Global Spare Parts Exploitation Costs Optymization and Its Reduction to Rectangular Knapsack Problem

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Abstract: Maintenance costs represent a significant portion of the budget in large enterprises. Unplanned downtime and breakdowns have a very negative impact on the financial result. Minimizing their number is today a huge challenge for systems supporting production management and maintenance processes. Due to intensive research in the field of reliability today, we know a lot about the optimal use of a single spare part. Unfortunately, the analytical methods used for this purpose do not work well in the case of simultaneous analysis of the entire production line. The article proposes a combination of analytical methods and discrete optimization methods for global management of spare parts operating costs. The purpose of the presented algorithm is to support decision-making that minimize the global cost of maintaining reliability in the entire production company.

1 INTRODUCTION

Despite the installation of an increasing number of sensors and monitoring the degree of wear of many systems and their subassemblies, there is still a large number of elements which usability status is determined in a binary manner: functional / non-functional. An example is an oil hose for which the performance level on the fly cannot be determined. Perhaps in the future, advanced vision systems will be able to automatically detect microcracks on its surface and track wear levels. At present, however, the operation of such cases is purely reactive. The exchange is made at a strictly defined moment or the failure is removed when it occurs. From the point of view of maintenance people, this is a rather uncomfortable situation. However, for each such element on the production line, we can determine the optimum working time. Optimum time is understood here as the time that minimizes the costs of servicing. Intuition suggests that if the removal of a failure is much more expensive than a scheduled replacement, the standard working time of such an element should be shorter than in a situation where the costs of removing a failure and scheduled replacement are similar. However, the question is: when should the replacement be made so that the long-term costs are as low as possible? This problem will be discussed in this article. In the following parts, it will be shown how modern IT tools combined with the classical theory of reliability can support the decision-making process in a manufacturing company.

The Figure 1 shows the Weibull distribution with parameters of $\alpha = \beta = 5$, respectively. This distribution was chosen as an example to illustrate the reliability of aging components. In practice, for each element, the α and β parameters should be estimated based on historical data (which will be presented in the following sections) or information from the manufacturer. It can be calculated that the average product lifetime for a distribution with these parameters is about 4.6 time units (e.g. months).

The Figure 1 shows that the systematic replacement of such an element every one time unit practically eliminates the problem of failure. Therefore, the manufacturer may recommend that users regularly replace them at a specific time period. However, is this approach economically justified? Unfortunately, there is no clear answer to this problem. The solution depends on the cost of replacing this element during standard service activities, and the cost of replace-

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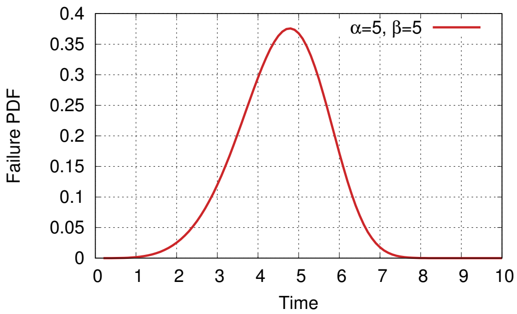


Figure 1: Probability density for the Weibull distribution with $\alpha = 5$ and $\beta = 5$.

ment in the event of a failure. It should be emphasized that the costs related to downtime should be added to the cost of the service replacement as well as to the cost of the replacement during a failure. This is a very important cost component that can cause the same element used at different points in the production line to be treated differently. In fact, any finite resource at the company's disposal that has any value can be considered as a cost. This can be time, money, energy, etc. In principle, cost should be understood to mean all the necessary resources that are consumed during replacement (both service replacement and failure replacement).

The problem of age replacement policy is the subject of many authors' publications. First (Barlow and Proschan, 1965) and (Barlow and Proschan, 1975) have analyzed the optimization of the expected operating costs and the impact of the replacement policy on these costs. Moreover, for several common failure distributions methods have been developed by (Glasser, 1967). On the other hand, (Fox, 1966) and (Ran and Rosenlund, 1976) consider the optimal replacement time, taking into account discounting - this issue is especially important in the case of rapidly changing prices. The publications (Scmeaffer, 1971), (Cléroux and Hanscom, 1974), (Cléroux et al., 1979) and (Subramanian and Wolff, 1976) present a generalized approach to the optimization of operating costs and preventive replacement. The confidence intervals with unknown failure distributions for the problem of minimizing the expected replacement costs were also quite intensively investigated, which can be found in the articles (Ingram and Scheaffer, 1976), (Frees and Ruppert, 1985), (Léger and Cléroux, 1992). Next, (Berg and Epstein, 1978) compares an age-related exchange policy with exchange policy at strictly defined periods. The articles (Christer, 1978) and (Ansell et al., 1984) analyze the asymptotic operating costs assuming the replacement policy according to the age of the element. A very extensive bibliography on the problem of optimization of maintenance costs was presented by (Nakagawa, 2003). In the review article (Zhao et al., 2017) an up-to-date summary of the state of knowledge in this field and perspectives for further research directions can be found. It should be emphasized that the problem of the replacement policy as a result of aging is also valid today, as evidenced by publications from recent years (Knopik and Migawa, 2017), (Fu et al., 2018), (Finkelstein et al., 2020), (Wang et al., 2021a), (Zhao et al., 2021). One problem in the practical use of the methods developed so far is focusing attention on a single element or spare part. In this article, a global approach will be proposed, the aim of which will be to optimize the expected operating costs of the entire production line. For this reason, under certain assumptions, analytical methods will be combined with discrete optimization methods reducing the problem to a two- or one-dimensional knapsack problem.

