The Impact of Demand Correlation on Bullwhip Effect in a Two-stage Supply Chain with Two Retailers

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Abstract: In a two-stage supply chain with two retailers, if they have correlated customer demand, forecasting based on their respective history order might cause significant forecast inaccuracy. Current forecast methods only use supply chain members' own history demand information. However, when there are multi-retailer's having correlated demand, the common forecasting methods ignore the forecast error caused by retailers' interaction. Then, a question comes up that what is the relation between this forecast error and the bullwhip effect. The present paper studies relation of multi-terminals' demand correlation and bullwhip effect in a two-stage supply chain with two retailers. Under centralized or decentralized information, (1) the impact of retailers' demand correlation on retailers'/supplier's bullwhip effect is studied; (2) the contrast of supplier's and retailers' bullwhip effect and the contrast of supplier's/ retailers' bullwhip effect under different information sharing condition are studied. The studies show that multi-terminals' demand correlation is a cause of supply chain's bullwhip effect.

1 BACKGROUND

Today, modern supply chain faces more diversified demands of customers, and more intense horizontal competition among the parties in the same level of a supply chain. Especially in a supply chain producing a homogeneous product, demands of the parties in the same level undoubtedly get affected by their interaction. However, this correlation is not considered in common forecasting methods, such as moving average, exponential smoothing, or empirical forecasting. For example, in one community, there are often more than one supermarket or convenience store, facing the same group of customers and providing products same in price, quality or service. It is obvious that demand of these terminals should be highly correlated. When the manager of such retail terminal makes order based on one of the cited forecast method, if he or she ignores this correlation, the forecast inaccuracy would cause a severe inventory backlog or stock-out.

What is the relationship between retail terminals' forecast inaccuracy caused by their demand correlation and the supply chain's bullwhip effect? Or more specifically, what characters of demand correlation are related to the bullwhip effect? Under what circumstances (such as centralized information or decentralized information) may terminal demand correlation cause bullwhip effect? Although substantial research has been done on bullwhip effect in vertical supply chain, not much research has been performed on bullwhip effect in supply chain having horizontal competition. In the present paper we focus on the relation between demand correlation and bullwhip effect.

2 LITERATURE REVIEW

Lee et al. (1997) prove the existence of bullwhip effect and describe it with AR (1) demand process. Later, Lee et al. (2000) prove that bullwhip effect can be reduced by supply chain information sharing.

Chen et al. (2000a) quantify the bullwhip effect in a two-echelon supply chain with a single manufacturer and a single retailer. They examine the impact of forecasting (moving average forecasting and exponential forecasting) and order lead time on the bullwhip effect, and conclude that bullwhip effect would exist if order lead time is not zero and that the bullwhip effect would become

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more severe with larger order lead time. Later, they extend the conclusion into a multi-stage supply chain, and reveal that information sharing reduce but not eliminate the bullwhip effect.

Luong (2007) use a forecasting procedure that minimizes the expected mean-square forecast error to estimate the lead time demand, and conclude that the variance of order will increase with increasing order lead time. In a later paper, Luong and Phien (2007) study the bullwhip effect based on a AR(2) demand process, and extend it into a AR(p) demand process. They find out that in different ranges of autoregressive coefficients, the relation between lead time and bullwhip effect become complicated that the bullwhip effect does not always exist and does not always increase when lead-time increases.

Li et al. (2006) research the impact of difference demand process on the bullwhip effect, and integrate a general ARIMA (p,d,q) demand process into the model to analyze the validity of the production-smoothing model. They find out the anti-bullwhip effect and the so-called ‘lead-time paradox’, and they also study the value of information sharing in supply chain.

3 MODEL DESCRIPTION

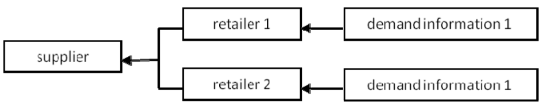


Figure 1: One Supplier and Two Retailers Structure.

In the above supply chain with one supplier and two parallel retailers, there exists demand correlation between the two suppliers. Here, the concept of correlation is:

(1) At any period t, retailer 1’s demand information is determined not only by its own history demand but retailer 2’s history demand.

(2) At any period t, the random error part of retailer 1’s demand information is correlated with that of retailer 2’s. However, the random error part of retailer 1’s demand information at period t1is independent with that of retailer 2’s at a different period t_2. This assumption is in form of

$$\begin{aligned} &Cov(\varepsilon_{i,t}, \varepsilon_{j,t}) = Cov(\varepsilon_{j,t}, \varepsilon_{i,t}) = \sigma_{12}, \\ &Cov(\varepsilon_{i,t_1}, \varepsilon_{j,t_2}) = 0 \end{aligned} \tag{1}$$

Generally, at the end of period t, the two retailers place order $O_{i,t}$ (i=1,2) to the supplier based on their current respective inventory position. The supplier will ship the product once it receives the order. Considering the transportation delay, we assume that the shipment will arrive at the retailer at the end of period (t+L), and here constant L means the same order lead time of the two retailers.

4 DEMAND FORECAST AND ORDER-UP-TO POLICY

As mentioned in the literature review, forecasting methods used in most of the previous research on bullwhip effect include the Moving Average (MA),the Exponential Smoothing (ES) and the optimal forecasting method (or Minimum Mean Square Error forecast, MMSE forecast) (zhang 2004, Heyman and Sobel 2003, Johnson and Thompson 1975, Chen et al. 2000). In practice, the MA is the most common forecasting method. The advantage of this method is that it is easy to use and that it is good enough to determine the current change of trend when accuracy is not strictly requested. The main disadvantage is that the moving averages are lagging indicators because the method assigns the same weight rather than greater weight to the more recent history data, while in practice the more recent changing trend is more important. The ES is relatively more suitable in short-to-medium term forecasting for that it is more sensitive to recent changing trend. However, it is not that easy to use because it can be complex to choose a proper smoothing factor. The optimal forecast method is the MMSE forecast, which is suitable in short-to-medium term forecast, sensitive to recent changing trend, high in forecasting accuracy and the most complex to use in comparison with other methods.

