

# T-Balance: A Unified Mechanism for Taxi Scheduling in a City-scale Ride-sharing Service

Jiyao Li and Vicki H. Allan

*Department of Computer Science, Utah State University, Logan, Utah, U.S.A.*

**Keywords:** Ridesharing Service, Task Assignment, Vehicle Repositioning, Q-learning.

**Abstract:** In this paper, we propose a unified mechanism known as T-Balance for scheduling taxis across a city. Balancing the supplies and demands in a city scale is a challenging problem in the field of the ride-sharing service. To tackle the problem, we design a unified mechanism considering two important processes in ride-sharing service: ride-matching and vacant taxi repositioning. For rider-matching, the Scoring Ride-matching with Lottery Selection (SRLS) is proposed. With the help of Lottery Selection (LS) and smoothed popularity score, the Scoring Ride-matching with Lottery Selection (SRLS) can balance supplies and demands well, both in the local neighborhood areas and hot places across the city. In terms of vacant taxi repositioning, we propose Q-learning Idle Movement (QIM) to direct vacant taxis to the most needed places in the city, adapting to dynamic change environments. The experimental results verify that the unified mechanism is effective and flexible.

## 1 INTRODUCTION

Thanks to the rapid development of wireless networks and the prevalence of portable smart devices, ride-sharing services have become an important part of our daily life. Ride-sharing services pool passengers with similar itineraries and time schedules together in a single taxi. Such services may provide positive impacts on society and the environment by reducing traffic congestion, emission of carbon dioxide, and taxi fare. A recent global market report found that there are a rapidly growing number of passengers participating in ride-sharing services such as Uber, Lyft and Didichuxing. From the Ride-Sharing Industry Statistics (Stasha, 2021), about one fourth of the U.S. population uses ride-sharing services and there are 3.8 million drivers worldwide working for Uber. A recent market report (Curley, 2019) also showed that global ride-sharing service is valued at \$61.3 billion and expected to reach \$218 billion by 2025. To increase future market shares, ride-sharing service corporations are willing to spend large sums of money optimizing service operations such as reducing travel cost, serving more riders with fewer taxis, and improving passengers' satisfaction.

If the distribution of taxis across the city could be coordinated for maximum efficiency, then the service rate of riders and taxi utilization could be improved, as well as reducing riders' response time (the time be-

tween requesting a ride and being picked up). Therefore, organizing taxis to meet demands across a city is a crucial problem in the ride-sharing service. However, balancing supply and demand is extremely challenging when there are a large amount of riders and taxis. Evaluating demand patterns in a city scale at a specific period is difficult, since the number of rider requests might fluctuate dramatically in as little as an hour. Also, some riders' destinations are in demand-sparse areas such that delivery will lead taxis away from demand-dense areas, leaving many passengers in busy areas unserved even though corporations employ a large number of taxis. In addition, each rider has a patience period, that is, they will cancel the request and change to other alternative services after their patience period has elapsed.

Many research papers that focus on how to schedule taxis in the ride-sharing industry have been published. However, most of the previous work has the following limitations: (i) Most existing studies did research on forecasting taxi demand patterns across a city. However, their works focused on predicting taxi demand patterns at a given timestamp rather than evaluating long-term demand (Zhang et al., 2017a) (Xu et al., 2018) (Liu et al., 2019); (ii) Many current works proposed various solutions of ride-matching in order to meet the balance between supply and demand, but they only consider balancing taxi distri-

bution in local areas rather than citywide (Banerjee et al., 2018) (Wang et al., 2018) (Xu et al., 2018) (Zhang et al., 2017b) (Li and Allan, 2019); (iii) Most research papers designed different repositioning strategies to guide vacant taxis to busy areas where drivers would have greater opportunities for picking up passengers, but such solutions often omitted balancing supply and demand in local areas, although taxis could be scheduled in a wider area (Wen et al., 2017) (Lin et al., 2018) (Qu et al., 2014) (Jha et al., 2018); (iv) In addition, several price mechanisms have been provided as incentives for drivers to move to needed places. However, experienced drivers typically are not tempted by such incentives. (Lu et al., 2018).