2 MINIMAL OPERATING COST FOR SINGLE ELEMENT

The problem of optimal selection of the replacement time will be presented in a more formal way. Let C_F be the cost of removing the failure of a given element, and C_M the cost of the scheduled service replacement. Moreover, let $a = C_F/C_M$. In addition, it is assumed that $C_F = C_M + \tau C_D$, where C_D is the downtime cost per unit of time and τ is the downtime required during a failure. It follows from the assumptions that the coefficient $a > 1$. It is then assumed that there will be regular maintenance replacements for this item at exactly T units of time since the previous replacement. If $F(t)$ is the failure probability at time t, and $G(t) = 1 - F(t)$ is the reliability at time t, then the expected cost of service activities for the given replacement time T is estimated as

$$C_T = C_F F(T) + C_M G(T). \tag{1}$$

In the formula above, the cost C_T is simply the weighted average cost of failure C_F that occurs with probability $F(T)$ and service cost C_M that occurs with probability $G(T) = 1 - F(T)$. If we consider that $C_F = a \cdot C_M$, we get the following equality

$$C_T = C_M\big(1 + (a-1)F(T)\big). \tag{2}$$

The calculated cost C_T is not enough. It does not yet take into account the frequency with which such changes will be made. There may be a situation in which the average cost of C_T is small, but the frequency with which it will be borne is so high that it is not an optimal solution. So let μ_T denote the mean time between replacements of a given element, both due to the failure and the planned service replacement. Then the value μ_T^{-1} determines the frequency of replacement of the element and allows to define the ε_T function which expresses the relative cost of maintaining the efficiency of the element per unit of time

$$\varepsilon_T = \frac{C_T}{C_M} \cdot \mu_T^{-1}. \tag{3}$$

Minimizing the ε_T function is our main task. Before we get down to that, it is necessary to define the formula for the expected replacement time μ_T. It is given by the following formula (Nakagawa, 2006)

$$\mu_T = \int_0^T t \, \mathrm{d}F(t) + TG(T) = \int_0^T G(t) \, \mathrm{d}t. \tag{4}$$

Thus, the ε_T function defining the relative cost of maintaining efficiency per unit of time is expressed as follows

$$\varepsilon_T = \frac{1 + (a-1)F(T)}{\int_0^T G(t) \, \mathrm{d}t}. \tag{5}$$

Nakagawa (Nakagawa, 2003) showed that T minimizing the value of ε_T meets one of the following conditions:

1. is finite and is given uniquely,
2. is infinite.

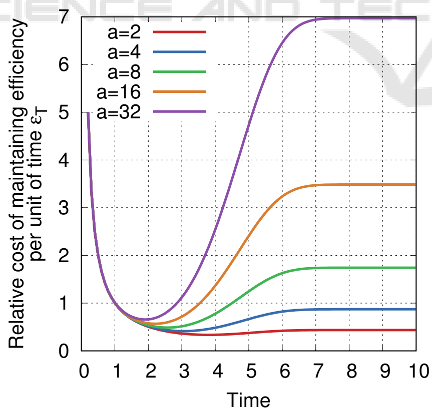


Figure 2: The functions of the relative cost of maintaining efficiency in the time unit ε_T for example values of the a parameter and the Weibull distribution with the parameters $\alpha = \beta = 5$.

It should be noted that in the absence of a finite value of T, all service replacements are unprofitable. In such cases, the only economically sensible strategy is to wait for a failure.

