Robust Gain-scheduling LPV Control for a Reconfigurable Robot

R. Al Saidi[1] and S. Alirezaee[2]

[1] *University of Windsor, Windsor, Canada*

[2] *Electrical and Computer Engineering Department, Windsor, Canada*

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Abstract: This paper develops a robust gain–scheduling linear parameter varying (LPV) control for a reconfigurable robot that combines as many properties of different open kinematic structures as possible and can be used for a variety of applications. The kinematic design parameters, i.e., the Denavit–Hartenberg (D–H) parameters, can be modified to satisfy any configuration required to meet a specific task. By varying the joint twist angle parameter (a configuration parameter), the presented model is reconfigurable to any desired open kinematic structure, such as ABB, FANUC and SCARA robotic systems. A robust LPV control is developed for on–line measured parameters of a perturbed LPV model of a Bosch Scara robot arm. This control achieves superior tracking performance in the presence of dynamic and parameter uncertainties.

1 INTRODUCTION

Robotics technology has been recently exploited in a variety of areas and various robots have been developed to accomplish sophisticated tasks in different fields and applications such as in space exploration, future manufacturing systems, medical technology, etc. In space, robots are expected to complete different tasks, such as capturing a target, constructing a large structure and autonomously maintaining in-orbit systems. The rapid changes and adjustments of the Reconfigurable Manufacturing Systems (RMS) structure must happen in a relatively short time ranging between minutes and hours and not days or weeks. These systems' reconfigurability calls for their components, such as machines and robots to be rapidly and efficiently modifiable to varying demands. In these missions, one fundamental task with the robot would be the tracking of changing workspace, paths, the grasping and the positioning of a target in Cartesian space. It is desirable and cost effective to employ a single versatile robot capable of performing tasks such as inspection, contact operations, assembly (insertion or removal parts), and carrying objects (pick and place). To satisfy such varying environments, a robot with changeable (kinematic structure) configuration is necessary to cope with these requirements and tasks. In the literature, LPV control is used for predefined (fixed) kinematic structure robots achieving specific tracking performance requirements. Methodologies for designing LPV control are given in (Packard, 1994), (Apkarian and Adams, 1997), (P. Gahinet, 1994) and (Kemin Zhou, 1996), where the gain scheduled controller was guaranteed for H_∞ performance of a class of LPV nonlinear systems. In (Zhongwei Yu, 2003) and (Seyed Mahdi Hashemi, 2009) a polytopic gain scheduled H_∞ control is developed for a two Degrees Of Freedom (DOF) robot considering nominal parameters without uncertainties. In (Y.Sun and Liang, 2018), an LPV controller is developed for a large number of affine scheduling dynamic parameters of a 2 DOF robot manipulator without considering uncertainties in their robotic model.

2 KINEMATICS DEVELOPMENT OF A RECONFIGURABLE ROBOT

Development of the general n–DOF Global Kinematic Model (n–GKM) is necessary for supporting any open kinematic robotic arm. The n–GKM model is generated by the variable D–H parameters as proposed in (A. Djuric, 2010). The D–H parameters presented in Table 1 are modeled as variable parameters to accommodate all possible open kinematic structures of a robotic arm. The twist angle variable α_i is limited to five different values, $(0^0, \pm 90^0, \pm 180^0)$,

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Table 1: D–H parameters of the n-GKM model.

i	d_i	θ_i	a_i	α_i
1	$R_1 d_{DH1} + T_1 d_1$	$R_1\theta_1 + T_1\theta_{DH1}$	a_1	$0^0, \pm 180^0, \pm 90^0$
2	$R_2 d_{DH2} + T_2 d_2$	$R_2\theta_2 + T_2\theta_{DH2}$	a_2	$0^0, \pm 180^0, \pm 90^0$
...	$\cdots$	$\cdots$	$\cdots$	$\cdots$
n	$R_n d_{DHn} + T_n d_n$	$R_n\theta_n + T_n\theta_{DHn}$	a_n	$0^0, \pm 180^0, \pm 90^0$

The subscript DHn implies that the d_i or θ_i parameter is constant.