To address the challenges and the limitations of ride-sharing service, we propose a unified mechanism known as T-Balance. The main contributions of the current work are described as following:

- The Lottery Selection (LS) selects appropriate riders to be served such that taxis can be scheduled to needed places in the process of ride-matching.
- Since neighboring zones impact each other, we calculate a new popularity score to estimate long-term demand density for each zone of the city. Equipped with Lottery Selection (LS) and the new popularity score, a ride-matching mechanism named Scoring Ride-Matching with Lottery Selection (SRLS) balances supply and demand across the city.
- In addition, we utilize a Q-learning Idle Movement (QIM) mechanism to reposition vacant taxis citywide, increasing the taxi service's ability to meet demand in the busiest areas of a city.

The remainder of the paper is organized as follows: we discuss recent related works of ride-sharing services in section 2; in section 3, the business model of ride-sharing service is introduced; in section 4, we propose a novel unified mechanism known as T-Balance for a city-scale ride-sharing service; in section 5, we compare our method to other approaches with the city-scale real data set; section 6 concludes our contributions and achievements.

## 2 RELATED WORK

We organize the previous works in terms of demand prediction, ride-matching, repositioning, and price scheme.

Demand forecast plays an important role in current smart transportation systems. It can help ride-

sharing platforms be smarter in scheduling taxis across a city. (Zhang et al., 2017a) proposed a deep learning-based approach known as Deep Spatio-Temporal Residual Networks (ST-ResNet) to predict the demand of a city. (Xu et al., 2018) designed a sequence learning model based on Recurrent Neural Networks (RNN) to predict rider requests in different areas of a city. (Yao et al., 2018) constructed a novel Deep Multi-View Spatial-Temporal Network (DMVST-Net) framework to model both spatial and temporal relations. Taxi demand can then be predicted with some semantic information. (Liu et al., 2019) proposed a novel Contextualized Spatial-Temporal Network (CSTN) that would effectively capture the diverse contextual information in order to learn the demand patterns. Most of these works focus on predicting future demand at a given timestamp, but do not consider providing the ride-sharing platform with a long-term model.

Ride-matching is a core building block in any ride-sharing platform. (Banerjee et al., 2018) proposed a Scaled MaxWeight (SMW) approach to schedule vehicles for demands in local areas. This method utilizes a closed queueing network and dispatches taxis from the queue with the most supplies. SMW has proven effective, but the authors do not offer a concrete solution for how to estimate demand weight. (Wang et al., 2018) optimizes the ride-matching policy by deep reinforcement learning. A Deep Q-network (DQN) with an action search was proposed. In addition, to speeding up the learning process, a knowledge transfer method was also used. While this solution can adapt to changes in the environment, coordination among vehicle agents is ignored.

(Xu et al., 2018) applied a two-step approach to solve the order dispatch problem. The offline learning step summarizes demand and supply patterns through historical data, then driver-order pairs are created by a combinatorial optimizing algorithm. (Zhang et al., 2017b) proposed Stochastic Gradient Descent (SGD) forecasting the combined probability of each rider and driver pairs and the Hill Climbing algorithm was used to maximize the global success rate. The drawback of both methods is that passengers may be forced to wait longer for service, although the serving rate is increased. (Li and Allan, 2019) proposed a polar coordinates based ride-matching method, but this method failed to improve performance in demand sparse places.

Repositioning techniques can direct vacant taxis from low demand areas to busier places. (Wen et al., 2017) proposed a model-free reinforcement learning approach for dispatching empty taxis across a city to

keep balance between supply and demand. The fleet size can be reduced by 14% with slightly increased extra taxi travel distance, but this method might lead to excess taxis in busy areas. (Lin et al., 2018) modeled the movement of vacant taxis as a multi-agent problem and proposed an approach known as contextual multi-agent actor-critic, which is a novel version of Multiagent Reinforcement Learning (MARL). (Qu et al., 2014) aimed to maximize the net profit of taxis. The authors created a graph representing road networks through historical data, then proposed a novel recursion approach to seek for the optimal route for vacant taxis. (Jha et al., 2018) tried to guide vacant taxis by the Driver Guidance System (DGS) depended on forecast data of road demand. The researchers did not show the performance of serving rate, although they argued that drivers' net profit could be maximized.