The introduced concepts will be illustrated on an exemplary Weibull distribution. The Figure 2 shows the graphs of the ε_T function for a few example values of a and the Weibull failure probability

$$F(t) = 1 - e^{-(t/\alpha)^{\beta}}, \tag{6}$$

with the following parameters: $\alpha = \beta = 5$. It can be seen from these graphs that the function of the relative cost of maintaining efficiency takes its minimum the sooner the higher the value of the a coefficient. This is in line with the intuition that the higher the cost of failure compared to the cost of service replacement, the shorter the time should be the component used. If the cost of removing the failure is twice as much as the cost of servicing, the element should be replaced after 3.8 units of time. When the cost of removing a failure exceeds four times the cost of regular service, the element should be replaced after 3.05 time units. A summary of all values for the above-mentioned graph is given in the Table 1.

Table 1: Replacement times that minimize the relative cost of maintaining the device in service for the specified values of a and the Weibull distribution of $\alpha = \beta = 5$.

a	α	β	Replacement time to minimize maintenance costs
2	5	5	3.80
4	5	5	3.05
8	5	5	2.57
16	5	5	2.21
32	5	5	1.91

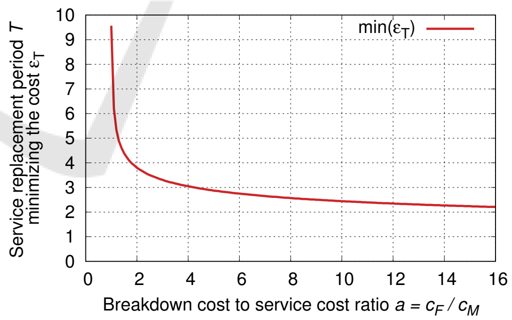
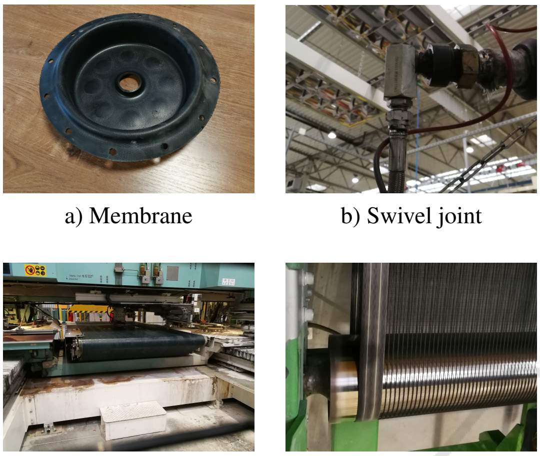


Figure 3: Values of T minimizing the relative cost of maintaining efficiency in the time unit ε_T for the given values of the parameter a.

Figure 3 shows how the maximum lifetime of a given component changes if the relative cost of failure increases. The presented ones concerned the Weibull distribution with the parameters $\alpha = \beta = 5$. If the aging process of a given element is approximated by the Weibull distribution with different values of these parameters, then the entire analysis should be performed again for the modified data.

3 EXAMPLES OF OPTIMISATION FOR REAL INDUSTRIAL ELEMENTS

For the purposes of this article, certain parts have been selected. They are: membrane, swivel joint, bando belt and IR belt. This parts are shown in Figure 4.



a) Membrane b) Swivel joint

c) Bando belt d) IR belt

Figure 4: Industrial spare parts used as examples for maintenance cost optimisation.

The data on the failure-free operation time of individual spare parts can be found in the Table 2.

Table 2: Uptime in days for each industrial spare part.

Spare part	Uptime (days)
Membrane	1, 40, 45, 16, 6, 29, 6, 76, 14, 34, 58, 91, 49, 42, 156, 28, 35, 88, 96, 106, 13, 17
Swivel joint	72, 208, 51, 153, 245, 119, 41, 692, 79, 235, 99, 342
Bando belt	14, 55, 107, 44, 146, 63, 18, 78, 42, 41, 40, 28, 59, 7, 106, 48, 24, 66, 23, 50, 3, 67, 7, 43, 72, 1, 75, 37, 27, 49, 34, 49, 6, 74, 91
IR belt	130, 373, 172, 178, 272, 138, 303, 85

They were used to determine the parameters for the Weibull distribution using the variable linearization method and the least squares method (Lai et al., 2006), (Lai, 2014). The approximation results were illustrated with probability graphs for each of the analyzed spare parts.

```
# Import libraries.
from reliability.Distributions import \
  Weibull_Distribution
from reliability.Fitters import Fit_Weibull_2P
import matplotlib.pyplot as plt

# Set uptime data.
data = [85,130,138,172,178,272,303,373]
# Prepare Probability Plot using least
# square method. It also determine Weibull
# distribution parameters.
fit = Fit_Weibull_2P(failures=data, \
  method='LS', linewidth=2.5)

# To get details about determined distribution
# use properties and functions of
#   fit.distribution
# object. It is possible to get both plots and
# numeric data.

# ... Add plot options and show ...
plt.show()
```

In this part, the use of open Python libraries for modeling the life expectancy of exemplary spare parts used in industry will be presented. For more information on using Python libraries to analyze and parameterize the Weibull distribution, see the article (Chmielowiec and Klich, 2021). The obtained results will be the basis for determining the optimal operating times for each part. They can be considered as the first level to support the parts replacement management process.