to maintain perpendicularity between joints' coordinate frames. Consequently, each joint has six distinct positive directions of rotation and/or translation. The reconfigurable joint is a hybrid joint that can be configured to be either revolute or prismatic type of motion, according to the required task. For the n–GKM model, a given joint's vector z_{i-1} can be placed in the positive or negative directions of the x, y, and z axis in the Cartesian coordinate frame. This is expressed in (1) and (2):

$$\text{Rotational Joints}: R_i = 1 \quad \text{and} \quad T_i = 0 \tag{1}$$

$$\text{Translational Joints}: R_i = 0 \quad \text{and} \quad T_i = 1 \tag{2}$$

The variables R_i and T_i are used to control the selection of joint type (rotational and/or translational). The orthogonality between the joint's coordinate frames is achieved by assigning appropriate values to the twist angles α_i. Their trigonometric functions are defined as the joint's reconfigurable parameters (K_{Si} & K_{Ci}) and expressed in (3) and (4):

$$K_{si} = \sin(\alpha_i) \tag{3}$$

$$K_{ci} = \cos(\alpha_i) \tag{4}$$

The kinematics of a reconfigurable robot are calculated by multiplication of the all homogeneous matrices from the base frame to the flange frame. The resulting homogeneous transformation matrix for the (n-GKM) model is given in (5):

$$A_{i-1} = \begin{bmatrix} \cos(R_i\theta_i + T_i\theta_{DHi}) & -K_{Ci}\sin(R_i\theta_i + T_i\theta_{DHi}) & K_{Si}\sin(R_i\theta_i + T_i\theta_{DHi}) & a_i\cos(R_i\theta_i + T_i\theta_{DHi}) \\ \sin(R_i\theta_i + T_i\theta_{DHi}) & K_{Ci}\cos(R_i\theta_i + T_i\theta_{DHi}) & -K_{Si}\cos(R_i\theta_i + T_i\theta_{DHi}) & a_i\sin(R_i\theta_i + T_i\theta_{DHi}) \\ 0 & K_{Si} & K_{Ci} & R_i d_{DHi} + T_i d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{5}$$

$i = 1, 2, ..., n$

The workspace of a reconfigurable robot manipulator with similar kinematic structure to the PUMA 560 is calculated and shown in Fig. 1. The resulting workspace shows a union of three layers which indicates the workspace variability property of any reconfigurable manipulator. The three workspace layers are calculated based on turning the third joint into a prismatic one (transnational motion). The variable workspace shown in Fig.1 was generated when the third joint has translated with 0.1, 0.2, 0.3 m.

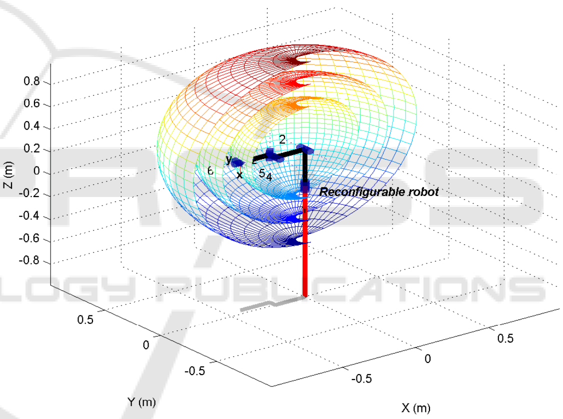

Figure 1: The workspace of a reconfigurable manipulator.

3 DYNAMIC PARAMETER PROPERTIES OF A RECONFIGURABLE ROBOT

The equation of motion of a reconfigurable robot can be described with a set of coupled differential equations in matrix form:

$$\tau = M(q)\ddot{q} + C(q,\dot{q}) + F\dot{q} + G(q) \tag{6}$$

where $q, \dot{q}$ and $\ddot{q}$ are respectively the vectors of generalized joint coordinates, velocities and accelerations. $M(q)$ is the joint–space inertia matrix, $C(q,\dot{q})$ is the Coriolis and centripetal coupling matrix, F is the friction force, $G(q)$ is the gravity loading, and τ is the vector of generalized actuator torques associated with the generalized coordinates q. The effect of varying the configurations of the dynamic parameters such as gravity and inertia are analyzed for the shoulder and elbow joints below.

3.1 Gravity Load Parameter

The gravity load term in (6) is generally a dominant term and present even when the manipulator is stationary or moving slowly. The torque on a joint due to gravity acting on the robot depends strongly on the robot's pose. The torque on the shoulder joint (q_2) is much greater when the robot is stretched out horizontally as shown in Fig.2, which indicates the relation between the gravity torque, shoulder (q_2) and elbow (q_3) joints.

