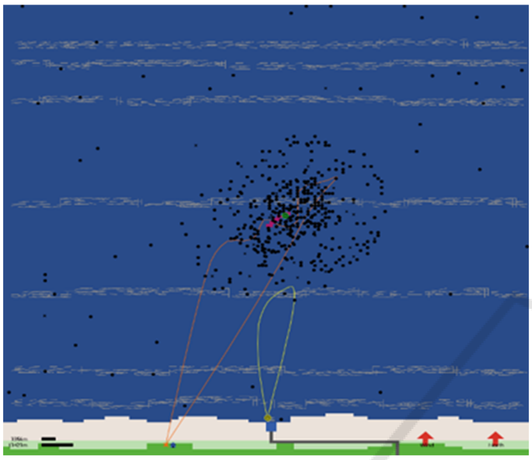

Figure 3: A helicopter (orange), boat (yellow), operations outpost (blue), and one buoy (green) searching the domain. Note the difference in travel distance in each searcher type. Pathfinder utilized each searcher's performance to find an optimal search plan. Also note how the search plans were affected by the target center-of-mass moving to the Northeast.

Figure 3 also demonstrates how the target movement affects the search plans. The target center-of-mass is moving to the Northeast and as agents encounter the currents in the North of the map, they immediately move East. Thus, the search plan skews North East.

One of the important features of Pathfinder is attaining searchers' safety since the optimization model can separate searchers by imposing constraints on the search trajectories. For example, the search plan visualized in figure 4, we use two helicopters based near each other for a search. Pathfinder found an optimal search plan below for which the helicopters never crossed paths.

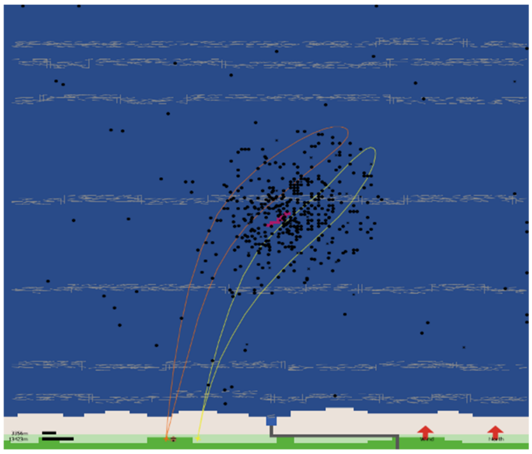

Figure 4: Two helicopters (yellow/orange) searching the domain. Note that the helicopters never crossed paths.

Another benefit of employing a computational methodology is that it can perform simulations on new SAR assets and methods. For example, Pathfinder can determine what would be better: a helicopter that is 10% faster or a helicopter that has 10% better sensors? We ran a few experiments with the prototype using the same initial search plans as figure 3 without the boat and buoy. Pathfinder could show, for example, that a 10% increase in speed gives us a search plan with a POD of 5.6% and a helicopter that is 10% better at detecting targets gives us a search plan with a POD of 6.2%. Thus, in this search scenario, a helicopter with 10% better sensors is more beneficial than those that are 10% faster. Thus, Pathfinder can find what assets are the most effective and consider the costs of using them. These are important questions to address (Biesecker, 2021) This methodology could eliminate a lot of the guesswork from appropriations and training.

5 CONCLUDING REMARKS

The obtained results have demonstrated that a methodology based on an ABM and optimization model is promising. This methodology can optimize teams of mobile and stationary searchers. The natural application of Pathfinder would be in assisting maritime searches. Pathfinder has the potential to improve the capabilities and functionality of current methodologies and could also be used for land searches. Using Pathfinder could advance both SAR and Anti-Submarine Warfare (ASW) operations. Even though the described results are related to the maritime SAR operations, we believe only a few modifications are sufficient for Pathfinder to be applied to ASW operations and land SAR operations.

For ASW planning, it may be necessary to add constraints so that the searcher could approach targets only from a certain direction, for example, from a blind spot behind submarines where their propellers are. Such constraints could be implemented.

