Interval-based Robot Localization with Uncertainty Evaluation

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Keywords: Landmark-based Localization, Uncertainty Estimation, Interval Analysis, Factor Graph, Probabilistic Uncertainty.

Abstract: Being able to provide trustworthy localization for a robot in a map is essential for various tasks with safety-related requirements. In contrast to classical probabilistic approaches that represent the uncertainty as a Gaussian distribution, we use interval error bounds for the uncertainty estimation of a localization problem. To tackle and identify the limitations of probabilistic localization uncertainty estimation, we carry out comparison experiments between an interval-based method and a factor graph-based probabilistic method. Different measurement error models are propagated by the two methods to derive the robot pose uncertainty estimates. Results show that the probabilistic approach can provide very good pose uncertainty when there is no non-Gaussian systematic sensor error. However, if the measurements have unmodeled systematic errors, the interval approach is able to robustly contain the true poses whereas the probabilistic approach gives completely wrong results.

1 INTRODUCTION

For safe navigation and operation, a mobile robot not only has to accurately determine its localization in the environment but also needs to provide trustworthy results. In indoor environments where no Global Positioning System (GPS) is available, the robot usually localizes itself by using sensory information (e.g. wheel odometry, 3D point cloud, or camera images) and updates its belief of position with respect to a given map. A variety of probabilistic techniques have been proposed to reach the localization goal on a prior map (Gutmann et al., 1998) (Wilbers et al., 2019). Nevertheless, most approaches focus on providing accurate point-wise solutions, while the quality of the uncertainty estimation of environmental perception and localization results are not thoroughly investigated.

In probabilistic localization, the Kalman filter and graph-based optimization approaches provide explicit estimates of robot poses efficiently, with the assumption that the measurement and pose uncertainty have zero-mean Gaussian distributions (Chen, 2012) (Kummerle et al., 2011). The Gaussian uncertainty assumption, however, could be inaccurate and unreliable due to the following reasons.

On one hand, there exist non-Gaussian sensor errors which are hard to be identified. For example, light detection and ranging (LiDAR) sensors are widely used for robot localization applications. It has been pointed out by (Voges, 2020) that stochastic distributions can not always truly represent the error model of LiDARs due to the presence of unmodeled systematic errors. Moreover, analysis in (Mallet et al., 2008) shows that the LiDAR measurement model is subject to mixed effects of geometric (e.g. the incidence angle between the laser beam and the surface) and radiometric object properties (e.g. different materials of the surface). These problems induce non-Gaussian measurement uncertainty, which can not be represented by the mean and variance. On the other hand, due to the nonlinearity of the system, a linearization process is required which can only give an approximation of the uncertainty estimate. Due to these limitations of probabilistic approaches, the resulting pose estimation and its uncertainty might be erroneous and may significantly differ from the true result.

In this paper, we aim to tackle and identify the limitations of probabilistic localization uncertainty when the Gaussian assumption is violated. First, we provide a solution to a 2D landmark-based localization problem using interval analysis (Jaulin et al., 2001). In

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Jiang, Y., Ehambram, A. and Wagner, B.
Interval-based Robot Localization with Uncertainty Evaluation.
DOI: 10.5220/0011143700003271
In *Proceedings of the 19th International Conference on Informatics in Control, Automation and Robotics (ICINCO 2022)*, pages 296-303
ISBN: 978-989-758-585-2; ISSN: 2184-2809

comparison to probabilistic methods that use Gaussian distributions for the measurement model (i.e. range and bearing) as well as the robot pose (position and heading), the interval method models them with interval boxes with lower and upper bounds, where no probability distribution is assumed. The interval-based solution can provide guaranteed uncertainty estimation for the robot poses that enclose the true values. We focus on the uncertainty estimation of the localization problem and compare the results from the interval-based method and a factor graph-based probabilistic method, to find advantages and limitations of both. In particular, we investigate the uncertainty propagation from the measurement model to the resulting robot pose.

The contributions of the paper are:

- We evaluate the uncertainty estimation of an interval-based solution and a factor graph-based probabilistic solution for a localization problem by designing four simulated experiments.
- We find failure cases where the probabilistic approach leads to incorrect uncertainty estimation with the presence of non-Gaussian measurement error, whereas the results of the interval-based method still hold true.