In addition, various price mechanisms have been proposed for ride-sharing services. A monetary constraint function was proposed by (Ma et al., 2014) to motivate passengers to participate in a ride-sharing service. (Kleiner et al., 2011) aimed to minimize the total travel distance of taxis and maximize the serving rate and the Sealed-Bid Second-Price Auction (SBSPA) was proposed to distribute taxi resources among passengers. (Asghari et al., 2016) allocate requesting customers through Sealed-Bid First-Price Auction (SBFPA) mechanism such that the profit of drivers could be maximized. A VCG (Wooldridge, 2009) auction approach was proposed to maximize the global utility function by (Zheng et al., 2019), and individual rationality and truthfulness can be ensured. (Chen, 2016) tried to maintain balance between supply and demand by utilizing a surge price, but failed to guarantee the efficiency of dispatching taxis across a city.

### 3 BACKGROUND

In the business model of ride-sharing service, a customer can send the service provider a request when they need a taxi. The request contains information like time, source, and destination. If the customer cannot get service within a certain period, he or she can give up the request and turn to an alternative service without any penalty.

On the other hand, each taxi sends the provider status message periodically. The status message contains information like: taxi identification, time, current location, and availability. With the information of customer request and taxis status, the provider can dispatch appropriate taxis to serve customers (Ride-

matching) and direct the vacant taxis to needed places (Idle Movement)

## 4 METHOD

To provide passengers with better service and increase drivers' earning, we propose an enhanced version of the hybrid solution called T-Balance based on (Li and Allan, 2020): The city consists of different zones. With the help of the Scoring Ride-Matching with Lottery Selection (SRLS) algorithm, rider groups whose destinations are located in busy areas will have more chances to be served, and available taxis from less popular zones will have more opportunities to be dispatched, thus simultaneously balancing supply and demand in both a local areas. In addition, considering the impact of the high ride request zones on other zones, a Q-learning Idle Movement (QIM) approach is applied to guide vacant taxis to busier zones such that taxi distribution could be balanced across the citywide.

### 4.1 Scoring Ride-matching with Lottery Selection (SRLS)

**Definition 1: Popularity Score.** The popularity score  $P(t, z)$  is defined as the summation of predicted rider demand from the specific time  $t$  to the future at a specific zone  $z$ , which is described as Eq.(1), where  $D$  is a demand predictor that could be implemented by (Yao et al., 2018) and evaluates the scale of demand at specific time and zone,  $\gamma_p$  being a decay and  $\gamma_p \in [0, 1]$ .

$$P(t, z) = \mathbb{E} \left( \sum_{k=0}^{+\infty} \gamma_p^k \cdot D(t+k, z) \right) \quad (1)$$

The popularity score estimates the degree of demand popularity at a specific zone over time. The impact of the future demand is determined by the scale of  $\gamma_p$ . If the value of  $\gamma_p$  approaches 1, then the future demand greatly impacts the current popularity score; the future demand has less impact on the current popularity score when the  $\gamma_p$  is approaches 0.

In (Li and Allan, 2020), riders were randomly selected from the rider list at each zone and then assigned to the appropriate taxi through Adjacent Matching. However, the approach failed to dispatch taxis to most needed places in the process of ride-matching since riders with different destinations have the same chance to be served. In other word, taxis are distributed to various places of the city. Therefore, riders in busy areas would lose opportunities to be served even though the quantity of taxis is enough.

In this work, we propose the Lottery Selection (LS) to select appropriate rider requests for drivers in the process of ride-matching, such that taxis would have more chances to be dispatched to the busier zones that need more supplies than others. The idea of the Lottery Selection is described as following: each rider is given a certain amount of lottery tickets  $L_r$  according to the popularity score  $P$  of their destination, and a lottery number  $LOTTERY\_NUM$  is drawn then the appropriate rider to be served is the rider who owns the  $LOTTERY\_NUM$  in his tickets. The solution is given in Algorithm.1, where  $M$  is a multiplier factor. Since Lottery Selection (LS) has stochastic property, taxis have more chances to be delivered to hot places in the process of driver-rider matching, and riders whose destination is less popular can still be served.

---

Algorithm 1: Lottery Selection (LS).