Combining an ABM and optimization model to find optimal search plans in a maritime domain has achieved several goals. The ABM can model target behavior and its effects on target movement. This is an improvement over current methods that only model target movement based on environmental factors. Then heterogeneous teams of searchers can be seamlessly optimized. The optimization model also gives us the flexibility to change penalties and searcher constraints based on naval aviator and SAR personal input. Finally, using the proposed methodology allows us to consider past search plans (successful and failed) and compare them to optimal search plans to refine Pathfinder's models. Thus, we believe Pathfinder has potential to enhance current search methodologies.

6 FUTURE RESEARCH

There are several research directions that can refine and improve the Pathfinder methodology.

To create quality search plans Pathfinder relies on accurate estimation of the parameters $\alpha(s_k, \tau(z_j^k), s_g, v)$ and $n(s_k, \tau(z_j^k), s_g, v)$ since they can influence accuracy of the search. Currently, there is not enough published data to derive these values for all target types, searcher types, terrain types, and visual ranges. In the future, we would like to gather these data points and then derive the values of α and n. One way of making the data collection less expensive is to use Virtual Reality (VR) to gather the data points. The cost of using VR to simulate a helicopter searching for a boat is significantly cheaper than renting a helicopter and boat to do experiments.

Likewise, more research is needed to collect data and perform analysis for the ABM. In the current state, the ABM needed several estimations for parameters. More research is needed to analyse these parameters to turn the ABM. In addition, we need more behavioural data, such as how often people in boats without power deploy their anchor or how often a missing kayaker will beach their kayak to conserve energy. This data needs to be collected and analyzed to fine tune the ABM. Machine learning techniques could be used to analyse this data and discover how targets make decisions.

Another important research question is, how many agents are sufficient for an accurate target trajectory description? The answer may depend on computing resources. In the future, employing parallel computing methods may change the dynamics of answering this question. With them, tens of thousands of agents over a multi-hour search operation could be modeled in seconds. But even in that case, Pathfinder's users or search managers may need some guidance. This line of research will continue as Pathfinder develops.

There is room for further improvement and fine-tuning of the optimization model. This includes refining the penalty weights and possibly the addition of more constraints and penalties. Collaboration with practitioners such as naval aviators and SAR personnel can help. We hope that such collaboration has a significant potential to produce search paths that are easy to implement and navigate.

Finally, we plan to add to Pathfinder's new searcher and target types that current methodologies cannot handle. That includes searchers that can transport and deploy other searchers such as, for example, USCG cutters that can transport helicopters. Another possibility is to model active targets that may change behavior depending on the searcher's movement, such as, for example, a submarine being searched by another submarine.

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REFERENCES

Abi-Seid, I., Morin, M., & Nilo, O. (2019). Decision Support for Planning Maritime Search and Rescue Operations in Canada. *PROCEEDINGS OF THE 21ST INTERNATIONAL CONFERENCE ON ENTERPRISE INFORMATION SYSTEMS*, 328-339.

Adlerstein, D. (2019, May). *Fisherman rescued, clinging to buoy*. Retrieved from Apalachicola Times: https://www.apalachtimes.com/news/20190525/fisherman-rescued-clinging-to-buoy

Ai, B., Li, B., Gao, S., Xu, J., & Shang, H. (2019). An Intelligent Decision Algorithm for the Generation of Maritime Search and Rescue Emergency Response Plans. *IEEE*, 155835-155850.

AMPL Optimization inc. (2021). *MINOS for AMPL*. Retrieved from AMPL: https://ampl.com/products/solvers/solvers-we-sell/minos/

Biesecker, C. (2021, March 2). *House Panel Wants Updated Coast Guard Fleet Analysis To Factor In New Demands.* Retrieved from Defense Daily: www.defensedaily.com/house-panel-wants-updated-coast-guard-fleet-analysis-factor-new-demands/congress/

Dabrowski, J. J., & Villiers, J. P. (2015). Maritime piracy situation modelling with dynamic Bayesian networks. *Information Fusion*, 116-130.

Deraeve, J., Anderson, M., & Low, M. (2010). Maritime counte-piracy study using agent-based simulation. *2012 Spring Simulation Multiconference (SpringSim '10)*, (pp. 1-8).

Ding, H., & Castanon, D. (2018). *Moving Object Search with Multiple Agents.* Miami FL: IEEE.

Eagle, J. (1984). *The Optimal Search for a Moving Target When the Search Path is Constrained.* JSTOR.