We assume that the two retailers use the MMSE forecast method to estimate the lead time demand. At the end of period t, history demand sequence of retail is $H_{i,t} = \{D_{i,1}, D_{i,2} \ldots D_{i,t-1}, D_{i,t}\}$. Through the MMSE forecast, we can get forecast of demand in next L periods (here L is the lead time), $\hat{F}_{i,t} = \{\hat{D}_{i,t+1}, \hat{D}_{i,t+2} \ldots \hat{D}_{i,t+L-1}, \hat{D}_{i,t+L}\}$, where conditional expect $\hat{D}_{t+i} = E(D_{t+i} | D_t, D_{t-1} \ldots D_0)$.

We assume that the two retailers follow order-up-to inventory policy. Their respective order-up-to points are determined by lead time demand forecast at the end of period t. Then we have $y_{i,t} = \hat{D}_{i,t+1} + \hat{D}_{i,t+2} + \ldots \hat{D}_{i,t+L} + Z_i \hat{\sigma}_{i,t}$, where $\hat{\sigma}_{i,t}$ is an estimate of the standard variance of retailer i’s forecast error during lead time L, and Z_i is a constant measuring retailer i's service level.

5 MODEL NOTATION

We assume that demand of the two retailers are correlated, which is a 2-dimension AR(1) process.

$$\begin{aligned} d_{1,t} &= a_1 + \rho_{11} d_{1,t-1} + \rho_{12} d_{2,t-1} + \varepsilon_{1,t}, d_{2,t} \\ &= a_2 + \rho_{21} d_{1,t-1} + \rho_{22} d_{2,t-1} + \varepsilon_{2,t} \end{aligned} \quad (2)$$

$\varepsilon_{1,t}, \varepsilon_{2,t}$ are i.i.d. following a distribution with mean 0, and satisfies

$$\begin{aligned} Var(\varepsilon_{i,t}) &= \sigma_{ii}^2, Cov(\varepsilon_{i,t}, \varepsilon_{j,t}) \\ &= Cov(\varepsilon_{j,t}, \varepsilon_{i,t}) = \sigma_{12}, Cov(\varepsilon_{i,t_1}, \varepsilon_{j,t_2}) = 0 \end{aligned} \quad (3)$$

It is obvious that expression (2) becomes two independent AR(1) processes when $\rho_{12} = \rho_{21} = \sigma_{12} = 0$.

For the stationary of AR process, we should choose proper $\rho_{11}, \rho_{12}, \rho_{21}, \rho_{22}$ to make the roots of $(x - \rho_{11})(x - \rho_{22}) = \rho_{12}\rho_{21}$ locate in the unit circle.

Let μ_1, μ_2 denote respectively the mean of the two retailers' demand, we have

$$\begin{aligned} \mu_1 &= \frac{(1-\rho_{22})a_1 + \rho_{12} a_2}{(1-\rho_{11})(1-\rho_{22}) - \rho_{12}\rho_{21}} \\ \mu_2 &= \frac{(1-\rho_{11})a_2 + \rho_{21} a_1}{(1-\rho_{11})(1-\rho_{22}) - \rho_{12}\rho_{21}} \end{aligned} \quad (4)$$

To ensure the positive value of μ1 and μ2, the following condition should be satisfied:

$$\begin{aligned} & a_1 > 0, a_2 > 0, (1-\rho_{22}) + \rho_{12} a_2 > 0, \\ & (1-\rho_{11}) + \rho_{21} a_1 > 0 \end{aligned} \quad (5)$$

To simplify expression (1), we make $z_{i,t} = d_{i,t} - \mu_i$, and (1) can be transferred as

$$\begin{aligned} z_{1,t} &= \rho_{11} z_{1,t-1} + \rho_{12} z_{2,t-1} + \varepsilon_{1,t}, \\ z_{2,t} &= \rho_{21} z_{1,t-1} + \rho_{22} z_{2,t-1} + \varepsilon_{2,t} \end{aligned} \quad (6)$$

Denote,

$$Z_{t+i} = \begin{pmatrix} z_{1,t+i} \\ z_{2,t+i} \end{pmatrix}, Y_{t+i} = \begin{pmatrix} y_{1,t+i} \\ y_{2,t+i} \end{pmatrix},$$

$$A = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}, \mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix},$$

$$D_{t+i} = \begin{pmatrix} d_{1,t+i} \\ d_{2,t+i} \end{pmatrix} = Z_{t+i} + \mu, \varepsilon_{t+i} = \begin{pmatrix} \varepsilon_{1,t+i} \\ \varepsilon_{2,t+i} \end{pmatrix}, \quad (7)$$

$$Var(\varepsilon_{t+i}) = \begin{pmatrix} \sigma_{11}^2 & \sigma_{12} \\ \sigma_{21} & \sigma_{22}^2 \end{pmatrix} = \Omega$$

We get the matrix form of (1) as below, where the characteristic root of A, $|\lambda| < 1$, or matrix E-A is invertible.

$$Z_{t+i+1} = AZ_{t+i} + \varepsilon_{t+i+1}, \forall i > 0 \quad (8)$$

5.1 Bullwhip Effect of the Two Retailers and the Supplier with Centralized Demand Information

Centralized demand information means that retailers share its history demand sequence $H_{i,t}$ with each other, so each retailer can forecast and make order decision based on both retailers' history demand.