In the remainder of this paper, we first introduce the related work in Section 2. In Section 3 we introduce the basics of interval analysis. Subsequently, we formulate our localization problem and establish our interval-based localization solution, as well as the state-of-the-art probabilistic solution in Section 4. In Section 5, we evaluate the uncertainty estimation of the two solutions by four small simulated experiments and analyze the results.

2 RELATED WORK

In the existing map-based localization literature, landmark-based methods are widely used by different sensor modalities benefiting from the identifiable landmarks in the environments like poles (Brenner, 2010) (Schaefer et al., 2019), visual features (Wilbers et al., 2019) and natural landmarks (Rohou et al., 2020). In this work, we also use a set of landmarks as the prior map. Typically, the landmarks observed by the sensor are associated with the map to track the robot in the map. The data association problem is challenging and false associations introduce outliers that need to be dealt with. Since data association is not the focus of this paper, we assume that the association between locally-observed landmarks and the map landmarks is known.

In recent years, interval methods have shown their potential in robot localization and simultaneous localization and mapping problems. The work in (Rohou et al., 2020) solves the localization problem for an underwater vehicle in a known map but with unreliable data association. By formulating a constraint network with a constellation contractor, the data association and state estimation problems are solved together. In an unmanned aerial vehicle (UAV) localization application (Kenmogne et al., 2017) interval has been used to compute the 6 DOF pose of the UAV by tracking known image features with bounded-error. The work in (Voges and Wagner, 2021) firstly proposes a guaranteed visual-LiDAR odometry for autonomous vehicles that uses LiDAR and a monocular camera to compute the 6 DOF pose estimate with bounded uncertainty. Evaluation of real data from large-scale experiments shows that the vehicle's real poses are enclosed in a guaranteed way. Authors in (Ehambram et al., 2021) extend this work (Voges and Wagner, 2021) by fusing information from LiDAR and stereo cameras to solve the dead-reckoning of a vehicle and use the interval contraction method for consecutive robot pose estimation. The above methods show that interval analysis can provide robust uncertainty estimates of the robot pose which always enclose the ground truth data. However, few investigations have been done to compare the interval results with those of the probabilistic methods. Interval analysis has the benefit of providing a guaranteed estimation of the robot pose, however, it can also suffer from pessimism due to large initial intervals, wrapping effect, and dependencies of parameters (Jaulin et al., 2001). Thus, we think it is important to validate the use of interval analysis by showing under what cases interval has advantages over probability approaches and also disadvantages in terms of uncertainty estimation.

Recently, graph-based optimization has become the standard solution for SLAM and widely applied for localization applications (Kummerle et al., 2011) (Kaess et al., 2011) (Wu et al., 2017) (Wilbers et al., 2019). The graph representation allows adding robot poses and landmark positions as nodes in the graph, while the constraints between the nodes are added as edges (or factors). A non-linear least-squares problem is formulated and the error function is minimized in a bundle-adjustment manner to determine the optimal robot pose and landmark positions. In this paper, we use the factor graph (Kaess et al., 2011) to formulate a 2D localization problem. We establish a factor graph-based localization solution as the state-of-the-art probabilistic approach. The uncertainty estimation of this method will be compared with the interval-based uncertainty and evaluated.

3 INTERVAL ANALYSIS

Interval analysis is a numerical tool based on the idea of enclosing real numbers in intervals (Jaulin et al., 2001). Instead of an exact value, variables are defined by an interval that is a subset of $\mathbb{R}$, denoted by $[x] = [\underline{x}, \overline{x}]$. The variable is bounded by a lower bound $\underline{x}$ and an upper bound $\overline{x}$, without any assumptions about its probability distribution. By using the simple error bounds, interval provides a guaranteed way to represent the uncertainty of a variable. For example, if we know that the accuracy of a distance measurement is $\pm 0.3\,\text{m}$, then the true value x^* is enclosed by an interval $x^* \in [x]$, with an uncertainty of the radius $r([x]) = (\overline{x} - \underline{x})/2 = 0.3\,\text{m}$. An interval box $[\mathbf{x}]$ is define as a vector of intervals.