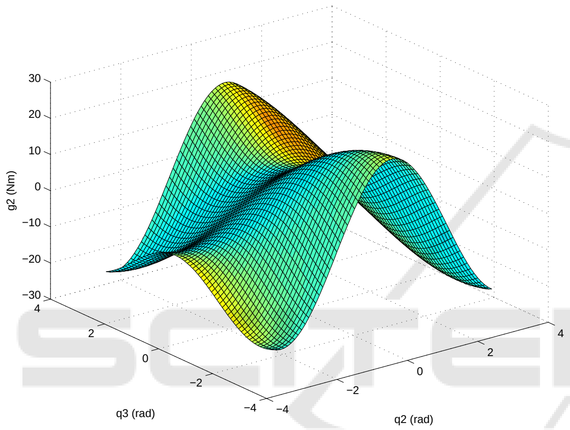

Figure 2: Gravity torque variation due to a different configuration pose (shoulder gravity load).

3.2 Inertia Matrix Parameter

The inertia matrix is a positive definite symmetric matrix in which the matrix elements are functions of the manipulator pose. The inertia matrix has diagonal elements M_{ij} that describe the inertia exerting the joint j by torque $\tau_j = M_{ij} q_j$. The first two diagonal elements, corresponding to the robot's waist and shoulder joints, are large since motion of these joints involves rotation of the heavy upper– and lower–arm links. The off-diagonal terms $M_{ij} = M_{ji}$, $i \neq j$ represent coupling of acceleration from joint j to the torques and forces on joint $j-1$. The variation of the inertia elements as a function of robot configurations are shown in Fig.3. The results indicate a significant variation in the value of M_{11} which changes by a factor of: max(M_{11}(:)) / min(M_{11}(:)) = 1.7683. The off–diagonal represents coupling between the angular acceleration of joint 2 and the torque on joint 1.

4 SYNTHESIS OF GAIN-SCHEDULED CONTROLLER

It has been shown from the analysis above that, the dynamic parameters of a reconfigurable robot are strongly dependent on robot configurations. These variable parameters completely define the operating point of the robot and are assumed to be measured in real time. This motivates the design of a control system that is scheduled with the measured parameters such as $M(q)$ and $F(\dot{q})$ to provide higher performance for large variation in these parameters. Gain scheduling or LPV techniques are used for controlling LPV systems. An LPV controller consists of designing a linear time invariant (LTI) controller that is adapting itself when the operating conditions change. In this control method, the system is assumed to depend affinely on a measured vector of time varying parameters. Assuming on–line measurements of these parameters, they can be fed to the controller to optimize the performance and robustness of the closed loop system.

4.1 Synthesis of LPV Polytopic Controllers

LPV control is applicable to time varying and nonlinear systems whose linearized dynamics are approximated by an affine parameter dependent system in the form of:

$$\begin{aligned} \dot{x} &= A(\theta(t))x + B_1(\theta(t))w + B_2(\theta(t))u \\ z &= C_1(\theta(t))x + D_{11}(\theta(t))w + D_{12}(\theta(t))u \\ y &= C_2(\theta(t))x + D_{21}(\theta(t))w + D_{22}(\theta(t))u \end{aligned} \quad (7)$$

where the time varying parameter vector $\theta(t)$ ranges within a known interval. The system (7) is further

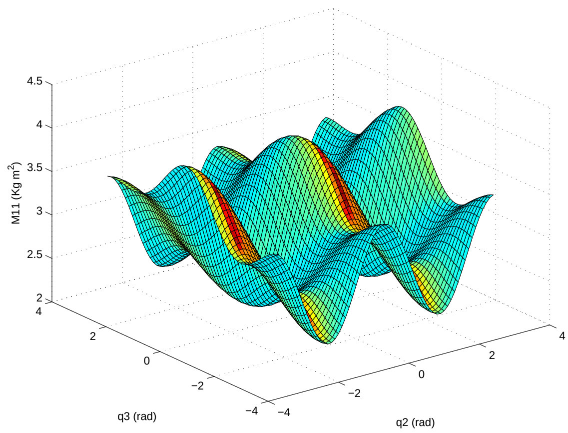

Figure 3: Variation of inertia matrix elements M_{11} with configuration.