We substitute $Z_{t+L-1} = AZ_{t+L-2} + \varepsilon_{t+L-1}$ for $Z_{t+L} = AZ_{t+L-1} + \varepsilon_{t+L}$ in expression (6), and continue this iteration to the end:

$$\begin{aligned} Z_{t+L} &= AZ_{t+L-1} + \varepsilon_{t+L} = A^2 Z_{t+L-2} + A\varepsilon_{t+L-1} + \varepsilon_{t+L} \\ &= \ldots \\ &= A^L Z_t + A^{L-1}\varepsilon_{t+1} + A^{L-2}\varepsilon_{t+2} + \ldots A\varepsilon_{t+L-1} + \varepsilon_{t+L} \end{aligned} \quad (9)$$

From $E(\varepsilon_{t+i}) = 0$, we can have $E(Z_{t+L}) = A^L Z_t$. Because for any ARMA process, MMSE forecast of demand of period t+i equals its conditional expectation, the MMSE forecasts of Z_{t+L}, $Z_{t+L-1} \ldots Z_{t+1}$ are

$$\bar{Z}_{t+L} = A^L Z_t, \bar{Z}_{t+L-1} = A^{L-1} Z_t \ldots, \bar{Z}_{t+1} = AZ_t \quad (10)$$

Then, the lead time demand forecast is

$$\begin{aligned} \sum_{i=1}^{L} \bar{D}_{t+i} &= \sum_{i=1}^{L} (\bar{Z}_{t+i} + \mu) \\ &= L\mu + (A + A^2 + \ldots + A^L) Z_t \\ &= L\mu + (E - A)^{-1}(A - A^{L+1}) Z_t \end{aligned} \quad (11)$$

The lead time demand forecast error is

$$\begin{aligned} & \sum_{i=1}^{L} (D_{t+i} - \bar{Z}_{t+i}) = \sum_{i=1}^{L} (Z_{t+i} - \bar{Z}_{t+i}) \\ & = (A^{L-1} + A^{L-2} + \ldots + E)\varepsilon_{t+1} + (A^{L-2} + \ldots + E)\varepsilon_{t+2} \\ & + \ldots + (A + E)\varepsilon_{t+L-1} + \varepsilon_{t+L} \end{aligned} \quad (12)$$

Variance of lead time demand forecast error is

$$\begin{aligned} & Var \sum_{i=1}^{L} (D_{t+i} - \bar{D}_{t+i}) = Var \sum_{i=1}^{L} (Z_{t+i} - \bar{Z}_{t+i}) \\ & = (A^{L-1} + A^{L-2} + \ldots + E)\Omega(A^{L-1} + A^{L-2} + \ldots + E)' \\ & + (A^{L-2} + \ldots + E)\Omega(A^{L-2} + \ldots + E)' \\ & + \ldots + (A + E)\Omega(A + E)' + \Omega \end{aligned} \quad (13)$$

Denote $O_t = \begin{pmatrix} o_{1,t} \\ o_{2,t} \end{pmatrix}$ as the matrix form of retailers' order quantity, and we have

Table 1: Parameter Description.

Parameter	Description
$d_{1,t}, d_{2,t}$	The demand of Retailer *1*,*2*at period t
$\hat{d}_{1,t}, \hat{d}_{2,t}$	The demand forecast of Retailer *1*, *2* at period *t*
$\varepsilon_{1,t}, \varepsilon_{2,t}$	The random variable of demand information faced by Retailer *1*, *2* respectively at period *t*. Here σ_{ii}^2 denote the variance of retailer i's random variable，and σ_{ij}, σ_{ji} denote the correlation of two retailers' random variable
$\varepsilon_{s,t}$	The random variable of demand information faced by Supplier， let $\varepsilon_{s,t} = \varepsilon_{1,t} + \varepsilon_{2,t}$
ρ_{11}, ρ_{22}	The autocorrelation coefficient of Retailer *1*, *2*
ρ_{12}, ρ_{21}	The correlation coefficient describing the correlation between Retailer 1and 2
$o_{1,t}, o_{2,t}$	The order quantity of Retail *1*, *2* at period *t*
$\hat{o}_{1,t}, \hat{o}_{2,t}$	The forecast order quantity of Retailer *1*, *2* at period *t*
$o_{s,t}$	The order quantity of Supplier at period *t*, let $o_{s,t} = o_{1,t} + o_{2,t}$
$\hat{o}_{s,t}$	The forecast order quantity of Supplier at period *t*, let $\hat{o}_{s,t} = \hat{o}_{1,t} + \hat{o}_{2,t}$
M_1^2, M_2^2, M_s^2	The measure of Bullwhip Effect of Retailer *1*, *2* and Supplier

$$\begin{aligned} O_t &= Y_t - Y_{t-1} + D_t = \sum_{i=1}^{L} \bar{D}_{t+i} - \sum_{i=1}^{L} \bar{D}_{t+i-1} + D_t \\ &= \sum_{i=1}^{L} (\bar{Z}_{t+i} + \mu) - \sum_{i=1}^{L} (\bar{Z}_{t+i-1} + \mu) + Z_t + \mu \\ &= A^{L+1} Z_{t-1} + (E-A)^{-1}(E - A^{L+1})\varepsilon_t + \mu \end{aligned} \tag{14}$$

Retailers' forecast order quantity is

$$\hat{O}_t = A^{L+1} Z_{t-1} + \mu \tag{15}$$

Variance of retailers' order quantity error is

$$Var(O_t - \hat{O}_t) = Var((E-A)^{-1}(E - A^{L+1})\varepsilon_t) \tag{16}$$

Let $B = (E-A)^{-1}(E - A^{L+1})$， and then we have

$$Var(O_t - \hat{O}_t) = BE(\varepsilon_t \varepsilon_t^{'})B^{'} = B \begin{pmatrix} \sigma^2_{11} & \sigma_{12} \\ \sigma_{21} & \sigma^2_{22} \end{pmatrix} B^{'} \tag{17}$$

Assume that retailers' order lead time L=1, and we have

$$\begin{aligned} &B = (E-A)^{-1}(E-A^2) = E + A = \begin{pmatrix} 1+\rho_{11} & \rho_{12} \\ \rho_{21} & 1+\rho_{22} \end{pmatrix} \\ &Var(O_t - \hat{O}_t) \\ &= \begin{pmatrix} 1+\rho_{11} & \rho_{12} \\ \rho_{21} & 1+\rho_{22} \end{pmatrix} \begin{pmatrix} \sigma^2_{11} & \sigma_{12} \\ \sigma_{21} & \sigma^2_{22} \end{pmatrix} \begin{pmatrix} 1+\rho_{11} & \rho_{21} \\ \rho_{12} & 1+\rho_{22} \end{pmatrix} \end{aligned} \tag{18}$$