Interval computation enables classical real arithmetic operations, $+, -, \times$, and $\div$, as well as elementary functions like $\sin, \cos, \exp\ldots$ on intervals based on set theory. Here is an example of addition operation on intervals:

$$[-2, 4] + [5, 6] = [-2+5, 4+6] = [3, 10] \quad (1)$$

For a 2D robot localization problem, where we consider the set of possible robot poses described in the interval domain $\mathbb{X}$, the problem can be characterized as:

$$\mathbb{X} = \{\mathbf{x} \in \mathbb{R}^3 \mid \mathbf{f}(\mathbf{x}) \in \mathbb{Y}\} = \mathbf{f}^{-1}(\mathbb{Y}), \quad (2)$$

where $\mathbf{f}: \mathbb{R}^3 \to \mathbb{R}^2$ is a non-linear function, which can be the measurement constraint, and $\mathbb{Y}$ the landmark positions. The robot pose $\mathbb{X}$ can be obtained by using Set Inverter Via Interval Analysis (SIVIA) (Jaulin et al., 2001) to calculate a subpaving solution set from an initial search domain.

Alternatively, this can also be solved by formulating a Constraint Satisfaction Problem (CSP) as a set of constraints and variables. We give an exemplary CSP:

$$\mathcal{H} : \begin{cases} \textbf{Variable :} & \mathbf{x}^n \\ \textbf{Constraint :} & \mathbf{g}(\mathbf{x}^n) = \mathbf{0}\,, \\ \textbf{Domain :} & [\mathbf{x}^n] \end{cases} \quad (3)$$

where $\mathbf{x}^n \in [\mathbf{x}^n]$ is the unknown variable vector, and $\mathbf{g}(\mathbf{x}^n)$ is the constraints to fulfill in the CSP $\mathcal{H}$.

Contractors (Jaulin et al., 2001) can be built to contract the variable's initial domain, to get a resulting interval box that satisfies all the constraints. Here the contraction means that the initial interval domain is reduced by removing all values that do not satisfy the constraints. The forward-backward contractor is the most convenient contractor to contract the domain of CSP $\mathcal{H}$ where all constraints are decomposed into primitive constraints (i.e. a sequence of functions containing only a single operator) and each of the primitive constraints is contracted until the interval boxes converge towards the smallest possible sizes. The benefit of contraction is that all possible solutions are guaranteed to be contained in the contracted domain.

In this work, interval analysis is used for solving a 2D robot localization problem. A CSP is formulated to compute the uncertainty of the robot pose. Contractors are built to propagate the uncertainty of the map landmarks and measurement constraints, resulting in a guaranteed robot pose uncertainty that satisfies the constraints in CSP $\mathcal{H}$.

4 PROBLEM FORMULATION

The goal is to track a moving robot in a known map of landmarks $\mathbb{M}$. As shown in Figure 1 the robot moves in a square-shaped indoor corridor from a starting point.

Figure 1: The 2D view of a robot moving in a $10\,\text{m} \times 10\,\text{m}$ square-shape corridor (bounded by two black boxes). The robot starts from the red point (0, 0) and follows the clockwise green trajectory (ground truth). Orange dots are landmarks to be measured.

The robot carries a LiDAR sensor and measures the radial distance r and horizontal angle θ of its surrounding landmarks to the robot itself. For each LiDAR scan a subset of the map landmarks $\mathbf{m}_l$ can be measured, where $l \in \{1, \ldots, L\}$. L is the number of landmarks the robot observes from its current scan. Note that this scenario is built in simulation, with the positions of map landmarks $\mathbb{M}$ known. The landmarks are assumed to be randomly distributed in the environment and can be perceived by the LiDAR accurately. Furthermore, the locally observed landmarks $\mathbf{m}_l$ are acquired according to the robot motion model and the geometry of the corridor which we establish by simulation. We assume that a feature extraction algorithm provides measurements of landmarks in the current scan and association between the locally observed landmarks $\mathbf{m}_l$ and the map $\mathbb{M}$ is already solved.