It should be noted that the choice of the Weibull distribution as a distribution modeling the life cycle of spare parts used in industry is not accidental. Since the introduction (Weibull, 1939), this distribution is widely used in numerous industrial issues: strength of glass (Keshevan et al., 1980), pitting corrosion (Sheikh et al., 1990), adhesive wear of metals (Queeshi and Sheikh, 1997), failure rate of shells (Almeida, 1999), failure frequency of brittle materials (Fok et al., 2001), failure rate of composite materials (Newell et al., 2002), wear of concrete elements (Li et al., 2003), fatigue life of aluminum alloys (Hemphill et al., 2012), fatigue life of Al-Si castings (Aigner et al., 2018), strength of polyethylene terephthalate fibers (Xie et al., 2020), failure of joints under shear (Bai et al., 2020) or viscosity-temperature relation for damping fluids (Chmielowiec et al., 2021).

On the basis of financial data, the optimal times for replacing individual elements were also determined. Although these times optimize the operating costs of each part separately, they do not allow to look at the problem from a global point of view – there are hundreds of such elements functioning in a manufacturing company. Therefore, in the next the use of rectangular knapsack problem algorithms to support the management process of maintenance costs will be discussed.

3.1 Membrane

The membrane is one of the key components of the pneumatic valve actuator, which is used to create a vacuum in an electric press. The press and the valve mounted in it operate in cycles of approximately 20 seconds. During this time, the membrane is subjected to forces causing its systematic wear. A rupture in the membrane causes the production line to stop completely for $\tau_1 = 4$ h. The cost of replacing an element is $C_{M,1} = 200$ EUR, and the cost of stopping a line is approximately $C_{D,1} = 650$ EUR h. Therefore, the cost of an emergency replacement of this element should be estimated at $C_{F,1} = C_{M,1} + \tau_1\, C_{D,1} = 2\,800$ EUR. This means the a ratio for the membrane is 14.0.

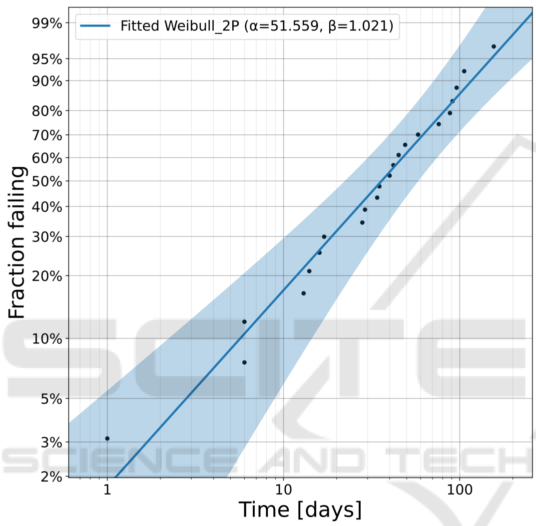


Figure 5: Membrane Weibull probability plot determined by coordinate linearization and less square method.

Using the data on the failure rate of the element, it is possible to estimate the parameters of the distribution. The Figure 5 shows the method of determining the parameters using the least squares method. The parameters obtained by this way have the value $\alpha_1 = 51.559$ and $\beta_1 = 1.021$. The Weibull distribution density plot for such parameters is presented in Figure 6. Determining the optimal replacement time as a value that minimizes the ε_T function given by the equation (5), we get $T_1 = 980$ days. Observing the historical data, it can be concluded that in practice it means replacement only in the event of a failure. This is due to the fact that the β parameter value for the membrane is very close to 1, and for this value, preventive replacements are unprofitable at all (Nakagawa, 2003).

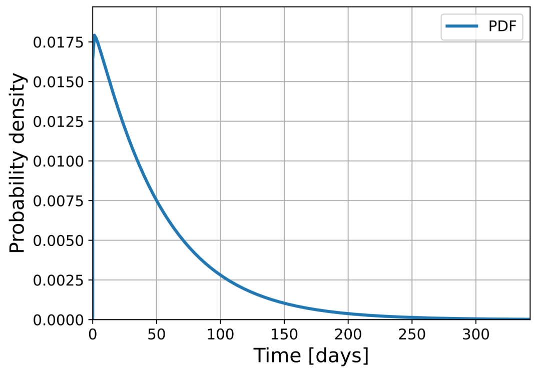


Figure 6: Membrane Weibull PDF for $\alpha = 51.559$ and $\beta = 1.021$.