Hence, we get the Bullwhip Effect of the two retailers as the below:

$$\begin{aligned} M_1^2 &= \frac{Var(O_{1,t} - \hat{O}_{1,t})}{Var\varepsilon_{1,t}} \\ &= \frac{(1+\rho_{11})^2 \sigma_{11}^2 + 2\rho_{12}(1+\rho_{11})\sigma_{12} + \rho_{12}^2 \sigma_{22}^2}{\sigma_{11}^2}, \\ M_2^2 &= \frac{Var(O_{2,t} - \hat{O}_{2,t})}{Var\varepsilon_{2,t}} \\ &= \frac{(1+\rho_{22})^2 \sigma_{22}^2 + 2\rho_{21}(1+\rho_{22})\sigma_{21} + \rho_{21}^2 \sigma_{11}^2}{\sigma_{22}^2} \end{aligned} \tag{19}$$

Also, the Bullwhip Effect of the supplier is

$$\begin{aligned} M_s^2 &= \frac{Var(o_{s,t} - \hat{o}_{s,t})}{Var(\varepsilon_{s,t})} \\ &= \frac{Var(o_{1,t} + o_{2,t} - \hat{o}_{1,t} - \hat{o}_{2,t})}{Var(\varepsilon_{1,t} + \varepsilon_{2,t})} \\ &= \begin{bmatrix} (1+\rho_{11}+\rho_{21})^2 \sigma_{11}^2 \\ +(1+\rho_{22}+\rho_{12})^2 \sigma_{22}^2 \\ +2(1+\rho_{11}+\rho_{21})(1+\rho_{22}+\rho_{12})\sigma_{12} \end{bmatrix} \Big/ \begin{pmatrix} \sigma_{11}^2 + \sigma_{22}^2 \\ +2\sigma_{12} \end{pmatrix} \end{aligned} \tag{20}$$

5.2 Bullwhip Effect of the Two Retailers and the Supplier with Decentralized Demand Information

Decentralized demand information means that retailers take each other as competitor and they do not share information of history demand sequence. Based on this assumption, each retailer can forecast and make order decision based on only its own history demand.

According to expression (6), we have

$$\begin{aligned} z_{1,t} &= \rho_{11} z_{1,t-1} + \rho_{12} z_{2,t-1} + \varepsilon_{1,t} \\ &= \rho_{11} z_{1,t-1} + \rho_{12}(\rho_{21} z_{1,t-2} + \rho_{22} z_{2,t-2} + \varepsilon_{2,t-1}) + \varepsilon_{1,t} \\ &= \rho_{11} z_{1,t-1} + \rho_{12}\rho_{21} z_{1,t-2} + \rho_{12}\rho_{22} z_{2,t-2} + \rho_{12}\varepsilon_{2,t-1} + \varepsilon_{1,t} \end{aligned} \tag{21}$$

Now substitute $z_{1,t-1} = \rho_{11} z_{1,t-2} + \rho_{12} z_{2,t-2} + \varepsilon_{1,t-1}$ for $z_{2,t-2}$ in the equation above, and we have

$$\begin{aligned} z_{1,t} &= (\rho_{11} + \rho_{22}) z_{1,t-1} \\ &+ (\rho_{12}\rho_{21} - \rho_{11}\rho_{22}) z_{1,t-2} \\ &+ \varepsilon_{1,t} - \rho_{22}\varepsilon_{1,t-1} + \rho_{12}\varepsilon_{2,t-1} \end{aligned} \tag{22}$$

Following the same procedure, we have

$$\begin{aligned} z_{2,t} &= (\rho_{11}+\rho_{22})z_{2,t-1} \\ &+(\rho_{12}\rho_{21}-\rho_{11}\rho_{22})z_{2,t-2} \\ &+\varepsilon_{2,t}-\rho_{11}\varepsilon_{2,t-1}+\rho_{21}\varepsilon_{1,t-1} \end{aligned} \tag{23}$$

Let

$$\beta_1=\rho_{11}+\rho_{22}, \beta_2=\rho_{12}\rho_{21}-\rho_{11}\rho_{22}, \\ v_{i,t}=\varepsilon_{i,t}-\rho_{jj}\varepsilon_{i,t-1}+\rho_{ij}\varepsilon_{j,t-1}$$

equation (22) and (23) become

$$z_{i,t}=\beta_1 z_{i,t-1}+\beta_2 z_{i,t-2}+v_{i,t} \tag{24}$$

Notice that each retailer only has its own history demand sequence. From equation (22) and (23), retailer i can estimate the auto-regression term in the equation and the auto-correlation part in the error term, while retailer i cannot estimate the correlation part in the error term. Hence, neither of the retailers can forecast the future demand based on its own history demand sequence.

Lemma 1

Retailer i can use a stable and invertible ARMA process to model its history demand.

$$z_{i,t}=\beta_1 z_{i,t-1}+\beta_2 z_{i,t-2}+\xi_{i,t}-\theta_i\xi_{i,t-1}, \forall i=1,2$$

$$\theta_i=\frac{-Var(v_{i,t})+\sqrt{\left[Var(v_{i,t})\right]^2-4\left[Cov(v_{i,t},v_{i,t-1})\right]^2}}{2Cov(v_{i,t},v_{i,t-1})},$$

where $|\theta_i|<1$, and the error term satisfies

（1） $E\xi_{i,t}=0$

（2） $E(\xi_{i,t}\xi_{i,t'})=0, \forall t\neq t'$, $E(\xi_{i,t}^2)=\sigma_{i\xi}^2=Var(v_{i,t})/(1+\theta_i^2)$

Based on Lemma 1, （22）and（23）become

$$z_{1,t}=\beta_1 z_{1,t-1}+\beta_2 z_{1,t-2}+\xi_{1,t}-\theta_1\xi_{1,t-1}, z_{2,t} \\ =\beta_1 z_{2,t-1}+\beta_2 z_{2,t-2}+\xi_{2,t}-\theta_2\xi_{2,t-1},$$

where $\xi_{1,t},\xi_{2,t}$ are i.i.d.