---

**Input** : Rider list  $l_z$  at zone  $z$  at timestamp  $t$   
**Output**: The the appropriate rider to be served

- 1 **for** each rider  $r$  in  $l_z$  **do**
- 2     Get rider  $r$ 's destination zone  $z_d$  and calculate the traveling time  $T_{trip}$ .
- 3      $L_r = M * P(t + T_{trip}, z_d)$
- 4 **end**
- 5  $LOTTERY\_NUM = rand\_range(1, sum(L_r))$
- 6  $sum = 0$
- 7 **for** each rider  $r$  in  $l_z$  **do**
- 8      $sum = sum + L_r$
- 9     **if**  $sum > LOTTERY\_NUM$  **then**
- 10         **return**  $r$
- 11     **end**
- 12 **end**

---

**Definition 2: Balanced Factor.** Given specific timestamp  $t$  and zone  $z$ , the balanced factor  $B$  is the proportion between the number of available taxis  $A(t, z)$  and the smoothed popularity score  $smooth(P(t, z))$ . It can be defined by the Eq.(2):

$$B(t, z) = \frac{A(t, z)}{1 + smooth(P(t, z))} \quad (2)$$

In (Li and Allan, 2020), the popularity score of each zone was estimated individually in the Supply-Demand Ratio, ignoring the impact of connectivity among zones. In this work, we smooth the popularity score  $smooth(P(t, z))$  considering the demand impact from its neighbor zones, the formula is shown in Eq.(3), where  $O(z)$  is a collection of neighbor zones of  $z$ , and  $\beta \in [0, 1]$  is the impact factor of neighboring zones.

$$smooth(P(t, z)) = P(t, z) + \beta \sum_{i \in O(z)} P(t, i) \quad (3)$$

Based on the Lottery Selection and Balanced Factor, a new ride-matching algorithm named Scoring Ride-matching with Lottery Selection (SRLS) is proposed. The main idea is to dispatch taxis from the least busy zone (highest Balanced Factor) in the neighborhood to serve rider requests, while taxis from the busier zones (lower Balanced Factor) are reserved for future use. The algorithm can be described as follows: the appropriate rider  $r$  is selected by Lottery Selection (LS) and the zone where the rider  $r$  is located can be retrieved by  $z(r)$ , we iterate the neighbor zones  $O(z(r))$  to find the zone with highest Balanced Factor and randomly select taxi  $v$  from that zone. As described by Algorithm 2:

---

Algorithm 2: Scoring Ride-matching with Lottery Selection (SRLS).

---

**Input** : Rider list  $l_z$  at zone  $z$  at timestamp  $t$   
**Output**: The selected taxi  $v$

- 1 Select the appropriate rider  $r$  through Lottery Selection.
- 2  $MAX\_BF = B(z(r), t)$
- 3  $Z = z(r)$
- 4 **for** each  $z$  in  $O(z(r))$  **do**
- 5     Estimate Zone  $z$ 's Balanced Factor  $B(t, z)$  at timestamp  $t$ .
- 6     **if**  $B(t, z) > MAX\_BF$  **then**
- 7          $MAX\_BF = B(t, z)$
- 8          $Z = z$
- 9     **end**
- 10 **end**
- 11 Randomly pick taxi  $v$  from zone  $Z$ .
- 12 **return**  $v$

---

## 4.2 Q-Learning Idle Movement (QIM)

The Scoring Ride-Matching with Lottery Selection (SRLS) can send taxis to needed places and effectively balance taxis distribution locally in the process of ride-matching. However, there are still a certain number of taxis that will fail to be dispatched during ride-matching. Such taxis wander across the city to seek riders aimlessly, leading to the problem that rider requests cannot be served on time and drivers' traveling cost would be increased. In previous work, a Greedy Idle Movement (GIM) was proposed to tackle the issue. However, this approach cannot adapt to the dynamic environment. For this reason, a flexible Idle Movement Strategy based on Q-learning (Watkins and Dayan, 1992) named Q-Learning Idle Movement (QIM) is proposed. Each vacant taxis can learn the movement strategy by itself according to the current environment.