assumed to be polytopic:

$$\begin{pmatrix} A(\theta(t)) & B_1(\theta(t)) & B_2(\theta(t)) \\ C_1(\theta(t)) & D_{11}(\theta(t)) & D_{12}(\theta(t)) \\ C_2(\theta(t)) & D_{21}(\theta(t)) & D_{22}(\theta(t)) \end{pmatrix}$$

$$\in Co\left(\begin{pmatrix} A_i & B_{1i} & B_{2i} \\ C_{1i} & D_{11i} & D_{12i} \\ C_{2i} & D_{21i} & D_{22i} \end{pmatrix}, i = 1,2,\cdots,r\right) \quad (8)$$

where $A_i, B_{1i}, \cdots,$ denote the values of $A(\theta(t)), B_1(\theta(t)), \cdots,$ at the polytope vertices. The system matrix dimensions are given by:

$$A(\theta(t)) \in \mathbb{R}^{n\times n}, D_{11}(\theta(t)) \in \mathbb{R}^{p_1\times m_1}, D_{22}(\theta(t)) \in \mathbb{R}^{p_2\times m_2} \quad (9)$$

An LPV system has a quadratic H_∞ performance γ if and only if there exists a single positive definite matrix $X > 0$ such that:

$$\begin{pmatrix} A(\theta(t))^T X + XA(\theta(t)) & XB(\theta(t)) & C(\theta(t))^T \\ B(\theta(t))^T X & -\gamma I & D(\theta(t))^T \\ C(\theta(t)) & D(\theta(t)) & -\gamma I \end{pmatrix} < 0 \quad (10)$$

for all admissible values of the parameter vector $\theta(t)$. An LPV polytopic controller with the same parameter dependence as the system is given as:

$$\begin{aligned} \dot{x}_k &= A_k(\theta(t))x + B_k(\theta(t))w \\ z &= C_k(\theta(t))x + D_k(\theta(t))w \end{aligned} \quad (11)$$

Then, the LPV controller (11) can be employed to assure the quadratic H_∞ performance γ of the resulting closed loop system:

$$\begin{aligned} \dot{x}_{cl} &= A_{cl}(\theta(t))x + B_{cl}(\theta(t))w \\ z &= C_{cl}(\theta(t))x + D_{cl}(\theta(t))w \end{aligned} \quad (12)$$

where

$$A_{cl}(\theta(t)) = \begin{bmatrix} A(\theta(t)) + B_2(\theta(t))D_k(\theta(t))C_2(\theta(t)) & B_2(\theta(t))C_k(\theta(t)) \\ B_k(\theta(t))C_2(\theta(t)) & A_k(\theta(t)) \end{bmatrix}$$

$$B_{cl}(\theta(t)) = \begin{bmatrix} B_1(\theta(t)) + B_2(\theta(t))D_k(\theta(t))D_{21}(\theta(t)) \\ B_k(\theta(t))D_{21}(\theta(t)) \end{bmatrix}$$

$$C_{cl}(\theta(t)) = \begin{bmatrix} C_1(\theta(t)) + D_{12}(\theta(t))D_k(\theta(t))C_2(\theta(t)) \\ D_{12}(\theta(t))C_k(\theta(t)) \end{bmatrix}$$

$$D_{cl}(\theta(t)) = D_{11}(\theta(t)) + D_{12}(\theta(t))D_k(\theta(t))D_{21}(\theta(t)) \quad (13)$$

Synthesis of an LPV control is to ensure the following:

- The resulting polytopic closed loop system (12) is enforced to be stable over the entire parameter polytope and for arbitrary parameter variations.
- The L_2–induced norm of the performance signals are bounded by γ for all possible trajectories within the parameter vector $\theta(t)$.