Assume that retailers' order lead time L=1, and we can get the lead time demand forecast and forecast error as below

$$\begin{aligned} \bar d_{i,t} &= \bar z_{i,t}+\mu_i=E(z_{i,t}|H_{i,t})+\mu_i \\ &=\beta_1 z_{i,t-1}+\beta_2 z_{i,t-2}-\theta_i\xi_{i,t-1}+\mu_i \\ d_{i,t}-\bar d_{i,t} &= z_{i,t}-\bar z_{i,t}=\xi_{i,t}, \forall i=1,2 \end{aligned} \tag{25}$$

Hence, we get the variance of two retailers' order lead time demand forecast error

$$Var(d_{i,t+1}-\hat d_{i,t+1})=\sigma_{i\xi}^2, \forall i=1,2 \tag{26}$$

Under decentralized information，retailer i's order quantity is

$$\begin{aligned} o_{i,t} &= \bar d_{i,t}-\bar d_{i,t-1}+d_{i,t}=\bar z_{i,t}-\bar z_{i,t-1}+z_{i,t}+\mu_i \\ &=\bar z_{i,t}+\xi_{i,t}+\mu_i=(\beta_1^2+\beta_2)z_{i,t-1}+\beta_1\beta_2 z_{i,t-2} \\ &+\beta_1\theta_i\xi_{i,t-1}+(1+\beta_1-\theta_i)\xi_{i,t}+\mu_i \end{aligned} \tag{27}$$

Retailers' forecast order quantity is

$$\begin{aligned} \hat o_{i,t} &= (\beta_1^2+\beta_2)z_{i,t-1}+\beta_1\beta_2 z_{i,t-2} \\ &+\beta_1\theta_i\xi_{i,t-1}+\mu_i \end{aligned} \tag{28}$$

Variance of retailers' order quantity error is

$$\begin{aligned} o_{i,t}-\hat o_{i,t} &= (1+\beta_1-\theta_i)\xi_{i,t} \\ Var(o_{i,t}-\hat o_{i,t}) &= (1+\beta_1-\theta_i)^2\sigma_{i\xi}^2 \end{aligned} \tag{29}$$

Hence, we get the Bullwhip Effect of the two retailers as below

$$M_i^2=\frac{Var(o_{i,t}-\hat o_{i,t})}{Var\varepsilon_{i,t}}=\frac{(1+\beta_1-\theta_i)^2\sigma_{i\xi}^2}{\sigma_{ii}^2} \tag{30}$$

Also, the Bullwhip Effect of the supplier is

$$\begin{aligned} M_s^2 &= \frac{Var(o_{s,t}-\hat o_{s,t})}{Var(\varepsilon_{s,t})}=\frac{Var(o_{1,t}+o_{2,t}-\hat o_{1,t}-\hat o_{2,t})}{Var(\varepsilon_{1,t}+\varepsilon_{2,t})} \\ &= \frac{(1+\beta_1-\theta_1)^2\sigma_{1\xi}^2+(1+\beta_1-\theta_2)^2\sigma_{2\xi}^2}{\sigma_{11}^2+\sigma_{22}^2+2\sigma_{12}} \end{aligned} \tag{31}$$

6 BULLWHIP EFFECT ANALYSIS AND COMPARISION

In this sector, we analyze the impact of demand correlation on retailer and supplier Bullwhip Effect. To eliminate the possible influence of other parameters, we assume that $\rho_{11}=\rho_{22}$, $\rho_{12}=\rho_{21}$, $\sigma_{11}=\sigma_{22}$. This assumption is reasonable in practice, because in the same local market there are often two retailers similar in both market share and products sold.

6.1 Numerical Analysis of M_i^2 under Centralized Information

With conditions of centralized information, L=1, and MMSE forecasting, the two retailers face bullwhip effect as below

$$
\begin{aligned}
M_1^2 &= \frac{Var(o_{1,t}-\hat{o}_{1,t})}{Var\varepsilon_{1,t}} \\
&= \frac{(1+\rho_{11})^2\sigma_{11}^2+2\rho_{12}(1+\rho_{11})\sigma_{12}+\rho_{12}^2\sigma_{22}^2}{\sigma_{11}^2} \\
M_2^2 &= \frac{Var(o_{2,t}-\hat{o}_{2,t})}{Var\varepsilon_{2,t}} \\
&= \frac{(1+\rho_{22})^2\sigma_{22}^2+2\rho_{21}(1+\rho_{22})\sigma_{12}+\rho_{21}^2\sigma_{11}^2}{\sigma_{22}^2}
\end{aligned}
\tag{32}
$$

When $\rho_{11}=\rho_{22}$, $\rho_{12}=\rho_{21}$, $\sigma_{11}=\sigma_{22}$, we get $M_1^2=M_2^2$.

Then, it is obvious that

$$
\begin{aligned}
\frac{dM_i^2}{d\rho_{ij}} &= \frac{2(1+\rho_{ii})\sigma_{ij}}{\sigma_{ii}^2}+\frac{2\sigma_{jj}^2}{\sigma_{ii}^2}\rho_{ij}, \\
\frac{dM_i^2}{d\sigma_{ij}} &= \frac{2(1+\rho_{ii})}{\sigma_{ii}^2}\rho_{ij}, i\neq j, and\ i,j=1\ or\ 2
\end{aligned}
\tag{33}
$$

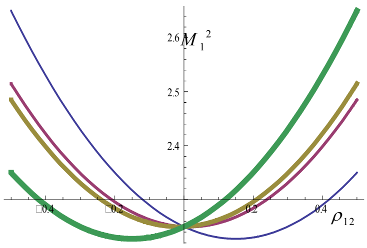
Let $\frac{dM_i^2}{d\rho_{ij}}=0$, we get $\rho_{ij}=\frac{-\sigma_{ij}(1+\rho_{ii})}{\sigma_{jj}^2}$。

From $|\rho_{ii}|<1, |\rho_{ij}|<1$, we get $0<1+\rho_{ii}<2$, so $dM_i^2/d\sigma_{ij}$ and ρ_{ij} have the same sign.