Considering each taxi as an autonomous agent, we want to let vacant taxis decide where to go. A Q-learning approach is applied to train taxi agents such that they are able to make a reasonable decision in idle mode. The movement of a vacant taxi is model as a Markov Decision Process (MDP) (Puterman, 2014). The components of the MDP are defined as follows:

**State:** The state  $s$  of a taxi is defined as a three tuple  $(t, z, \delta)$ , where  $t \in T$  is a timestamp,  $z \in Z$  is a zone index, and  $\delta$  indicates the maximum Balanced Factor difference between  $z$  and its adjacent zones, which can be formulated as Eq.(4), where  $O(z)$  are adjacency zones of zone  $z$ , and we round up the value to  $b$  decimals.

$$\delta = \text{round}(B(t, z) - \min_{z' \in O(z)} B(t, z'), b) \quad (4)$$

We also constrain the value of  $\delta$  to the range  $[\delta_{min}, \delta_{max}]$ . If the value of  $\delta$  is inside the range, then it is unchanged. If  $\delta < \delta_{min}$ , then  $\delta = \delta_{min}$ , and  $\delta = \delta_{max}$  if  $\delta > \delta_{min}$ .

**Action:** A taxi driver may implement two types of actions. One action is staying at the current zone  $z$ ; the other is moving to an adjacent zone with the lowest updated Balanced Factor  $\text{argmin}_{z' \in O(z)} B'(t, z')$ .

**Reward:** At each timestamp  $t$ , if taxi agents can serve rider requests, then they will receive a positive reward  $R_c$ , where  $R_c$  is a positive constant. If taxi agents stays at the current zone without receiving rider requests, they will receive a penalty  $-R_c$ . If the taxi moves to its adjacent zone without receiving rider requests, they will receive a penalty  $-2R_c$ , which considers travel cost.

**Discount Factor:** The discount factor  $\gamma_q \in [0, 1]$ .

At each simulated cycle, the Q-learning is applied to learn the action value  $Q(s, a)$  indicating the sum of reward from now to future that an agent may achieve given a specific state  $s$  and action  $a$ . At start, the values in the Q-table are initialized as 0. At each timestamp, a taxi selects an action based on  $Q(s, a)$ , it would get the reward and transfer to another state  $s'$ , then  $Q(s, a)$  can be updated. The details are described in Algorithm 3, where the  $\alpha$  is the learning rate.

When a taxi is in idle mode, it will select an action according to  $\text{argmax}_{a \in A} q(s, a)$ . If  $a = 0$ , it would not move and stop at the current place, otherwise, it will move to the adjacency zone with minimized  $\phi'(t, z)$ .

---

Algorithm 3: Q-Learning Idle Movement (QIM).

---

```

1 Select action  $a$  from current state  $s$  using
   $\epsilon$  – greedy policy derive from  $Q$ .
2 Take action  $a$ , then the state is transfer from  $s$ 
  to  $s'$ .
3 if receive rider request then
4   |  $R = R_c$ 
5 else
6   | if Action is move then
7     |  $R = -2R_c$ 
8   | else
9     |  $R = -R_c$ 
10  | end
11 end
12  $Q(s, a) =$ 
    $Q(s, a) + \alpha_q (R + \gamma_q \max_{a'} Q(s', a') - Q(s, a))$ 
13  $s' = s$ 
14 return  $a$ 

```

---

## 5 EXPERIMENT

The experiment is performed using the taxi data records of the city of Chicago (cit, 2018). Every record contains the timestamp, the start zone ID, the end zone ID, and payment. In our setting, the interval of one simulated cycle  $\delta_t$  is 3 minutes, the traveling cost of a taxi to adjacent area is 1, while it is 0.5 if it just drives around within its current area, and the money spent per unit of travel cost is \$2.5. The patience period is 20 minutes (in other words, after 20 minutes waiting in our simulation, customers will change to another service). To verify the effectiveness of the T-Balance, 43,764 rider requests during busy hours (from 11:00 to 23:59) of a weekday is used. SMW in (Banerjee et al., 2018) and the Hybrid Solution in (Li and Allan, 2020) are implemented as a baseline. We suppose that all the three methods are equipped with the cluster algorithm in (Li and Allan, 2020). Furthermore, we also study the impact of Lottery Selection on the quantity of unserved riders and the average online running time of each algorithm.