This localization problem can be formulated by the state estimation equations:

$$\begin{cases} \textbf{Measurement}: & \mathcal{G}(\mathbf{x}, \mathbf{z}, \mathbf{m}_l) = \mathbf{0} \\ \textbf{Association}: & \mathbf{m}_l \in \mathbb{M} \end{cases}, \quad (4)$$

where we denote $\mathbf{x} = (x \ y \ \psi)^T$ as the unknown robot pose, $\mathbf{z} = (r \ \theta)^T$ as the measurement for each $\mathbf{m}_l$ in current scan, and $\mathbf{m}_l = (m_{l_x} \ m_{l_y})^T$ the position of the landmark. More specifically, the measurement constraint in Equation 4 can be broken down as follows:

$$\mathcal{G}(\mathbf{x}, \mathbf{z}) = \begin{pmatrix} x + r \cdot \cos(\psi + \theta) - m_{l_x} \\ y + r \cdot \sin(\psi + \theta) - m_{l_y} \end{pmatrix}. \quad (5)$$

We illustrate the above measurement model in Figure 2 where the robot measures its distance and azimuthal angle with respect to a landmark.

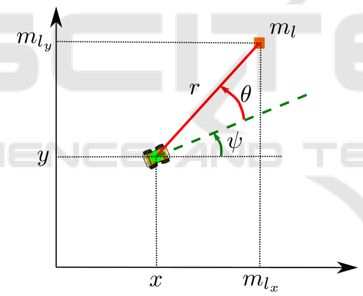

Figure 2: The robot measurement model.

Next, we will introduce our interval-based solution and factor graph-based probabilistic solution for the aforementioned localization problem, respectively.

4.1 Interval-based Localization

All variables in Equation 4 are described in the interval domain. This allows us to formulate a CSP $\mathcal{L}$:

$$\mathcal{L}: \begin{cases} \textbf{Variable}: \mathbf{x}, \mathbf{z}, \mathbf{m}_l, \mathbb{M} \\ \textbf{Constraint}: \begin{matrix} \mathcal{G}(\mathbf{x}, \mathbf{z}, \mathbf{m}_l) = \mathbf{0} \\ \mathbf{m}_l \in \mathbb{M} \end{matrix} \\ \textbf{Domain}: [\mathbf{x}], [\mathbf{z}], [\mathbf{m}_l], [\mathbb{M}] \end{cases}, \quad (6)$$

where $[\mathbf{x}]$ is the set of feasible robot poses that we want to determine and $[\mathbf{z}] = ([r] \ [\theta])^T$ is our landmark measurement. Interval boxes are suitable to represent the uncertainty of LiDAR measurements because we usually only know the accuracy of the distance and beam divergence (Voges, 2020). Besides the measurement uncertainty, the uncertainty of known map landmarks $\mathbb{M}$ is also specified. The uncertainty of map landmarks can be provided by the accuracy of the given map or feature extraction algorithm. In this CSP we assume no information about the prior robot position, thus its initial domain is set to $[\mathbf{x}] = ([-\infty, \infty] \ [-\infty, \infty] \ [-\pi, \pi])^T$.

Then, we build forward-backward contractors for solving each constraint in CSP $\mathcal{L}$ and get the intersection to contract the interval pose. Before using the contractors, we first break down our constraints into simple equations according to Equation 5 and 6 (Rohou et al., 2020):

$$\begin{cases} a = \psi + \theta \\ d_x = r \cdot \cos(a) \\ d_y = r \cdot \sin(a) \\ m_{l_x} = x + d_x \\ m_{l_y} = y + d_y \\ \mathbf{m}_l \in \mathbb{M} \end{cases}, \quad (7)$$

where a and $\mathbf{d} = (d_x \ d_y)^T$ are intermediate variables for decomposing complex constraints into elementary equations. Subsequently, we apply addition contractor $\mathcal{C}_+$ (e.g. $\mathcal{C}_+([a], [\psi], [\theta])$) for addition equations and polar contractor $\mathcal{C}_{polar}([d_x], [d_y], [r], [a])$ for the polar equations in Equation 7 (Desrochers and Jaulin, 2016). As a result, related interval domains are reduced until smallest interval boxes are reached.