4.2 Application of LPV Polytopic Control

A Bosch Scara robot with RRT kinematic structure is considered where the first two joints and links are shown in Fig.4 (above). The third joint is considered to be mechanically decoupled from the motions of the other joints. The inertia and Coriolis matrices are derived as follows:

$$M(\theta) = \begin{bmatrix} I_1 + 2I_2\,cos(\theta_2) & I_3 + I_2\,cos(\theta_2) \\ I_3 + I_2\,cos(\theta_2) & I_3 \end{bmatrix} \quad (14)$$

$$C(\theta,\dot{\theta}) = \begin{bmatrix} -2I_2\,sin(\theta_2)\dot{\theta}_1\dot{\theta}_2 - I_2\,sin(\theta_2)\dot{\theta}_1^2 \\ I_2\,sin(\theta_2)\dot{\theta}_1^2 \end{bmatrix} \quad (15)$$

The complete system model of the first two links is shown in Fig.4 (below), including servo motors, and the dynamic cross–coupling torques (Coriolis effects). A spring–damper is introduced to model the torsion stiffness of the robot shaft between each DC motor and the link. The dynamic coupling appears in the joint systems as torques on the joint axes and is considered as an independent disturbance torque. The nominal values of the robot parameters are estimated at the zero position of the first two joints. The state equations of the first link are derived as follows:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{J_{L1}}(K_s x_3 + D_s N_1 x_4 - F_{v1}x_2 + \tau_{DL}) \\ \dot{x}_3 &= N_1 x_4 - x_2 \\ \dot{x}_4 &= \frac{1}{J_{m1}}(K_{m1}x_5 - N_1 k_5 x_3 - F_{v1}x_4) \\ \dot{x}_5 &= \frac{1}{L_{m1}}(-R_{m1}x_5 - K_{m1}x_4 + x_6 + K_{p12}u - K_{p12}x_5) \\ \dot{x}_6 &= -k_{i12}x_5 + k_{i12}u \\ y &= x_1 \end{aligned} \quad (16)$$

The derivations of the state equations for the second link are omitted for brevity but are similar to the derivations above.

4.3 Application of LPV Control

In this subsection, an LPV polytopic control is designed for the LPV linear system given in (16) with two parameters configuration dependent $M(q)$ and $F(\dot{q})$. These two coefficients, satisfy $M(q) \in [M_{\min}, M_{\max}]$ and $F(\dot{q}) \in [F_{\min}, F_{\max}]$. Assuming that $M(q)$ and $F(\dot{q})$ are on–line measurable parameters, the controller is allowed to incorporate these measurements in the same fashion as the system. The resulting LPV controller exploits all the available measurements of $M(q)$ and $F(\dot{q})$ to provide a smooth and automatic gain scheduling. The LPV control structure displayed in Fig.5 consists of the LPV linear system $P(\theta(t))$ and three LPV polytopic controllers. The reference velocity $\dot{q}_d$ and position q_d are fed directly to the LPV feedforward controllers $K_1(\theta(t))$ and $K_2(\theta(t))$, respectively, while the robot configuration position q is fed back to the LPV feedback controller $K_3(\theta(t))$. The LTI performance function W_p is chosen to weight the resulting error e between the reference position q_d and measured joint angle position q. The function $K_3(\theta(t))S(\theta(t))$ is weighted using an LTI input weighting function W_u to ensure robustness against unmodeled dynamics. The LPV control objectives are as follows:

1. To get internal stability of the closed loop system.
2. To enforce the performance and robustness requirements by minimizing the L_2–gain of the closed loop performance channel.

These objectives should be satisfied for the time varying trajectories $M(q)$ and $F(\dot{q})$. The design procedure is performed with describing the LPV system $P(\theta(t))$ (16) by two affine parameter–dependent models. Using the LPV loop shaping procedure, the resulting LPV polytopic system is placed within a polytope convex hull of four vertex systems $Co\{P_i, i = 1,\cdots,4\}$. The vertices P_i are the values of $P(\theta(t))$ at the four vertices (the four corners $P_1,\cdots,P_4$) of the following parameter box:

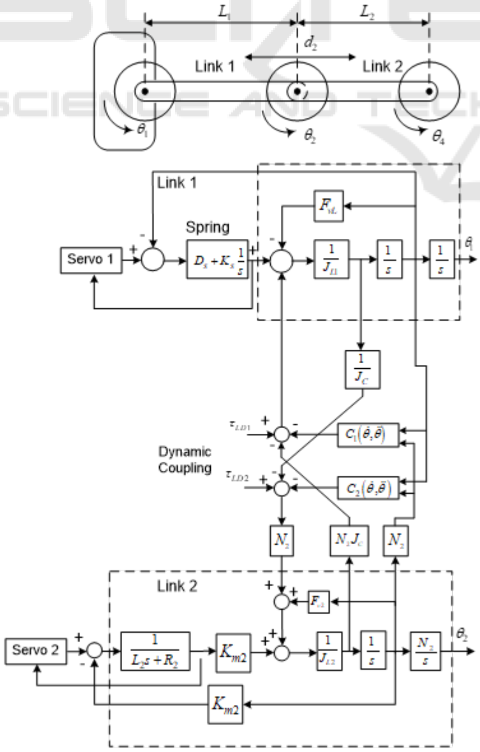

Figure 4: Schematic top view of the Bosch Scara robot in zero position (above). Model of first two links and servo motors of the Bosch Scara robot (below).