Figure 1 and Figure 2 present the performance evaluation on the service rate and taxi utilization. The service rate reflects what percentage of passengers are served by the fleet of taxis, and taxi utilization indicates the percentage of time when taxis are used to serve passengers rather than wandering to seek passengers. From the plots, we see that increasing taxi fleet volume can serve more passengers, but on the other hand, decreases the utilization of taxis. (in other words, an individual driver would have less chance to serve riders and earn less). We also discovered that

T-Balance is better than the two other methods. The main reason is that the Lottery Selection (LS) helps the service rate due to dispatching taxis to busy zones for future use in ride-matching process, and the Q-learning Idle Movement (QIM) improves both metrics by guiding vacant taxis to the most needed places to serve demands.

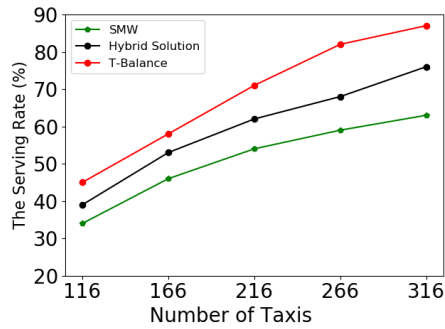


Figure 1: The Service Rate.

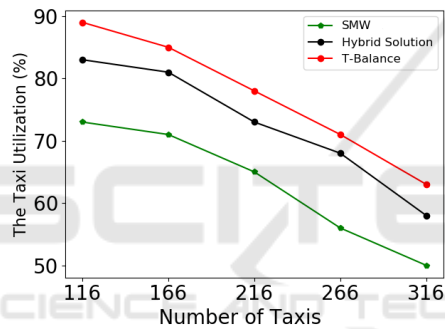


Figure 2: The Taxi Utilization Rate.

Figure 3 and Figure 4 reflect the economic side of the ride-sharing service. Figure 3 shows how much profit a taxi driver can earn on each solution while Figure 4 plots the total income of the ride-sharing service provider. Both Figures assume that all drivers work continuously for 13 hours of that day. We can see that a large fleet can help the provider earn more money, but thin down the profit of each individual driver. This is mainly because a larger number of taxis can serve more passengers such that provider's revenue can be increased, but shrink the taxi utilization, causing drivers to have more idle hours and have less chances to earn money from passengers. Also, the T-Balance work much better than other two methods when there are fewer taxis, this is because the Lottery Selection (LS) and Q-learning Idle Movement (QIM) can schedule taxis to needed places precisely under the circumstances that the supplied resources are limited.

Figure 5 reflects the expected response time of each approach along with various number of taxis. The response time is the time interval between the

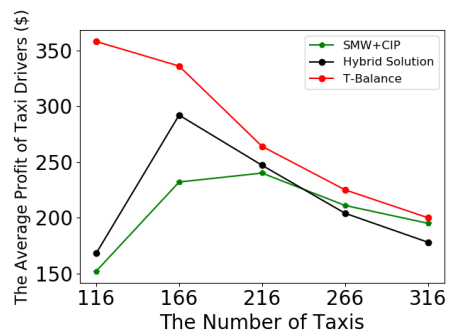


Figure 3: The Average Profit of Taxi Drivers.

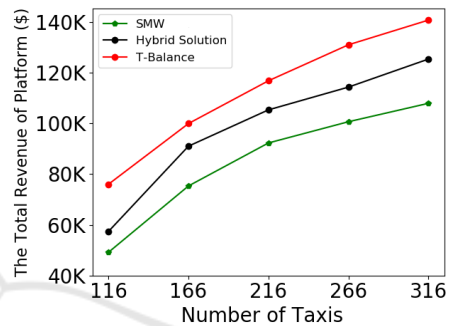


Figure 4: The Total Revenue of Provider.

timestamp when a rider request is send out and the timestamp when the customer can be picked up by a taxi. We may observe that larger number of taxis can help to shrink the response time in all three approaches. It seems that all three methods can schedule taxi across the city well as long as supplied resource is enough. We also observe that the response time of T-Balance is the least. This is mainly because the Lottery Selection (LS) and the Q-learning Idle Movement (QIM) can schedule taxis to places where there would be large amount of demands in current or future such that riders can be served in a short time.

We also study how the Lottery Selection (LS) affects the number of unserved riders in several hypothetical taxi services employing a varying number of cars. As shown in the Figure 6, the efficiency of the Lottery Selection (LS) seemed significant when dealing with a small fleet of taxis, but as the number of cars increased, the effort from the LS is not obvious. The main reason is that the LS can still arrange supplied resources for future use well especially under the circumstance where resources are limited.