3.2 Swivel Joint on an Industrial Robot

The swivel joint is an element installed on the head of an industrial robot that applies a seal to the car window. It connects the material feeding hose (polyurethane mass) with the movable robot head. The tool on which the swivel is installed travels around the three edges of the glass approximately every 70 seconds. During this time, the joint rotates 270 degrees and returns to the zero position at the end of the robot cycle. Swivel joint wear occurs as a result of repeated rotation, and its sign is loss of tightness or blockage of movement. Damage of the swivel causes the production line to stop completely for $\tau_2 = 1$ h. The cost of replacing it is $C_{M,2} = 60$ EUR, and the cost of stopping the line is the same as membrane, approximately $C_{D,2} = 650$ EUR h. Therefore, the cost of an emergency replacement for this item should be

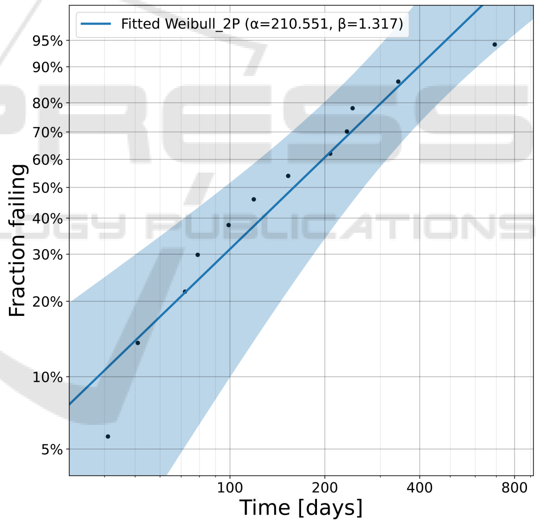


Figure 7: Swivel joint Weibull probability plot determined by coordinate linearization and less square method.

estimated at $C_{F,2} = C_{M,2} + \tau_2\, C_{D,2} = 710$ EUR. This means the a factor for a swivel joint is 11.833.

Using the data on the failure rate of the swivel joint, it is possible to estimate the parameters of the distribution. The Figure 7 shows the process of determining the parameters using the least squares method. The parameters calculated with this method have the value $\alpha_2 = 210.551$ and $\beta_2 = 1.317$. The Weibull distribution density plot for the determined parameters is presented in Figure 8. Determining the optimal replacement time for this item, we get $T_2 = 87$ days. After this time, the swivel joint should therefore be serviced.

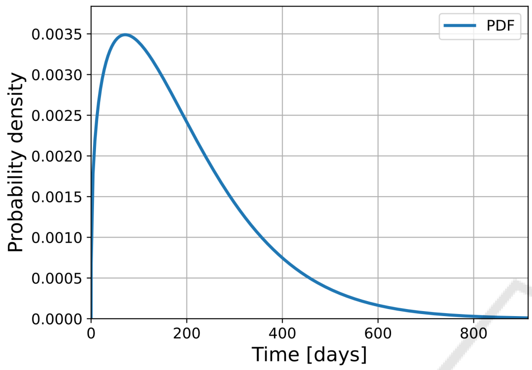


Figure 8: Swivel joint Weibull PDF for $\alpha = 210.551$ and $\beta = 1.317$.

3.3 Bando Belt

Bando belt is an element that transports glass waste after the breaking operation. It works in a fast cycle, every 16 seconds, starting and braking with great torque. The belt works at ambient temperature and is exposed to high tensile forces. The symptom of failure is partial or complete breaking of the belt. Damage to the Bando belt causes a complete production line stop at $\tau_3 = 4$ h. The cost of replacing it is $C_{M,3} = 1\,000$ EUR, and the cost of stopping the line is the same as the memabrane and the swivel, approximately $C_{D,3} = 650$ EUR h. Therefore, the cost of an emergency replacement of this element should be estimated at $C_{F,3} = C_{M,3} + \tau_3\, C_{D,3} = 3\,600$ EUR. This means that the a ratio for a Bando belt is 3.6.

Using the Bando belt failure rate data it is possible to estimate distribution parameters. The Figure 9 shows the process of determining the parameters using the least squares method. The parameters obtained with this method have the value $\alpha_3 = 54.849$ and $\beta_3 = 1.157$. The Weibull distribution density plot for the determined parameters is presented in Figure 10. Determining the optimal replacement time for this item, we get $T_3 = 237$ days. After this time, the Bando belt should be serviced.