$$p_1 = \begin{bmatrix} M_{\max} \\ F_{\max} \end{bmatrix}, \qquad p_2 = \begin{bmatrix} M_{\min} \\ F_{\max} \end{bmatrix}$$
$$p_3 = \begin{bmatrix} M_{\max} \\ F_{\min} \end{bmatrix}, \qquad p_4 = \begin{bmatrix} M_{\min} \\ F_{\min} \end{bmatrix} \tag{17}$$

The LPV synthesis problem illustrated in Fig.5 is solved using the Matlab/Linear Matrix Inequality (LMI) control toolbox. To solve this problem, the input should be parameter independent (Apkarian and Adams, 1997). This condition is satisfied by pre–filtering the control input u with a high pass filter W_f as follows:

$$W_f = \frac{(s + 2\pi * 100 * 0.4)}{(1/2000\, s + 2\pi * 100)} \tag{18}$$

The optimization problem is to find an LPV controller to minimize:

$$\left\| \begin{matrix} W_p S \\ W_u KS \end{matrix} \right\|_{L_2} \tag{19}$$

The performance weighting functions W_p and W_u are designed to enforce the performance and robustness specifications in (19). An appropriate scaling of the system has been performed so that the input is less than or equal to one in magnitude, and therefore a simple input weight $W_u = 1$ is selected. The performance weight is chosen as follows:

$$W_p = \frac{(s/M + w_c)}{s + w_c A} \tag{20}$$

The value $w_c = 10$ has been selected to achieve approximately the desired crossover frequency w_c of 10 rad/s. The steady state error requirement is determined by the selection of the parameter value of A, which is chosen to be $A = 10^{-4}$.

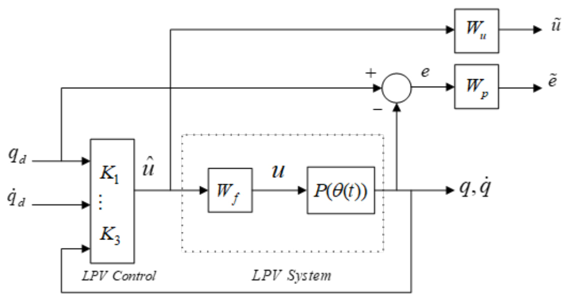

Figure 5: LPV control structure includes the LPV system $P(\theta(t))$, performance weighting function W_p, robustness function W_u, LPV polytopic controllers $(K_1(\theta(t)),\cdots,K_3(\theta(t)))$ and input filter W_f.

5 SYNTHESIS OF ROBUST LPV CONTROLLERS WITH LFT SYSTEM DESCRIPTION

In the following, a robust LPV control with LFT system description is developed for a perturbed time–varying system where the control structure is shown in Fig.6. The robust control objective is to guarantee some closed loop performance $\gamma > 0$ for the L_2–gain of $w_p \to z_p$ against the time varying measured $\Theta(t)$ and uncertain (not measured) parameters Δ. The robust LPV control synthesis for LPV systems (both described with LFT interconnection) should guarantee that the resulting closed loop system is internally stable for all time varying and uncertain parameters. Also, the induced L_2–norm of the closed loop system satisfies:

$$\left\| F_l\left(F_u\left(P, \begin{pmatrix} \Delta & 0 \\ 0 & \Theta(t) \end{pmatrix}\right), F_l\left(K, \Theta(t)\right)\right)\right\|_\infty < \gamma \tag{21}$$

where F_u and F_l are the upper and lower closed loop system. To solve the above robust LPV synthesis problem, the uncertainties of the time varying parameter vector $\Theta(t)$ of both LPV system and controller are gathered into a single uncertainty block with the uncertainty Δ as shown in Fig.6. Here the uncertainty Δ satisfies $\Delta(iw) \in \mathbf{\Delta}$, where $\mathbf{\Delta}$ is defined as:

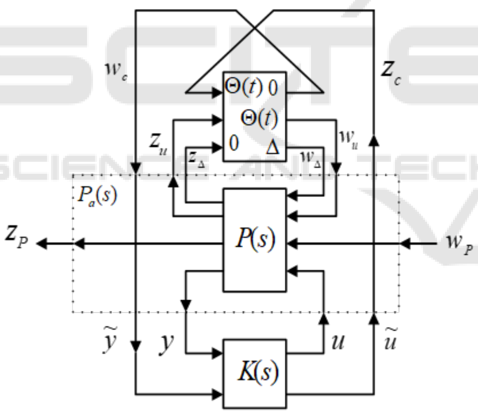

Figure 6: The robust LPV control structure, LPV system $P_s(s)$ with the time varying parameters $\Theta(t)$ and uncertainty block $\mathbf{\Delta}$.

$$\mathbf{\Delta} = \{\Delta = \mathrm{diag}[\delta_1 I_{r_1}, \ldots, \delta_s I_{r_S}, \Delta_1, \ldots, \Delta_F] : \delta_i \in \mathbb{C}, \\ \Delta_j \in \mathbb{C}^{m_j \times m_j}, \|\Delta\| < 1\} \tag{22}$$

We introduce the new LTI state–space description of the system $P_a(s)$:

$$\begin{pmatrix} \dot{x} \\ z_u \\ z_c \\ z_\Delta \\ z_p \\ y \\ \tilde{y} \end{pmatrix} = \left(\begin{array}{c|cccccc} A & B_u & 0 & B_\Delta & B_p & B_1 & 0 \\ \hline C_u & D_{uu} & 0 & D_{u\Delta} & D_{up} & D_{u1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_r \\ C_\Delta & D_{\Delta u} & 0 & D_{\Delta\Delta} & D_{\Delta p} & D_{\Delta 1} & 0 \\ C_p & D_{pu} & 0 & D_{p\Delta} & D_{pp} & D_{p1} & 0 \\ C_1 & D_{1u} & 0 & D_{1\Delta} & D_{1p} & D_{11} & 0 \\ 0 & 0 & I & 0 & 0 & 0 & 0 \end{array}\right) \begin{pmatrix} x \\ w_u \\ w_c \\ w_\Delta \\ w_p \\ u \\ \tilde{u} \end{pmatrix} \tag{23}$$

and the LTI controller with a form of :

$$\dot{x}_c = A_c x_c + B_c \begin{pmatrix} y \\ \tilde{y} \end{pmatrix}$$
$$\begin{pmatrix} u \\ \tilde{u} \end{pmatrix} = C_c x_c + D_c \begin{pmatrix} y \\ \tilde{y} \end{pmatrix} \tag{24}$$

so the closed loop system that maps exogenous input w_p to controlled output z_p can be expressed as:

$$F_u\left(F_l(P_a(s), K(s)), \Delta_G\right) \tag{25}$$

where

$$\Delta_G = \begin{pmatrix} \Delta & 0 & 0 \\ 0 & \Theta(t) & 0 \\ 0 & 0 & \Theta(t) \end{pmatrix}, \quad \Delta \in \mathbb{C}, \Theta(t) \in \mathbb{R} \tag{26}$$

The compatible scaling matrix to the block diagonal uncertainty matrix of (26) is:

$$D_G = \begin{pmatrix} D_\Delta & 0 & 0 \\ 0 & D_{11} & D_{12} \\ 0 & D_{21} & D_{22} \end{pmatrix} \tag{27}$$

where the scaling structure is given as:

$$D_\Delta \Delta = D_\Delta \Delta, \quad \forall \Delta$$
$$\begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix} \begin{pmatrix} \Theta(t) & 0 \\ 0 & \Theta(t) \end{pmatrix} = \begin{pmatrix} \Theta(t) & 0 \\ 0 & \Theta(t) \end{pmatrix} \times \\ \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix}, \forall \Theta(t) \tag{28}$$

The corresponding robust performance problem can be solved by finding a robust LTI control in form of (24) for the nominal LTI system $P_a(s)$ of (23) against the norm bounded structured uncertainties Δ_G. A sufficient condition for robust performance is provided by the small gain theorem. This statement is equivalent to: Given $\gamma > 0$, an uncertainty structure Δ_G, find a scaling matrix D_G and an LTI controller $K(s)$ such that the closed loop system is internally stable and

$$\left\| D_G^{1/2} F_l(P_a(s), K(s)) D_G^{-1/2} \right\|_\infty < \gamma \tag{29}$$

Thus the original LPV control problem is replaced by a robust control problem admitting some degree of conservatism.