In addition, we estimate the online running time of the Scoring Ride-matching with Lottery Selection (SRLS) and Q-learning Idle Movement (QIM), and compare them to the ARDL and GIM in (Li and Allan, 2020) separately, as shown in Table 1. The online running time indicates how much time we need to run

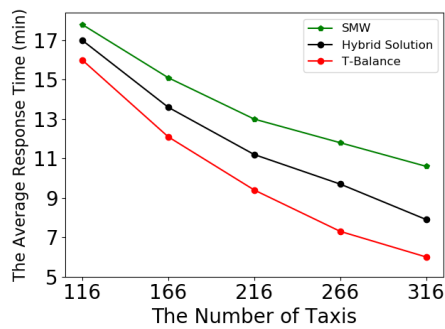


Figure 5: The Average Response Time of Riders.

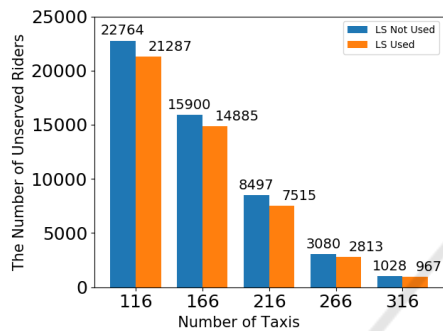


Figure 6: Unserved Riders with or without Lottery Selection.

an algorithm once. Although the methods in the T-Balance cost more several millisecond, they achieve much better performance on effectiveness as above shown.

Table 1: The Average Online Running Time of Each Algorithm(sec).

	T-Balance	Hybrid Solution
	Method/Time	Method/Time
Ride-Matching	SRLS/0.039	ARDL/0.034
Idle Movement	QIM/0.065	GIM/0.050

From the above experiments and comparisons, the T-Balance works better than the two other methods across various performance metrics. This indicates that the Scoring Ride-matching with Lottery Selection (SRLS) can balance the supplies and demands well. The Q-learning Idle Movement (QIM) is effective in directing vacant taxis to the most needed places adapting to the change of the dynamic environment. Therefore, T-Balance is more flexible and adjustable in various scenarios such that taxis can be frequently sent to the most needed places without wasting too much traveling cost.

## 6 CONCLUSION

This work has four contributions. First, we design a Lottery Selection (LS) algorithm which delivers taxis to areas of high need by selecting high priority riders, while low priority riders can still have chances to be served. Second, using Lottery Selection (LS) and the smoothed popularity score computed from among neighbor zones, the Scoring Ride-matching with Lottery Selection (SRLS) keeps balance between supplies and demands in each local neighborhood and hot places. Third, the Q-Learning Idle Movement (QIM) directs vacant taxis to the most needed places adapting to the change of the dynamic environment. Fourth, comparing our current work to state-of-the-art methods, the results verify the effectiveness and flexibility of the T-balance.

## REFERENCES

- (2018). *Chicago data portal: Taxi Trips*. City of Chicago. <https://data.cityofchicago.org/Transportation/Taxi-Trips/wrvz-psew>.
- Asghari, M., Deng, D., Shahabi, C., Demiryurek, U., and Li, Y. (2016). Price-aware real-time ride-sharing at scale: an auction-based approach. In *Proceedings of the 24th ACM SIGSPATIAL international conference on advances in geographic information systems*, pages 1–10.
- Banerjee, S., Kanoria, Y., and Qian, P. (2018). State dependent control of closed queueing networks. *ACM SIGMETRICS Performance Evaluation Review*, 46(1):2–4.
- Chen, M. K. (2016). Dynamic pricing in a labor market: Surge pricing and flexible work on the uber platform. In *Proceedings of the 2016 ACM Conference on Economics and Computation*, pages 455–455.
- Curley, R. (2019). Global ride sharing industry valued at more than \$61 billion. Technical report.
- Jha, S. S., Cheng, S.-F., Lowalekar, M., Wong, N., Rajendram, R., Tran, T. K., Varakantham, P., Trong, N. T., and Rahman, F. B. A. (2018). Upping the game of taxi driving in the age of uber. In *Thirty-Second AAAI Conference on Artificial Intelligence*.
- Kleiner, A., Nebel, B., and Ziparo, V. A. (2011). A mechanism for dynamic ride sharing based on parallel auctions. In *IJCAI*, volume 11, pages 266–272.
- Li, J. and Allan, V. H. (2019). A ride-matching strategy for large scale dynamic ridesharing services based on polar coordinates. In *2019 IEEE International Conference on Smart Computing (SMARTCOMP)*, pages 449–453. IEEE.
- Li, J. and Allan, V. H. (2020). Balancing taxi distribution in a city-scale dynamic ridesharing service: A hybrid solution based on demand learning. In *2020 IEEE International Smart Cities Conference (ISC2)*, pages 1–8.