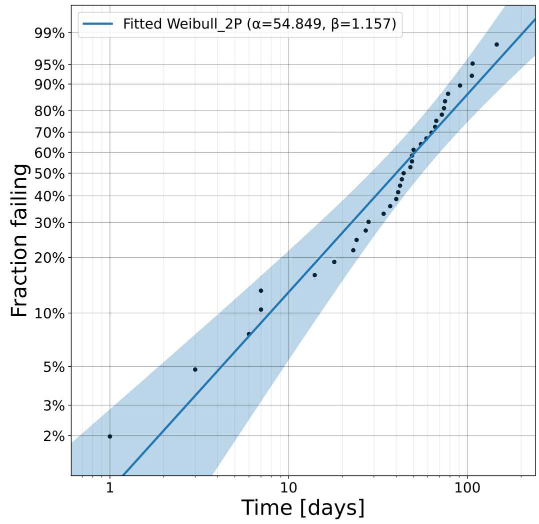


Figure 9: Bando belt Weibull probability plot determined by coordinate linearization and less square method.

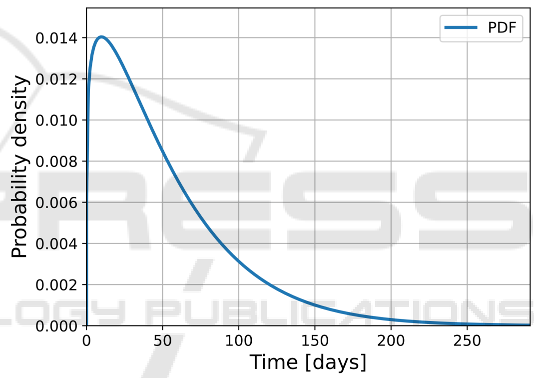


Figure 10: Bando belt Weibull PDF for $\alpha = 54.849$ and $\beta = 1.157$.

3.4 IR Belt

The IR belt is a transport mesh that transports the glass through the paint drying zone after the screen printing process. The belt works with constant dynamics (speed and acceleration), but in an environment of increased temperature - 180 degrees Celsius. Belt failure usually manifests as belt stretching beyond the adjustment value (critical length) or breaking completely. Damage to the IR belt causes the production line to stop completely for $\tau_4 = 8$ h. The cost of replacing it is $C_{M,4} = 5\,200$ EUR, and the cost of stopping the line is the same as for all other elements, and is approximately $C_{D,4} = 650$ EUR h. Therefore, the cost of an emergency replacement of this element should be estimated at $C_{F,4} = C_{M,4} + \tau_4\, C_{D,4} =$

10 400 EUR. This means that the a ratio for a Bando belt is 2.0.

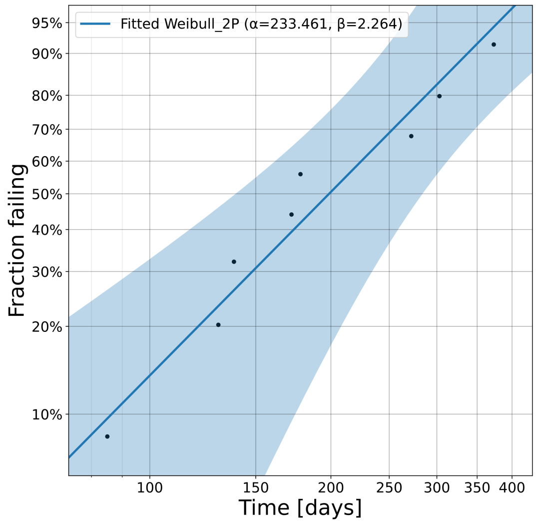


Figure 11: IR belt Weibull probability plot determined by coordinate linearization and less square method.

Using the IR belt failure rate data it is possible to estimate distribution parameters. The Figure 11 shows the process of determining the parameters using the least squares method. The parameters obtained by this method have the value $\alpha_4 = 233.461$ and $\beta_4 = 2.264$. The Weibull distribution density plot for the determined parameters is presented in Figure 12. Determining the optimal replacement time for this

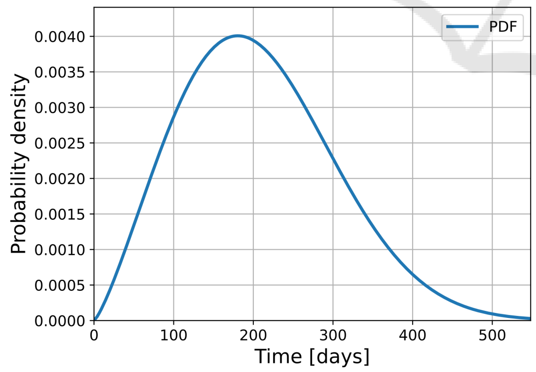


Figure 12: IR belt Weibull PDF for $\alpha = 233.461$ and $\beta = 2.264$.

item, we get $T_4 = 222$ days. After this time, service replacement of the IR belt should take place.