In our localization problem, the CSP $\mathcal{L}$ will be solved for each robot pose, which means that whenever the robot measures landmarks from one LiDAR scan, the contractors will be applied to contract the initial box $[\mathbf{x}]$ for the contracted robot pose. The resulted interval box represents the uncertainty of the robot pose.

4.2 Factor Graph-based Localization

A factor graph includes variables and factors which represent constraints between variables in a graph and solve a maximum a posteriori (MAP) estimate problem (Kaess et al., 2011). Here we illustrate the factor graph of our localization problem in Figure 3.

As a probabilistic approach, all quantities and factors involved are labeled with a probability distribution. Given that all measurements are affected by Gaussian noise, the goal of our localization problem is to estimate a Gaussian approximation of the posterior of the robot poses. In the following, we formulate our localization problem as an optimization problem

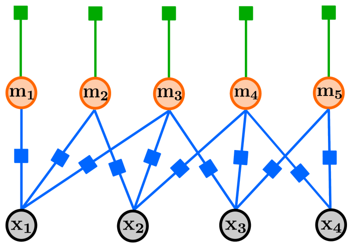

Figure 3: Factor graph representation of the localization problem. We have robot poses $\mathbf{x}$ (grey circles), observed landmarks $\mathbf{m}_l$ (orange circles) as variables. Blue boxes are the measurement factors between poses and observed landmarks $\mathbf{m}_l$. Green boxes represent the map factors between the observed landmarks and map $\mathbb{M}$.

that determines the robot poses and landmark positions based on a set of measurements.

Let $\mathbf{v} = (\mathbf{x}\ \mathbf{m}_l)^T$ be the state vector that consists of robot poses $\mathbf{x} = (x_1 \dots x_T)$ and landmarks $\mathbf{m}_l = (m_{l_1} \dots m_{l_K})$ where T, K are the numbers of all poses and landmarks, respectively. Here we define the robot poses as $\mathbf{x}_t \in SE(2)$ and landmarks as $\mathbf{m}_{l_k} \in \mathbb{R}^2$. The optimization problem can be represented as a MAP estimate according to Bayes' law:

$$\mathbf{v}^* = \arg\max_{\mathbf{v}} p(\mathbf{v}\,|\,\mathbf{z}) = \arg\max_{\mathbf{v}} p(\mathbf{z}\,|\,\mathbf{v})\, p(\mathbf{v}) \quad (8)$$

where $\mathbf{z}$ is the measurements gathered by the robot. The term $p(\mathbf{v})$ is the prior over all states which require to be given. With the assumption of independent Gaussian noise on the measurements, Equation 8 can be written as:

$$\mathbf{v}^* = \arg\min_{\mathbf{v}} \sum_i \mathbf{e}_i(\mathbf{v}, \mathbf{z}_i)^T \, \Omega_i \, \mathbf{e}_i(\mathbf{v}, \mathbf{z}_i) + \mathcal{F}^{map}(\mathbf{m}_l) \quad (9)$$

where $\mathbf{e}_i(\mathbf{v}, \mathbf{z}_i)$ is the error function that measures how well the state vector $\mathbf{v}$ satisfy the constraint $\mathbf{z}_i$, Ω_i is the information matrix of measurement $\mathbf{z}_i$ and $\mathcal{F}^{map}$ is the prior knowledge of the map landmarks. In our localization problem, apart from defining measurement factors which can be described by the left part on the right side of Equation 9, we also define the map factors, which is a Gaussian prior about the landmark positions in the known map (similarly, in the interval approach we define the prior of map landmarks as interval boxes). To solve Equation 9 and compute the Gaussian approximation of the robot poses as well as landmark positions, we use the standard least-squares optimization method Levenberg-Marquardt using the GTSAM solver for factor graphs (Dellaert, 2012).

5 EXPERIMENTAL EVALUATION

Our simulated experiments are designed to 1) support the claim that the interval-based localization method can ensure a guaranteed uncertainty estimation without assuming any probability distribution over the measurement; 2) compare the uncertainty estimation of the interval-based method and the factor graph-based probabilistic method; 3) to find out the advantages and disadvantages of using interval method and probabilistic method when there are unidentified non-Gaussian measurement errors.