- Lin, K., Zhao, R., Xu, Z., and Zhou, J. (2018). Efficient large-scale fleet management via multi-agent deep reinforcement learning. In *Proceedings of the 24th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining*, pages 1774–1783.
- Liu, L., Qiu, Z., Li, G., Wang, Q., Ouyang, W., and Lin, L. (2019). Contextualized spatial-temporal network for taxi origin-destination demand prediction. *IEEE Transactions on Intelligent Transportation Systems*, 20(10):3875–3887.
- Lu, A., Frazier, P., and Kislev, O. (2018). Surge pricing moves uber’s driver partners. Available at SSRN 3180246.
- Ma, S., Zheng, Y., and Wolfson, O. (2014). Real-time city-scale taxi ridesharing. *IEEE Transactions on Knowledge and Data Engineering*, 27(7):1782–1795.
- Puterman, M. L. (2014). *Markov decision processes: discrete stochastic dynamic programming*. John Wiley & Sons.
- Qu, M., Zhu, H., Liu, J., Liu, G., and Xiong, H. (2014). A cost-effective recommender system for taxi drivers. In *Proceedings of the 20th ACM SIGKDD international conference on Knowledge discovery and data mining*, pages 45–54.
- Stasha, S. (2021). *Ride-Sharing Industry Statistics to get you going in 2021*. <https://policyadvice.net/insurance/insights/ride-sharing-industry-statistics/>.
- Wang, Z., Qin, Z., Tang, X., Ye, J., and Zhu, H. (2018). Deep reinforcement learning with knowledge transfer for online rides order dispatching. In *2018 IEEE International Conference on Data Mining (ICDM)*, pages 617–626. IEEE.
- Watkins, C. J. and Dayan, P. (1992). Q-learning. *Machine learning*, 8(3-4):279–292.
- Wen, J., Zhao, J., and Jaillet, P. (2017). Rebalancing shared mobility-on-demand systems: A reinforcement learning approach. In *2017 IEEE 20th International Conference on Intelligent Transportation Systems (ITSC)*, pages 220–225. IEEE.
- Wooldridge, M. (2009). *An introduction to multiagent systems*. John Wiley & Sons.
- Xu, J., Rahmatizadeh, R., Bölöni, L., and Turgut, D. (2018). Real-time prediction of taxi demand using recurrent neural networks. *IEEE Transactions on Intelligent Transportation Systems*, 19(8):2572–2581.
- Xu, Z., Li, Z., Guan, Q., Zhang, D., Li, Q., Nan, J., Liu, C., Bian, W., and Ye, J. (2018). Large-scale order dispatch in on-demand ride-hailing platforms: A learning and planning approach. In *Proceedings of the 24th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining*, pages 905–913.
- Yao, H., Wu, F., Ke, J., Tang, X., Jia, Y., Lu, S., Gong, P., Ye, J., and Li, Z. (2018). Deep multi-view spatial-temporal network for taxi demand prediction. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 32.
- Zhang, J., Zheng, Y., and Qi, D. (2017a). Deep spatio-temporal residual networks for citywide crowd flows prediction. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 31.
- Zhang, L., Hu, T., Min, Y., Wu, G., Zhang, J., Feng, P., Gong, P., and Ye, J. (2017b). A taxi order dispatch model based on combinatorial optimization. In *Proceedings of the 23rd ACM SIGKDD international conference on knowledge discovery and data mining*, pages 2151–2159.
- Zheng, L., Cheng, P., and Chen, L. (2019). Auction-based order dispatch and pricing in ridesharing. In *2019 IEEE 35th International Conference on Data Engineering (ICDE)*, pages 1034–1045. IEEE.