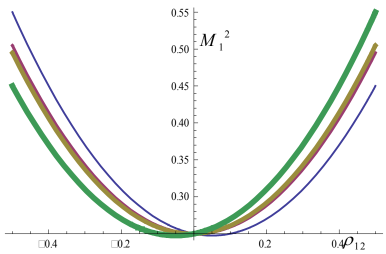
Figure 2, Figure 3 and Figure 4 display the relation between Bullwhip Effect and demand correlation under centralized information.

Let $\sigma_{11}=\sigma_{22}=10, \rho_{11}=0.5,-0.5$. Notice that to ensure the stability, let $\rho_{12}\in(-0.5,0.5)$. M_1^2 varies with ρ_{12} as shown in Figure2.

Let $\sigma_{11}=\sigma_{22}=10, \rho_{11}=\rho_{22}=0.5,-0.5$. To ensure the stability, $\rho_{12}\in(-0.5,0.5)$. M_s^2 varies with ρ_{ij} as shown in Figure3.

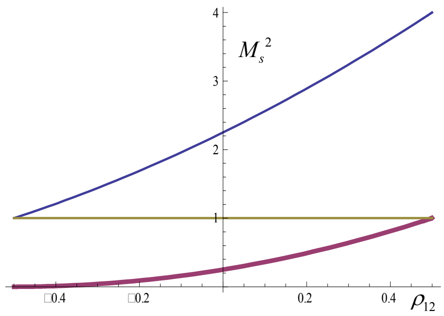


$\rho_{11}=0.5$, $\sigma_{12}=-10,-1,1,10$ (thin->thick)



$\rho_{11}=-0.5$, $\sigma_{12}=-10,-1,1,10$ (thin->thick)

Figure 2: Retailer's Bullwhip Effect under Centralized demand information.



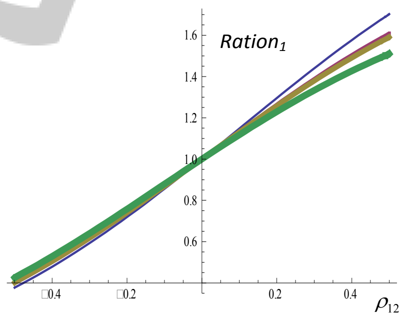
$\rho_{11}=\rho_{22}=0.5,-0.5$ (thin->thick)

Figure 3: Supplier's Bullwhip Effect under Centralized demand information.

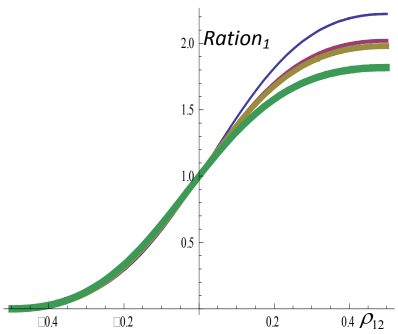
Next, compare the retailers' bullwhip effect with the supplier's under centralized demand information, when $\rho_{11}=\rho_{22}$, $\rho_{12}=\rho_{21}$, $\sigma_{11}=\sigma_{22}$:

$$
\begin{aligned}
Ratio_1 &= \frac{M_s^2}{M_i^2} \\
&= \frac{(1+\rho_{12}+\rho_{22})^2}{[(1+\rho_{11})^2\sigma_{11}^2+2\rho_{12}(1+\rho_{11})\sigma_{12}+\rho_{12}^2\sigma_{22}^2]/\sigma_{11}^2}
\end{aligned}
\tag{34}
$$

Let $\sigma_{11}=\sigma_{22}=10, \rho_{11}=\rho_{22}=0.5,-0.5$, $Ratio_1$ varies with ρ_{12} as shown in Figure 4.



$\rho_{11}=0.5$, $\sigma_{12}=-10,-1,1,10$ (thin->thick)



$\rho_{11}=-0.5$, $\sigma_{12}=-10,-1,1,10$ (thin->thick)

Figure 4: Bullwhip Effect Contrast under Centralized demand information.

6.2 Numerical Analysis of M_i^2 under Decentralized Information

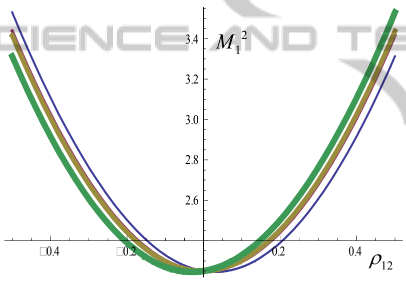
With conditions of decentralized information, L=1, and MMSE forecasting, the two retailers face bullwhip effect as below

$$M_i^2=\frac{Var(o_{i,t}-\hat{o}_{i,t})}{Var\varepsilon_{i,t}}=\frac{(1+\beta_1-\theta_i)^2\sigma_{i\xi}^2}{\sigma_{ii}^2} \tag{35}$$

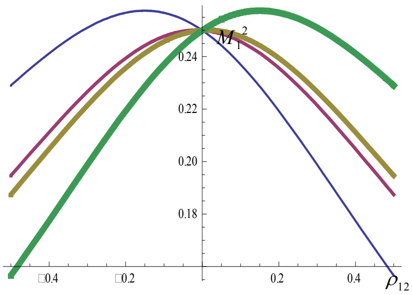
where

$$\begin{aligned}&\beta_1=\rho_{11}+\rho_{22}\\&\theta_i=\frac{-V(v_{i,t})+\sqrt{\left[V(v_{i,t})\right]^2-4\left[Cov(v_{i,t},v_{i,t-1})\right]^2}}{2Cov(v_{i,t},v_{i,t-1})},|\theta_i|<1\\&\sigma_{i\xi}^2=\frac{V(v_{i,t})+\sqrt{\left[V(v_{i,t})\right]^2-4\left[Cov(v_{i,t},v_{i,t-1})\right]^2}}{2}\end{aligned} \tag{36}$$

When $\sigma_{11}=\sigma_{22}=10,\rho_{11}=\rho_{22}=0.5,-0.5$, M_1^2 varies with ρ_{12} as shown in Figure 5.