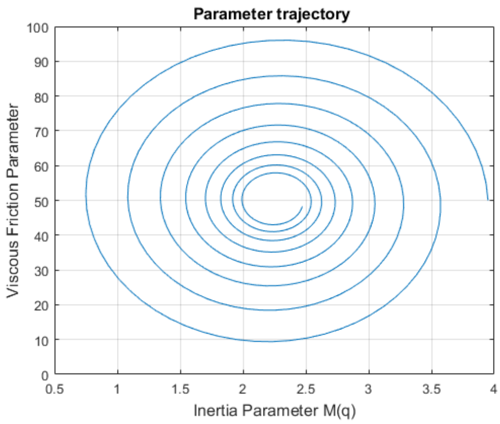

Figure 7: Parameter trajectory.

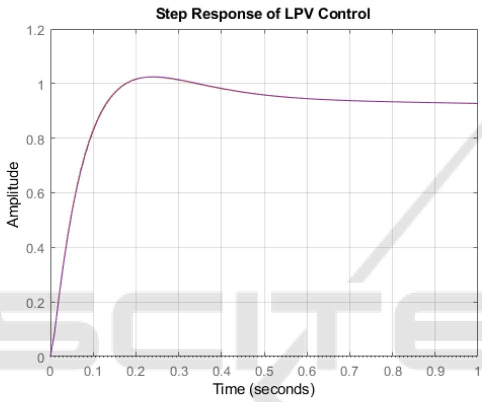

Figure 8: Step response of the LPV closed loop system.

5.1 Simulation Results of LPV and Robust LPV Applications

The two parameter $M(q)$ and $F(\dot{q})$ are frozen to some values in the parameter box specified in (17). The LPV close loop system is simulated for frozen parameters 10%, 30%, 60% and 90% of their nominal values and along the following spiral parameter trajectory shown in Fig.7.

$$M(q) = 2.25 + 1.7e^{-4t}cos(100t) \qquad (30)$$
$$V(q) = 50 + 49e^{-4t}sin(100t)$$

The step response using LPV control (11) in the presence of real and dynamic uncertainties is shown in Fig.8. The step performance is degraded due to the slow response and high steady state error. The step response using the robust LPV control (24) shown in Fig.9 indicates that the performance requirements are satisfied in terms of the steady state and speed of response in the presence of same uncertainties.

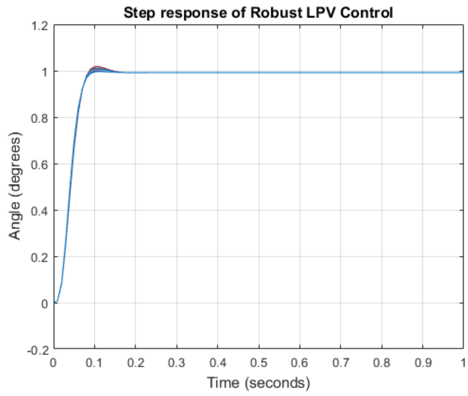

Figure 9: Step response of the LPV closed loop system with the presence of uncertainty $\mathbf{\Delta}$.

6 CONCLUSIONS

In our research, we developed an LPV control for a reconfigurable manipulator with features such as variable twist angles, length links and hybrid (transnational/rotational) joints. The kinematic design parameters, i.e., the D-H parameters, are variable and can generate any required configuration to facilitate a specific application. The dynamic parameters such as inertia, torque and gravity of a reconfigurable robot are dependent on the robot configuration and modeled as time-varying and can be measured online. An LPV controller is developed that adapts itself with varying operating conditions. A robust LPV controllers were developed using LFT control structure for a perturbed time-varying system. The closed loop responses using robust LFT control was proved to satisfy the stability and tracking performance requirements. This research is intended to serve as a foundation for future studies in reconfigurable control systems.

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