4 REDUCTION OF THE OPTIMIZATION COST PROBLEM FOR MULTIPLE PARTS TO A RECTANGULAR KNAPSACK PROBLEM

The examples presented in the previous section clearly show that the optimal replacement times vary greatly between the different parts, as it is shown in Figure 13. Therefore, it becomes practically impossi-

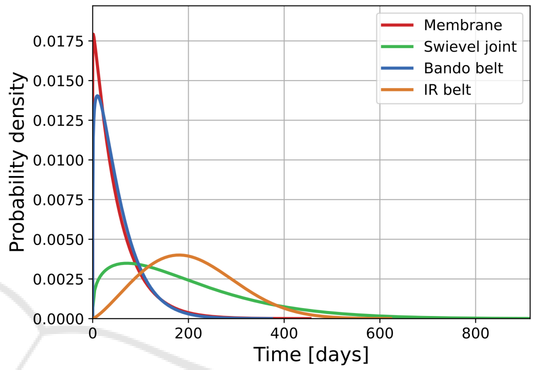


Figure 13: Common Weibull PDF plot for all presented parts.

ble to optimize the replacement time for all elements, which may be even several hundred in the enterprise. However, the question arises whether the downtime caused by a failure cannot be used to replace other parts as well, and not only the damaged one. Computer aided calculations can estimate the profitability of immediate replacement of every spare part operating in the factory. Thus, the IT system may propose preventive replacement of other parts to the maintenance department at the time of failure. The effect of using this type of approach will be the reduction of the total emergency downtime of the line, which will contribute to the reduction of maintenance costs in the enterprise.

The presented proposal is a definitely new approach to the problem of optimization of operating costs in an enterprise. The classical methods are limited to the use of analytical methods of the probability theory and reliability theory. This article presents the global optimization method, which combines analytical and discrete methods. The cost reduction method presented here is one of many possible approaches to optimizing the performance of maintenance teams. However, it fits perfectly into modern trends in predictive maintenance for the needs of large-scale production of automotive industry companies. It also opens up wide possibilities of using machine learning mechanisms to better estimate the failure rate function $F(t)$ and to determine global minima for the operating cost function.

In order to create an algorithm to support the decision-making process in the maintenance system, it is necessary to define a common measure that determines the condition of the production line. Such a measure will be the expected cost of keeping the line operational from now until the next planned downtime. Suppose the line uses n spares $U_1, U_2, \ldots, U_n$. Each of the U_i parts has a set of parameters that define it. It can therefore be said that

$$U_i = (t_i, \alpha_i, \beta_i, F_i, G_i, \mu_i(t), C_{M,i}, C_{D,i}, \tau_i, \sigma_i), \quad (7)$$

where t_i is the current time of failure-free operation of the element, α_i, β_i are the Weibull distribution parameters defining the failure rate of F_i and the reliability of G_i, $\mu_i(t)$ is the expected uptime of the element after working t time units, $C_{M,i}$ is the cost of component replacement during planned downtime, $C_{D,i}$ is the cost of production line downtime, τ_i is the downtime required to replace the component, and σ_i is the part of the human resources that is used during the replacement of a given item. If it is assumed that the next scheduled stoppage will take place for T units of time, then the expected cost of operating a part of U_i at that time is

$$C(U_i, T) = \bar{k}_i C_{F,i} + G_i(t_i')C_{M,i}, \quad (8)$$

where $C_{F,i} = C_{M,i} + \tau_i C_{D,i}$ is the cost of an emergency replacement part, $\bar{k}_i = 1 + k_i + F_i(t_i')$ is the expected number of failures in time to the scheduled downtime, and $G_i(t_i')$ is the probability of failure-free operation of the part from the time of the last forecast failure to the scheduled downtime. The expected number of

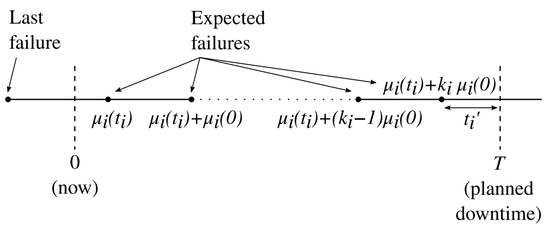


Figure 14: Expected failures in the case of not carry out preventive replacement of the element.

failures can be obtained by assuming the following values for k_i and t_i'

$$k_i = \left\lfloor \frac{T - \mu_i(t_i)}{\mu_i(0)} \right\rfloor, \quad (9)$$

$$t_i' = \min\left(T - \mu_i(t_i) - k_i\mu_i(0), T\right). \quad (10)$$

Note that if $T < \mu_i(t_i)$, then $k_i = -1$ and the expected number of failures is $\bar{k}_i = F_i(T)$. The interpretation of k_i and t_i' is shown in Figure 14.