El-Hadidy, M., & Alfreedi, A. (2020). Minimizing the expected search time of finding the hidden object by maximizing the discount effort reward search. *JOURNAL OF TAIBAH UNIVERSITY FOR SCIENCE*, 479-487.

Frost, J., & Stone, L. (2001, September). *Review of Search Theory: Advances and Applications to Search and Rescue Decision Support.* Retrieved from Defense Technical Information Center: https://apps.dtic.mil/dtic/tr/fulltext/u2/a397065.pdf

Harris, S., Dixon, D., Dunn D.L., & Romich, A. (2013). Simulation modeling for maritime port security. *Journal of Defense Modeling and Simulation*, 193-201.

Iida, K. (1993). Inverse Nth power detection law for washburn's lateral range curve. *Journal of the Operations Research Society of Japan* , 90-101.

Koopman, B. (1946 (declassified in 1958)). *Search and Screening.*

Kratzke, T., Stone, L., & Frost, J. (2010). Search and Rescue Optimal Planning System. *2010 13th International Conference on Information Fusion.* Edinburgh, UK .

Lin, L., & Goodrich, M. A. (2010). A Bayesian approach to modeling lost person behaviors based on terrain features in Wilderness Search and Rescue. *Computational and Mathematical Organization Theory*, 300-323.

Marchione, E., Johnson, S. D., & Wilson, A. (2014). Modelling Maritime Piracy: A Spatial Approach. *Journal of Artificial Societies and Social Simulation*.

Mohibullah, W., & Julie, S. J. (2013). Developing an Agent Model of a Missing Person in the Wilderness. *2013 IEEE International Conference on Systems, Man, and Cybernetics.* Manchester, UK: IEEE Xplore.

Motor Lifeboat (MLB). (n.d.). Retrieved from Textronsystems.com: https://www.textronsystems.com/products/motor-lifeboat-mlb

Murtagh, B., & Saunders, M. (1978). Large-scale linearly constrained optimization. *Mathematical Programming*, 14: 41–72.

N. C. Edwards, J. T. (1981). *Factors Affecting Coast Guard SAR Unit Visual Detection Performance.* Croton, CT 063: USCG R&D Center.

Onggo, B. S., & Karatas, M. (2016). Test-driven simulation modelling: A case study using agent-based maritime search-operation simulation. *European Journal of Operational Research*, 517-531.

Pike, J. (n.d.). *HH-60J Jayhawk Specification*. Retrieved from GlobalSecurity.org.

Przemieniecki, J. S. (2000). Mathematical Methods in Defense Analysis. In J. S. Przemieniecki, *Mathematical Methods in Defense Analysis* (pp. 278-279). Washington DC: AIAA.

Richardson, H. R., & Discenza, J. (1980). The United States Coast Guard Computer-Assisted Search Planning System (CASP). *Naval Research Logistics Quarterly, Vol. 27*, 141-157.

Sato, H., & Royset, J. O. (2010). Path Optimization for the Resource-Constrained Searcher. *Navel Research Logistics* , 422-440.

Simon, H. (1982). Models of Bounded Tationality . Cambridge MA: MIT Press.

Steele, M. J. (2004). *Agent-Based Simulation of Unmanned Surface Vehicles: A Force in the Fleet.* Monterey CA: Naval Postgraduate School.

Stone, L. (2011). In Search of Air France Flight 447. *Informs ORMS-Today*.

Stone, L., Royset, J., & Washburn, A. (2016). *Optimal Search for Moving Targets.* Springer.

Sullivan, P. J. (2016). *Evaluating the Effectiveness of Waterside Security Alternatives for Force Protection of Navy Ships and Installations Using X3D Graphics and Agent-Based Simulation.* MONTEREY CA: NAVAL POSTGRADUATE SCHOOL.

USCG. (2013). *USGS Addendum to the US National Search and Rescue Supplement (NSS).* Washington DC: USCG.

Walton, D., Paulo, E., McCarthy, C., & Vaidyanathn, R. (2005). Modeling force response to small boat attack against high value commercial ships. *Proceedings of the Winter Simulation Conference* . Orlando FL USA: IEEE.

Wilensky, U. (1999). *Netlogo*. Retrieved from NorthWestern: https://ccl.northwestern.edu/netlogo/