ρ_{11}=0.5， σ_{12}=-10,-1,1,10 （thin->thick）



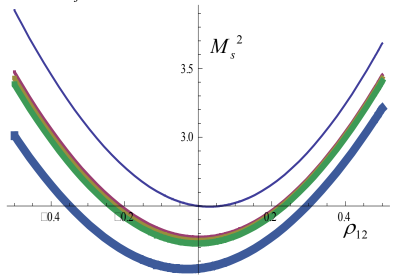
ρ_{11}=-0.5， σ_{12}=-10,-1,1,10 （thin->thick）

Figure 5: Retailer's Bullwhip Effect under Decentralized demand information.

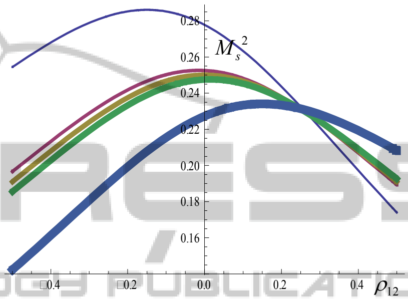
Under decentralized information, bullwhip effect of the supplier is

$$M_s^2=\frac{(1+\beta_1-\theta_1)^2\sigma_{1\xi}^2+(1+\beta_1-\theta_2)^2\sigma_{2\xi}^2}{\sigma_{11}^2+\sigma_{22}^2+2\sigma_{12}}=\frac{2(1+\beta_1-\theta_1)^2\sigma_{1\xi}^2}{\sigma_{11}^2+\sigma_{22}^2+2\sigma_{12}} \tag{37}$$

When $\sigma_{11}=\sigma_{22}=10,\rho_{11}=\rho_{22}=0.5,-0.5,\rho_{12}=\rho_{21}$，$M_s^2$ varies with ρ_{ij} as shown in Figure 6.



ρ_{11}=0.5， σ_{12}=-10,-1,0,1,10 （thin->thick）



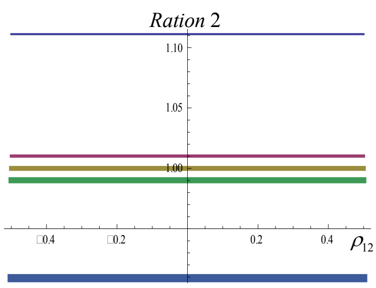
ρ_{11}=-0.5， σ_{12}=-10,-1,0,1,10 （thin->thick）

Figure 6: Supplier's Bullwhip Effect under Decentralized demand information.

Under decentralized information, if ρ_{11}=ρ_{22}, ρ_{12}=ρ_{21}, σ_{11}=σ_{22}, we have M_1^2=M_2^2 .Now we compare the retailers' bullwhip effect with the supplier's.

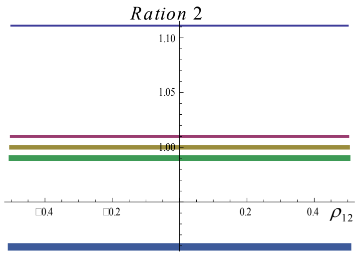
$$\begin{aligned}Ration_2&=\frac{M_s^2}{M_i^2}=\frac{M_s^2}{M_1^2}\\&=\frac{[(1+\beta_1-\theta_1)^2\sigma_{1\xi}^2+(1+\beta_1-\theta_2)^2\sigma_{2\xi}^2]/(\sigma_{11}^2+\sigma_{22}^2+2\sigma_{12})}{[(1+\beta_1-\theta_1)^2\sigma_{1\xi}^2]/\sigma_{11}^2}\\&=\frac{2\sigma_{11}^2}{\sigma_{11}^2+\sigma_{22}^2+2\sigma_{12}}\end{aligned} \tag{38}$$

When $\sigma_{11}=\sigma_{22}=10,\rho_{11}=\rho_{22}=0.5,-0.5,\rho_{12}=\rho_{21}$, $Ration_2$ varies with ρ_{ij} as shown in Figure 7.



ρ_{11}=0.5， σ_{12}=-10,-1,0,1,10 （thin->thick）

Figure 7: Bullwhip Effect Contrast under Decentralized demand information.



ρ_{11}=-0.5， σ_{12}=-10,-1,0,1,10 （thin->thick）

Figure 7: Bullwhip Effect Contrast under Decentralized demand information (cont.).

6.3 Bullwhip Effect Contrast between Centralized and Decentralized Demand Information

In this section, we analyze the retailers'/supplier's bullwhip effect contrast between centralized and decentralized information.

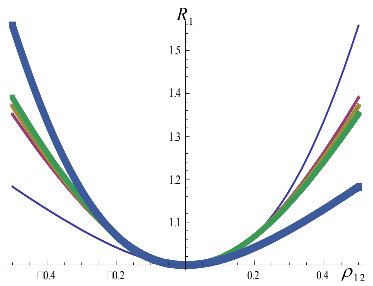
Let Ri represent the ratio of retailer i's bullwhip effect under centralized information to that under decentralized information

$$R_i = \frac{M_i^2(decentralized)}{M_i^2(centralized)} = \frac{[(1+\beta_1-\theta_i)^2\sigma_{i\xi}^2]/\sigma_{ii}^2}{[(1+\rho_{ii})^2\sigma_{ii}^2+2\rho_{ij}(1+\rho_{ii})\sigma_{ij}+\rho_{ij}^2\sigma_{jj}^2]/\sigma_{ii}^2} = \frac{(1+\beta_1-\theta_i)^2\sigma_{i\xi}^2}{(1+\rho_{ii})^2\sigma_{ii}^2+2\rho_{ij}(1+\rho_{ii})\sigma_{ij}+\rho_{ij}^2\sigma_{jj}^2} \tag{39}$$

When $\sigma_{11}=\sigma_{22}=10, \rho_{11}=\rho_{22}=0.5,-0.5, \rho_{12}=\rho_{21}$, we get $R_1=R_2$, and R_1 varies with ρ_{12} as shown in Figure 8.