Additionally, we define the alternative expected cost of operating the U_i part

$$C'(U_i, T) = \bar{s}_i C_{F,i} + (1 + G_i(t_i''))C_{M,i}, \quad (11)$$

for which $\bar{s}_i = s_i + F_i(t_i'')$, $s_i = \lfloor T/\mu_i(0) \rfloor$ and $t_i'' = T - s_i\mu_i(0)$. This function expresses the expected alternative costs that will be incurred in the event that part of U_i has been replaced at present on the occasion of downtime due to failure of another part. The interpretation of s_i and t_i'' is shown in Figure 15.

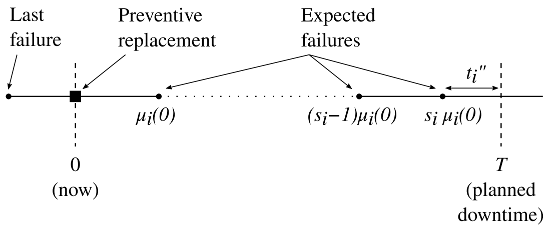


Figure 15: Expected failures in the event of a preventive replacement of an element.

For the functions C and C' defined in this way, we create a sequence of values $V_i = C(U_i, T) - C'(U_i, T)$, which express the expected profit on the replacement of U_i during the emergency downtime. If we assume that the part with index j failed, then the optimization problem can be reduced to a two-dimensional knapsack problem. It should be assumed that the knapsack has the shape of a rectangle with a width of τ_j and a height of $1 - \sigma_j$, and the elements arranged in it are rectangles of width τ_i, height σ_i and values V_i, where $i \neq j$. It should also be emphasized that the elements placed in the knapsack cannot be twisted, as time and human resources are not interchangeable. Figure 16 illustrates the problem described above.

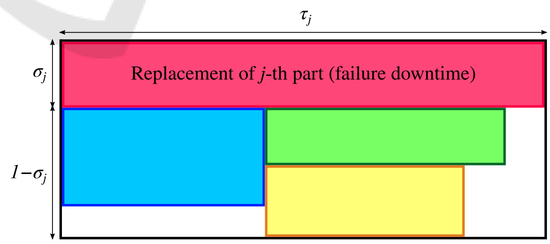


Figure 16: Optimum use of downtime for the preventive replacement of additional parts.

The literature devoted to this problem is very rich. The best review article to date is (Neuenfeldt Jr. et al., 2022). During a systematic review of bibliographic items, the authors distinguished four basic approaches to the construction of algorithms for solving the two-dimensional rectangular knapsack problem: approximation, exact, heuristic and hybrid. The article presents a very in-depth bibliometric analysis indicating which publications have the most significant impact on the development of algorithms solving this problem. The review includes over one hundred publications issued by 2020. The intensity of research in this area is also evidenced by numerous publications issued in the last two years: (Zhou et al., 2019), (Fırat and Alpaslan, 2020), (Wei et al., 2020), (Li et al., 2021) and (Wang et al., 2021b).

5 CONCLUSIONS

The article presents how the methods of analytical and discrete optimization can be used to minimize the operating costs of a production line. The third part presents analytical methods of cost optimization for a single spare part. The MES (Manufacturing Execution System) equipped with this type of mechanisms can keep the maintenance manager informed about the probability of failure of a given part and the expected costs of its removal. It provides the current knowledge of what events should be prepared for and what to expect in the near future. On the other hand, the equations (8) and (11) introduced in the fourth part constitute the basis for estimating the actual value of the service activity. The algorithm of conduct presented in this section is to support the decision-making process in the area of maintenance from the perspective of the entire enterprise. The complexity of the two-dimensional knapsack problem indicates that the global optimization problem is extremely computationally difficult. Therefore, the actual functioning of global methods of minimizing operating costs in real time requires significant computing power. Therefore, the presented approach requires further research in the field of, for example, simplification of the model, which will result in a reduction of computational complexity.

Nevertheless, the use of heuristic methods, which relatively quickly indicate the first approximation of the solution to the problem, will be appropriate in this case. We should remember that the system is supposed to support decision-making, and not completely eliminate the human factor. Therefore, even suggestions of possible solutions will give very valuable information for the maintenance manager and can be successfully used in modern MES systems.

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