We use simulated data (as explained in Section 4) for our experiments to leave out the influence of inaccurate data association and outliers so that we can investigate how the measurement model and uncertainty propagation strategy can affect the final uncertainty estimation. Firstly, we introduce important parameters that we use for the experiments.

We assume the robot moving in a $10\,\mathrm{m} \times 10\,\mathrm{m}$ square-shaped indoor corridor with an initial position at (0, 0) as illustrated in Figure 1. To calculate comparable uncertainty estimations for the interval-based and factor graph-based methods, we carefully choose the uncertainty of map landmarks and measurements. For the factor graph based method, we define the standard deviations for the prior map landmarks as $\sigma_{\mathbb{M}_x} = \sigma_{\mathbb{M}_y} = 0.01\,\mathrm{m}$, and for the angle and distance measurements as $\sigma_\theta = 0.01°$, $\sigma_r = 0.1\,\mathrm{m}$. For the interval-based method, we intend to define the interval box region of the map and measurements as such, that it will always include the 99% confidence region of Gaussian error ellipse. According to the χ^2 distribution, we choose 3σ as the radius of the intervals. Specifically, for the map landmarks the uncertainty is $[\Delta_{\mathbb{M}_x}] = [\Delta_{\mathbb{M}_y}] = [-0.03, 0.03]\,\mathrm{m}$, and for the measurements the uncertainties are $[\Delta_\theta] = [-0.03, 0.03]°$, $[\Delta_r] = [-0.3, 0.3]\,\mathrm{m}$.

In the following sections, we design four small experiments to compare the uncertainty estimation results of the two methods. We assign non-Gaussian measurement noises in the measurement model to find out how the two methods handle it. For the interval method, the measurement model always follows the interval uncertainty. For the probabilistic methods, no matter there is or not non-Gaussian noise in the measurements, it always assume it to be Gaussian. First, we compare the interval uncertainty with the probabilistic uncertainty when the measurements of given map landmarks follow Gaussian distributions. The non-zero Gaussian measurement is just a perfect assumption, while in real cases, the LiDAR measurements could contain systematic errors that are hard to identify. Thus in the second and third experiment, we simulate systematic errors for both methods and show how the two approaches handle the uncertainty propagation. In the last experiment, we suppose that the actually measured map landmarks are uniformly distributed in the interval uncertainty box of map landmarks, then apply the two methods with interval and probabilistic measurement models as in the first experiment.

5.1 Comparison of the Uncertainty Estimation

Our first experiment is designed to show the comparison of the robot pose uncertainty estimation using the interval-based localization algorithm and the factor graph-based optimization algorithm. Here the measurements are acquired according to the given map landmark positions. The measurements in the factor graph are inserted using Gaussian distribution while the measurements in the interval method follow interval uncertainty. We show in Figure 4 the result of both methods.

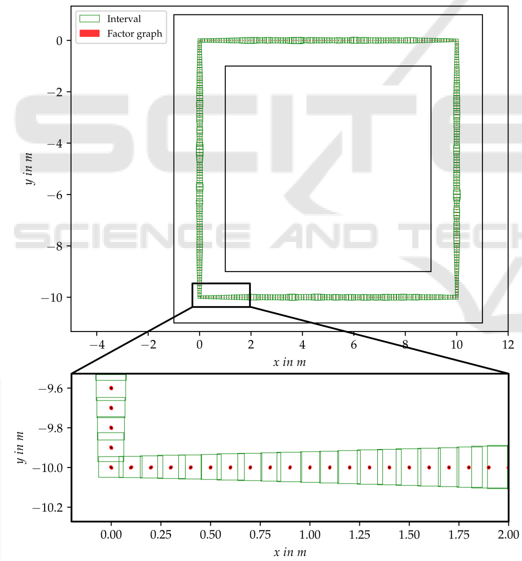

Figure 4: Uncertainty of robot poses given by interval method and factor graph method. The green boxes represent the interval pose uncertainty. Red ellipses represent the probabilistic pose uncertainty which is the 99% confidence region. The black dots inside the red ellipses are the ground truth value.