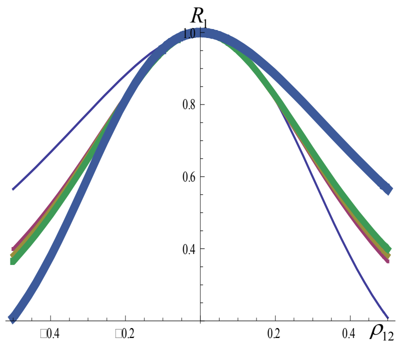
Let S represent the ratio of supplier's bullwhip effect under centralized information to that under decentralized information

$$S=\frac{M_s^2(decentralized)}{M_s^2(centralized)}=\frac{(1+\beta_1-\theta_1)^2\sigma_{1\xi}^2}{(1+\rho_{11}+\rho_{21})^2(\sigma_{11}^2+\sigma_{12})} \tag{40}$$



ρ_{11}=0.5， σ_{12}=-50,-5,0,5,50 （thin->thick）

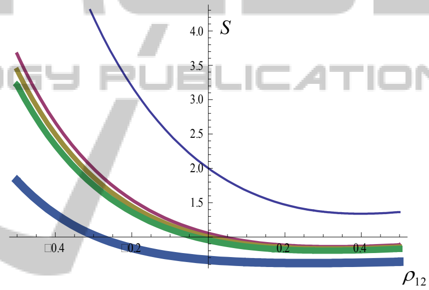
Figure 8: Retailer's B.E. Contrast Between Centralized And Decentralized D-I.



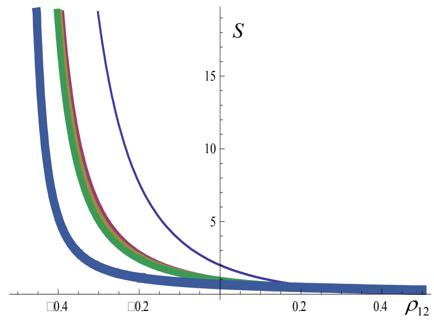
ρ_{11}=-0.5， σ_{12}=-50,-5,0,5,50 （thin->thick）

Figure 8: Retailer's B.E. Contrast Between Centralized And Decentralized D-I (cont.).

When $\sigma_{11}=\sigma_{22}=10, \rho_{11}=\rho_{22}=0.5,-0.5, \rho_{12}=\rho_{21}$, S varies with ρ_{ij} as shown in Figure 9.



ρ_{11}=0.5， σ_{12}=-50,-5,0,5,50 （thin->thick）



ρ_{11}=-0.5， σ_{12}=-50,-5,0,5,50 （thin->thick）

Figure 9: Supplier's B.E. Contrast Between Centralized And Decentralized D-I.

7 CONCLUSIONS AND INSIGHTS

7.1 Main Conclusions

(1) Under decentralized information, when ρ_{ii}>0:

$M^2{}_i$, $M^2{}_s$ is monotone increasing as absolute value of ρ_{12} increases.

$Ration_2$ varies around 1, and its monotone decreasing as σ_{12} increases is not significant. This situation indicates that σ_{12} is not strongly related to the amplification of bullwhip effect going up the supply chain. σ_{12} has little impact on bullwhip effect.

(2) Under centralized information, when ρ_{ii} >0：

Bullwhip effect of retailer/supplier is monotone increasing as ρ_{12} increases, and the amplification is significant.

When ρ_{12} >0, $Ration_1 = M^2{}_{centr\,s} / M^2{}_{centr\,i}$ >1, and $Ration_1$ is monotone increasing as ρ_{12} increases. It means that the amplification of variance of order in supplier stage is larger than that in retailer stage, and this difference increases with the value of ρ_{12}. This situation indicates that larger ρ_{12} will increase the amplification of variance of order quantity spreading to the upstream supply chain.

When ρ_{12} <0, $Ration_1$ <1, and $Ration_1$ is monotone increasing as ρ_{12} increases. It means that the amplification of variance of order in supplier stage is smaller than that in retailer stage, and this difference decreases as ρ_{12} increases. The impact of number of stages of supply chain on bullwhip effect is not effected by ρ_{12}.

σ_{ij} has little impact on bullwhip effect.

(3) When ρ_{ii}<0, ρ_{12} and σ_{12} both have little impact on bullwhip effect.

7.2 Management Insights

To sum up, what we should pay attention to are as following:

(1) When retailers' demands are positive correlated, no matter under centralized or decentralized information, this correlation has significant impact on retailers'/supplier's bullwhip effect.

(2) Under decentralized information, both retailers' and supplier's bullwhip effect increases as the absolute value of retailers' demand correlation increases, and bullwhip effect in supplier stage and retailer stage are almost the same.

(3) Under centralized information, when retailers' demands are positive correlated, both retailers' and supplier's bullwhip effect increases as retailers' demand correlation increases, and bullwhip effect level in supplier stage is larger than that in retailer level. It indicates that under centralized information the impact of number of supply chain stages on bullwhip effect is related with the retailers' demand correlation.

(4) Under centralized information, when and only when retailers' demands are negative correlated ($\rho_{ij} < 0$), the supplier's bullwhip effect will be less than retailers'. It indicates that under centralized information supplier's demand forecast become more accurate as the result of retailers' competition.

Hence, when retailers' demands are correlated, besides the well-known causes of bullwhip effect (such as lead time, number of supply chain stages), any member in the supply chain should consider the impact of multi-terminals' demand correlation on bullwhip effect when making production plan. Furthermore, under centralized information, when retailers' demand are positive correlated, the bullwhip effect in supplier stage is higher than that in retailers' stage; on the contrary, under centralized information, when retailers' demand are negative correlated, the bullwhip effect in supplier stage is lower than that in retailers' stage. These conclusions provide theoretical reference about bullwhip caused by terminals' demand correlation for enterprises to make production plan.

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