As can be seen, the green boxes represent the interval pose uncertainty and the red error ellipses which are the 99% confidence region represent the probabilistic uncertainty. It is clear that throughout the trajectory the probabilistic uncertainty is much smaller than the interval uncertainty. Moreover, 100% ground-truth poses are enclosed by both the interval and probabilistic pose uncertainties. The results show that both methods can give guaranteed results while the probabilistic method can provide more accurate pose estimation (i.e. the mean value) than the interval method, as the interval results can only give a region of all possible robot positions, but not explicit poses like the probabilistic method do.

5.2 Impact of 1-σ Systematic Error

In the second experiment, we add a positive 1-σ_r systematic error to the distance measurements of both methods. This is to resemble unidentified systematic errors in the range measurements from a LiDAR. The results are shown in Figure 5.

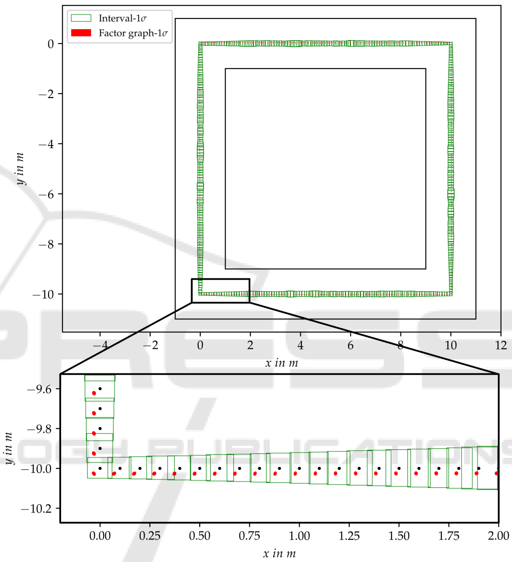

Figure 5: Green boxes represent the interval pose uncertainty. Red ellipses represent the 99% confident region of probabilistic uncertainty. Black dots are the ground truth value.

Note that interval uncertainty boxes still enclose the ground truth. Also, the interval boxes shrink slightly because solutions that are not consistent with the new measurement constraints are removed. The probabilistic uncertainty (red ellipse) shows an obvious shift to the outer bound of the interval box, and no longer includes the ground-truth value. The results imply that the probabilistic method underestimates the uncertainty and gives wrong estimates, while the interval method is still able to enclose the ground truth.

5.3 Impact of 2-σ Systematic Error

In the third experiment, we decide to further investigate the influence of systematic errors of the measurements. We add a positive 2-σ_r systematic error to the distance measurements of both methods. The results are shown in Figure 6.

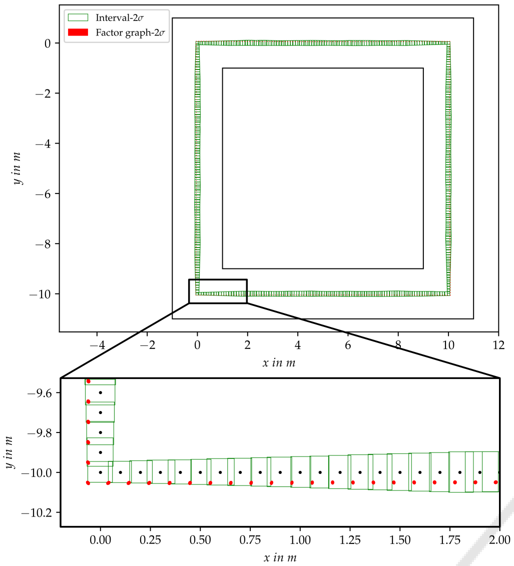

Figure 6: Green boxes represent the interval pose uncertainty. Red ellipses represent the 99% confident region of probabilistic uncertainty. Black dots are the ground truth value.

We can clearly see that interval boxes are still enclosing the ground truth, however, most of the probabilistic error ellipses go even further away from the true value. We believe that in this case, the probabilistic method gives very wrong uncertainty estimation that some of the error ellipses are even partly outside of the guaranteed interval uncertainty.

To conclude, we find that the interval method is robust to systematic errors of measurements, while the probabilistic method becomes significantly unreliable. The results of the second and third experiment show that the probabilistic method tends to underestimate the uncertainty of pose estimation and can even give totally wrong uncertainty estimation in the presence of large systematic errors. The interval method, on the contrary, can guarantee to provide trustworthy uncertainty estimates and not underestimate the pose uncertainty.

5.4 Impact of a Uniformly Distributed Measurement

In this part, we assume our measured map landmarks are no longer on the given positions, but uniformly distributed in the interval uncertainty box we defined for the map landmarks. This is a more realistic assumption since the actually measured landmarks by a LiDAR sensor could reside in the close vicinity of the real landmark positions. In other words, we create our map landmark positions that follow a uniform distribution inside the interval region $[\mathbb{M}_i] = \mathbb{M}_i + [-0.03, 0.03]m \times [-0.03, 0.03]m$ and create the measurements which follow interval uncertainty and Gaussian uncertainty according to this new map. In this experiment, the non-Gaussian measurement error comes from the uncertainty of measured map landmarks, but not the uncertainty in the measurement models, as in the previous two experiments. The purpose of this design is to see how the probabilistic method copes with a known map with interval uncertainty. Results are displayed in Figure 7.

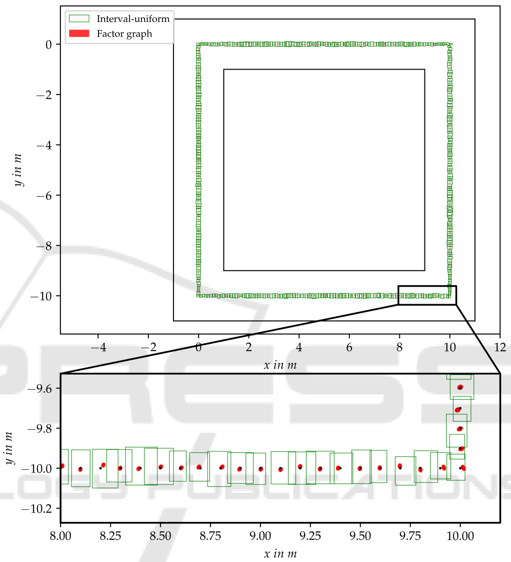

Figure 7: Green boxes represent the interval pose uncertainty. Red ellipses represent the 99% confident region of probabilistic uncertainty. Black dots are the ground truth value.

As seen from the result, the interval uncertainty boxes encloses the ground truth poses throughout the whole trajectory. However, the probabilistic error ellipses are not always able to include the ground truth poses with some black dots completely outside the red ellipse region, which shows that the probabilistic uncertainty is overconfident. This experiment implies that, when the uncertainty of map landmarks is an interval, then modeling the measurements as Gaussian can lead to underestimated pose uncertainty. Unlike the interval-based method that gives a worst-case uncertainty but ensures the inclusion of real result, the probabilistic method tends to underestimate the uncertainty.

6 CONCLUSIONS

We compare an interval-based solution and a factor graph-based probabilistic solution for a 2D localization problem for robot pose uncertainty estimation. For this purpose, we design four comparison experiments with different measurement models. From the results, we find out that the probabilistic method can provide more accurate pose estimation and smaller uncertainty estimation than the interval method given Gaussian measurement noise. However, the unidentified non-Gaussian errors in measurements can significantly impede the performance of the probabilistic approach, causing wrong pose estimates and underestimated uncertainty. In comparison, the interval method is not sensitive to systematic measurement errors or uncertainty of measured map positions, which is able to robustly provide guaranteed pose uncertainty estimation. It also shows that the interval method usually gives pose estimation with larger uncertainty compared to the probabilistic method. However, the benefits of using intervals for solving robot localization problems is that no prior information of the robot pose is required, and the ability to provide all possible solutions with guarantee.

In the further work, we aim to apply the interval-based method to real data. We plan to tackle the landmark extraction and association problem which can introduce more sources of error that will influence the uncertainty estimation. Besides, we would like to investigate again the uncertainty estimation performance of interval methods and probabilistic methods with noisy real measurement data for which the real distribution is unknown.

ACKNOWLEDGEMENTS

This work was supported by the German Academic Exchange Service (DAAD) as part of the Research Training Group i.c.sens [RTG 2159].

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